A Locking-free and Pressure Oscillation-free Elasto-plastic Large Deformation Analysis using F-barES-FEM-T4

Yuki ONISHI, Ryoya IIDA, Kenji AMAYA
Tokyo Institute of Technology, Japan

Motivation

Motivation

We want to accurately and stably analyze **severe large deformation** of solids in **any shape** with finite elements.

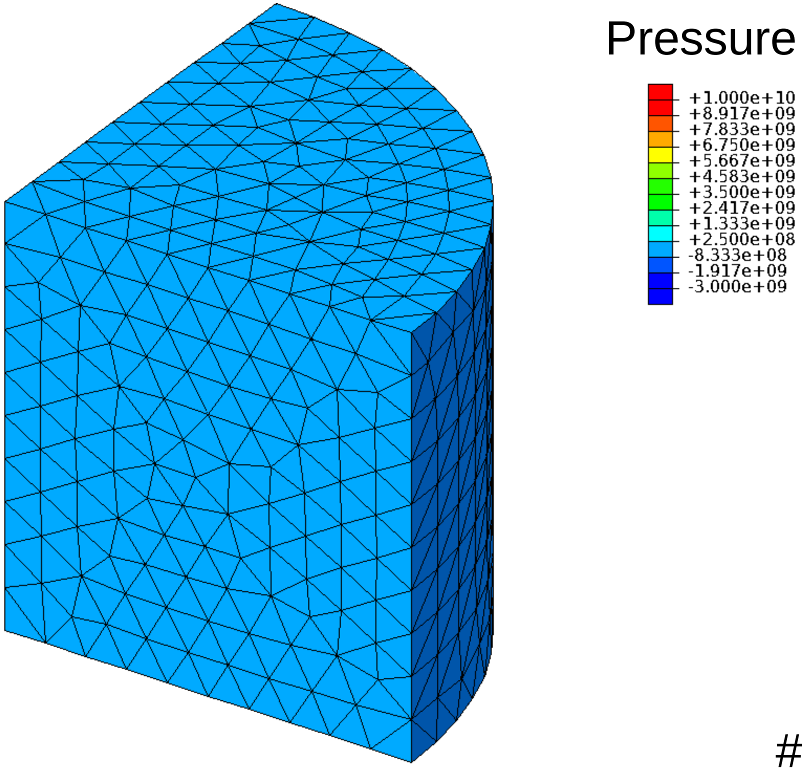
Issues

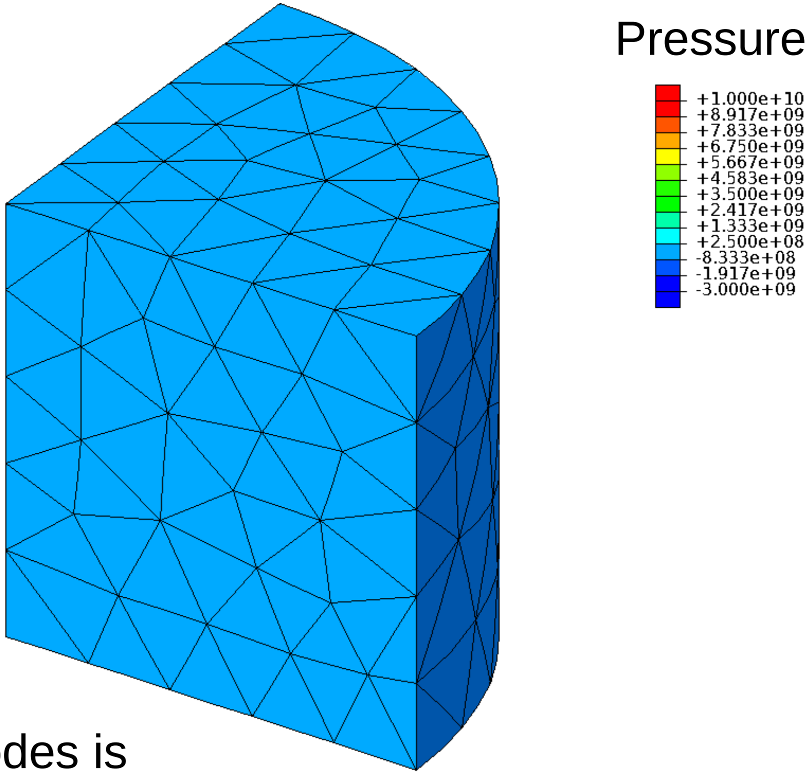
- Only tetra mesh is available for arbitrary body shape.
- The standard 1st / 2nd order tetrahedral element are poor especially when incompressibility is present. Also, all the other advanced tetrahedral elements (e.g., C3D4H, C3D10H, C3D10MH in ABAQUS) have some issues:
  - shear/volumetric locking,
  - pressure oscillation, etc.

Researches on FE formulations for 1st order tetra (T4) are still active especially for rubber-like or elasto-plastic materials.

An Example for Rubber-like Material

Material: neo-Hookean **hyperelastic**, $\nu_{\mathrm{ini}} = 0.49$





# of Nodes is almost the same.

<u>1st order hybrid T4 (C3D4H)</u>

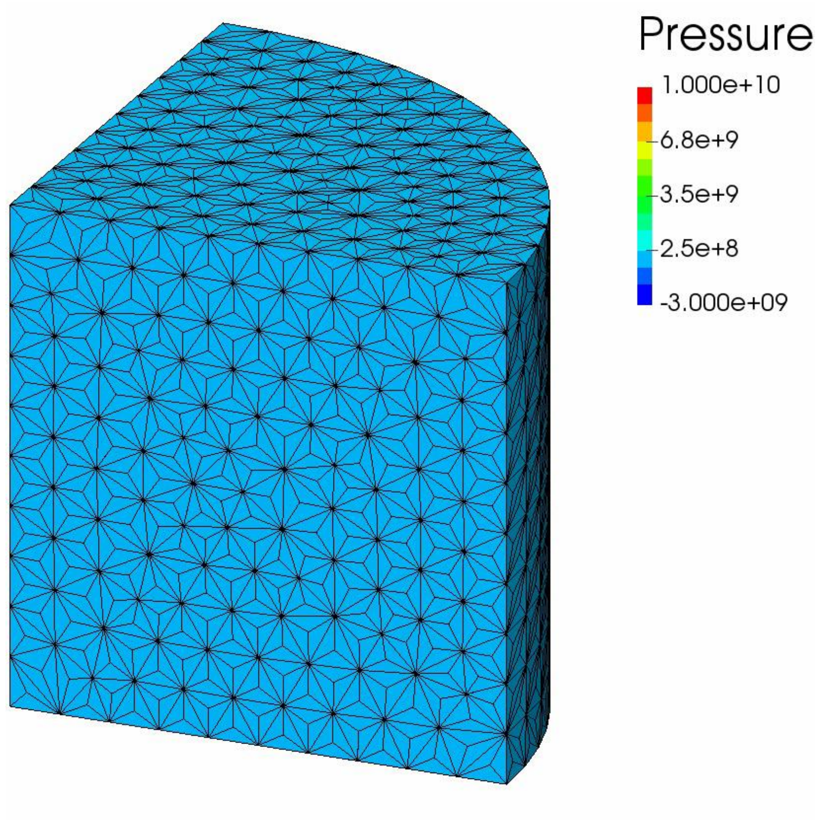
✓ No shear/volumetric locking
✗ Pressure oscillation
✗ Corner locking

<u>2nd order modified hybrid T10 (C3D10MH)</u>

✓ No shear/volumetric locking
✗ Low interpolation accuracy
✗ Early convergence failure

An Example for Rubber-like Material

Material: neo-Hookean **hyperelastic**, $\nu_{\mathrm{ini}} = 0.49$



# of Nodes is exactly the same as the C3D4H case.

Note:
F-barES-FEM-T4 is a pure displacement-based formulation.

F-barES-FEM-T4 (a new type of smoothed finite element method (**S-FEM**))

- ✓ No shear/volumetric locking
- ✓ Less pressure oscillation
- ✓ Less corner locking

F-barES-FEM-T4 has excellent accuracy on **rubber-like** materials.
How about it on **elasto-plastic** materials?

Objective

Apply the new type of S-FEM,
F-barES-FEM-T4,
to large deformation problems
of elasto-plastic materials.

Note: Elasto-plastic materials may have near incompressibility after yielding.

Table of Body Contents

- Methods: Quick introduction of F-barES-FEM-T4
- Results: A few example analyses
- Summary

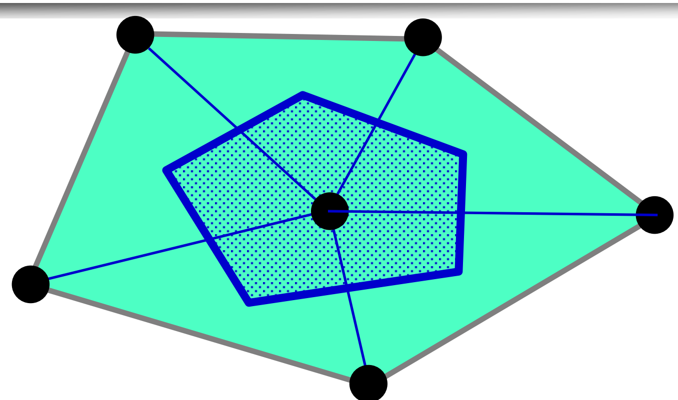
Methods

Quick introduction of F-barES-FEM-T4

(F-barES-FEM-T3 in 2D is explained for simplicity.)

Quick Review of Node-based S-FEM (NS-FEM)

For triangular (T3) or tetrahedral (T4) elements.



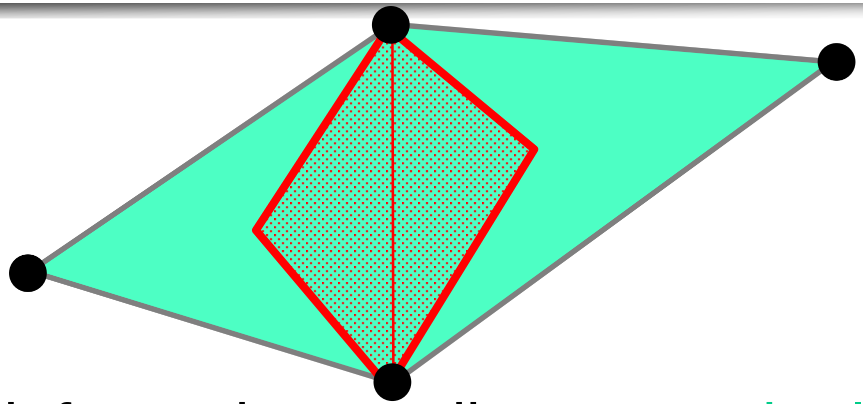
Algorithm:

1. Calculate the deformation gradient at each element, ${}^{\mathrm{Elem}}\boldsymbol{F}$, as usual.
2. Distribute ${}^{\mathrm{Elem}}\boldsymbol{F}$ s to the connecting nodes with area weights to make ${}^{\mathrm{Node}}\tilde{\boldsymbol{F}}$ at each node.
3. Use ${}^{\mathrm{Node}}\tilde{\boldsymbol{F}}$ s to calculate the stress, nodal force and so on.

NS-FEM avoids shear & volumetric locking in T3/T4 elements and also **alleviates pressure oscillation**.
Yet, it suffers from spurious low-energy modes, corner locking and **minor pressure oscillation….**

Quick Review of Edge-based S-FEM (ES-FEM)

For triangular (T3) or tetrahedral (T4) elements.



Algorithm:

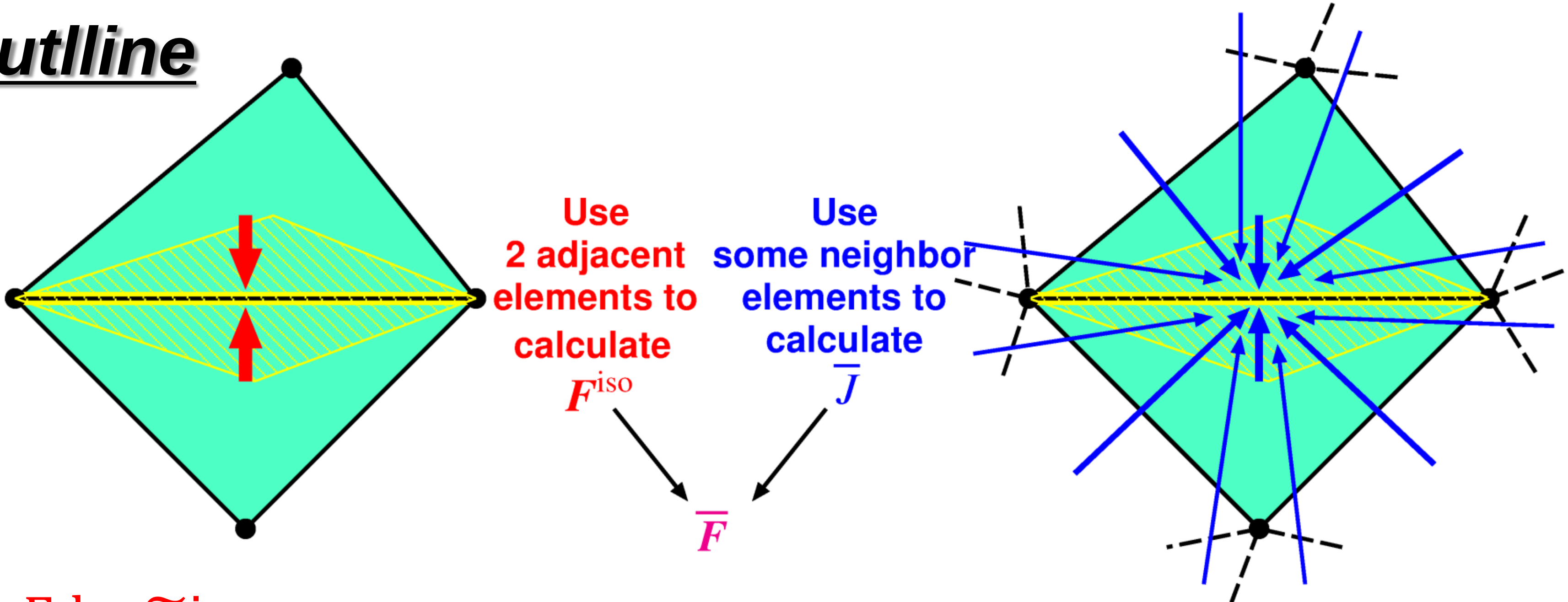
1. Calculate the deformation gradient at each element, ${}^{\mathrm{Elem}}\boldsymbol{F}$, as usual.
2. Distribute ${}^{\mathrm{Elem}}\boldsymbol{F}$ s to the connecting edges with area weights to make ${}^{\mathrm{Edge}}\widetilde{\boldsymbol{F}}$ at each edge.
3. Use ${}^{\mathrm{Edge}}\widetilde{\boldsymbol{F}}$ s to calculate the stress, nodal force and so on.

ES-FEM avoids shear locking in T3/T4 elements.
Yet, it suffers from volumetric locking, corner locking,
and **major pressure oscillation…**

Quick Introduction of F-barES-FEM

Concept: combine ES-FEM and NS-FEM using F-bar method

Outlline



- $^{\mathrm{Edge}}\tilde{F}^{\mathrm{iso}}$ is given by ES-FEM.
- $^{\mathrm{Edge}}\bar{J}$ is given by cyclically applied NS-FEM.
- $^{\mathrm{Edge}}\bar{F}$ is calculated in the manner of F-bar method:

$$^{\mathrm{Edge}}\bar{F} = {^{\mathrm{Edge}}\bar{J}}^{1/3}\ {^{\mathrm{Edge}}\tilde{F}^{\mathrm{iso}}}.$$

Outline of F-barES-FEM

Brief Formulation

A kind of low-pass filter for J

1. Calculate ${}^{\mathrm{Elem}}J$ as usual.
2. Smooth ${}^{\mathrm{Elem}}J$ at nodes and get ${}^{\mathrm{Node}}\tilde{J}$.
3. Smooth ${}^{\mathrm{Node}}\tilde{J}$ at elements and get ${}^{\mathrm{Elem}}\tilde{J}$.
4. Repeat 2. and 3. as necessary (c times).

Cyclic Smoothing of J

⋮ (c layers of ~)

5. Smooth ${}^{\mathrm{Elem}}\tilde{\tilde{J}}$ at edges to make ${}^{\mathrm{Edge}}\bar{J}$.
6. Combine ${}^{\mathrm{Edge}}\bar{J}$ and ${}^{\mathrm{Edge}}\boldsymbol{F}^{\mathrm{iso}}$ of ES-FEM as

$${}^{\mathrm{Edge}}\bar{\boldsymbol{F}} = {}^{\mathrm{Edge}}\bar{J}^{1/3}\ {}^{\mathrm{Edge}}\boldsymbol{F}^{\mathrm{iso}}.$$

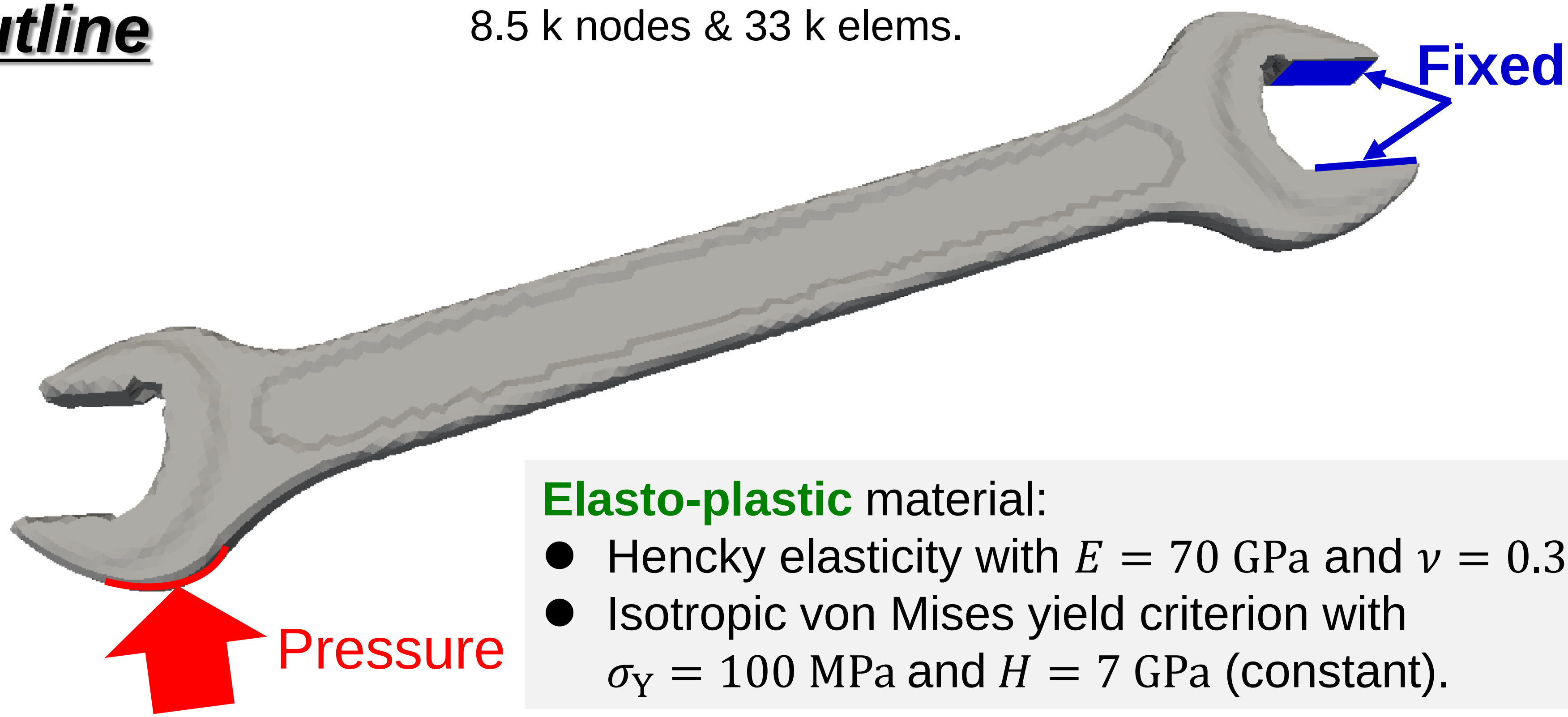
Hereafter, F-barES-FEM-T4 with c cycles of smoothing is called “F-barES-FEM-T4(c)”.

Results

A few example analyses

Bending of Elasto-Plastic Spanner

Outline



Elasto-plastic material:

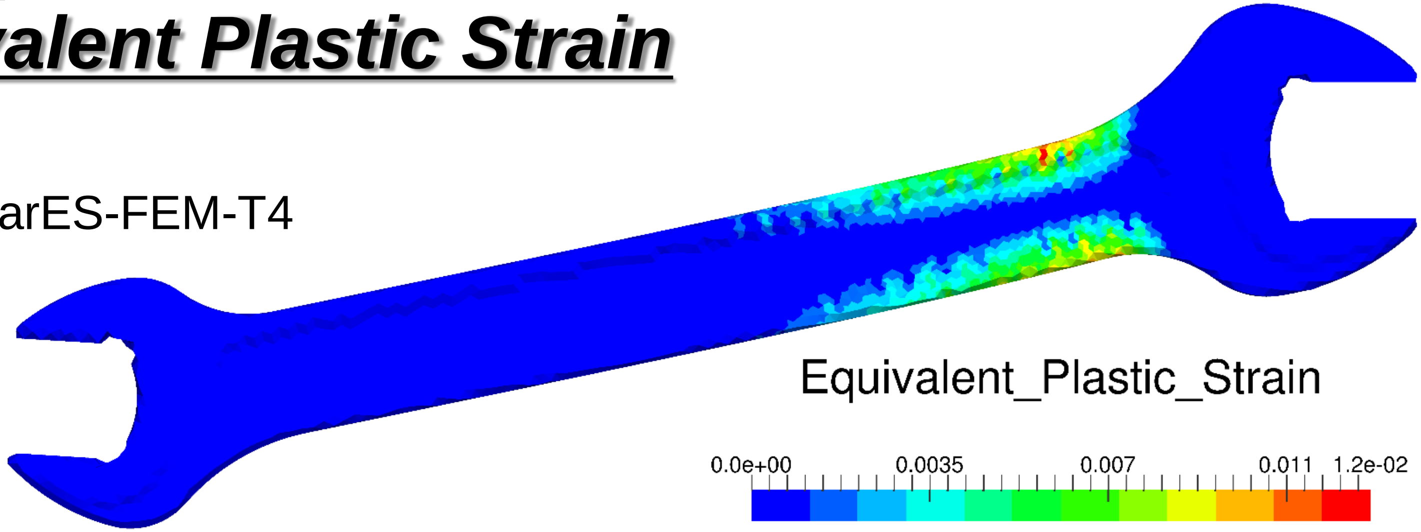
- Hencky elasticity with $E = 70\ \mathrm{GPa}$ and $\nu = 0.3$.
- Isotropic von Mises yield criterion with $\sigma_{\mathrm{Y}} = 100\ \mathrm{MPa}$ and $H = 7\ \mathrm{GPa}$ (constant).

- 2 faces are perfectly constrained.
- Pressure is applied to a side part of the spanner.
- Compared to ABAQUS C3D4H with the same unstructured T4 mesh.

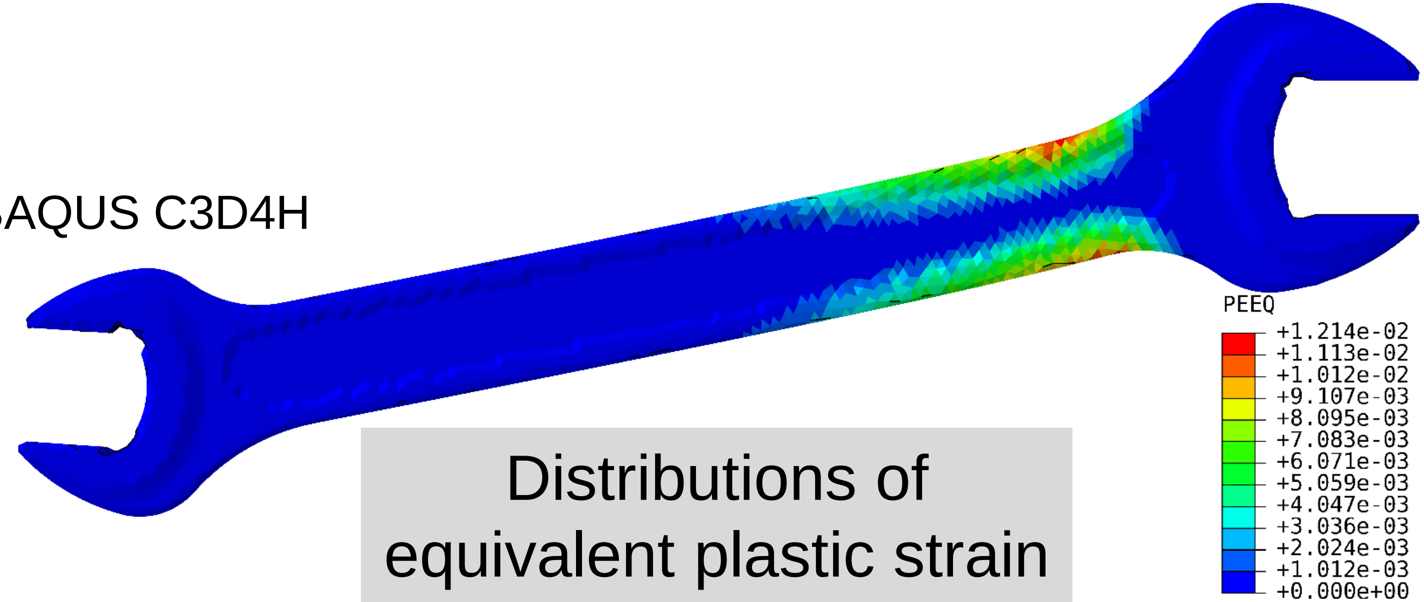
Bending of Elasto-Plastic Spanner

Equivalent Plastic Strain

F-barES-FEM-T4



ABAQUS C3D4H

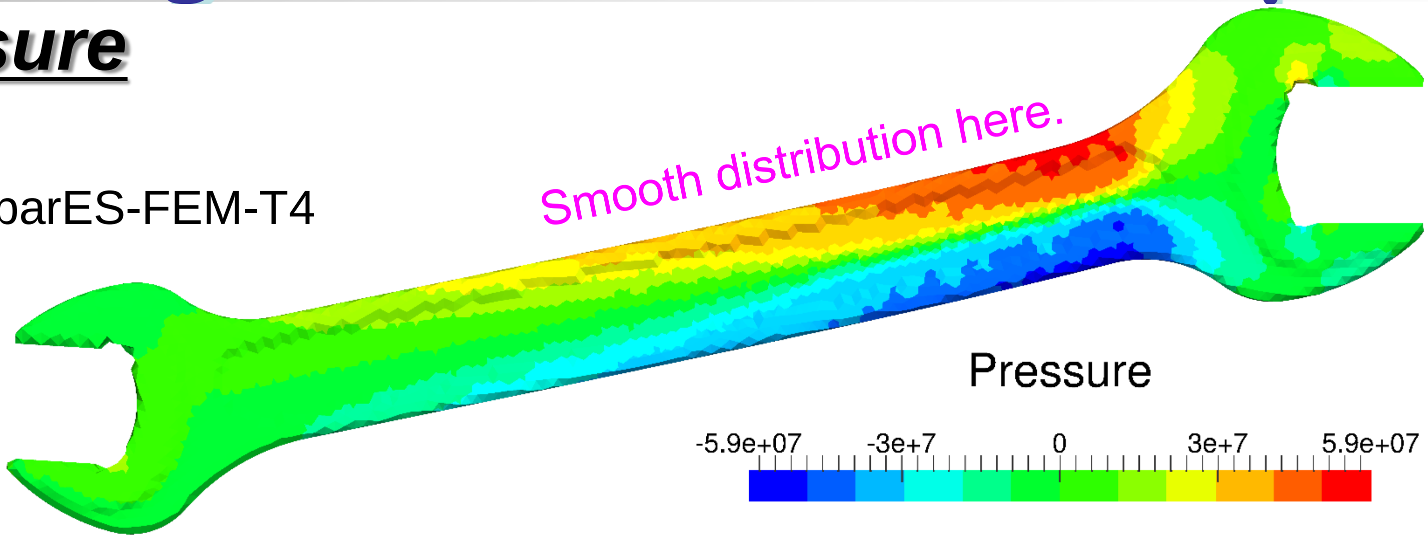


Distributions of equivalent plastic strain are about the same.

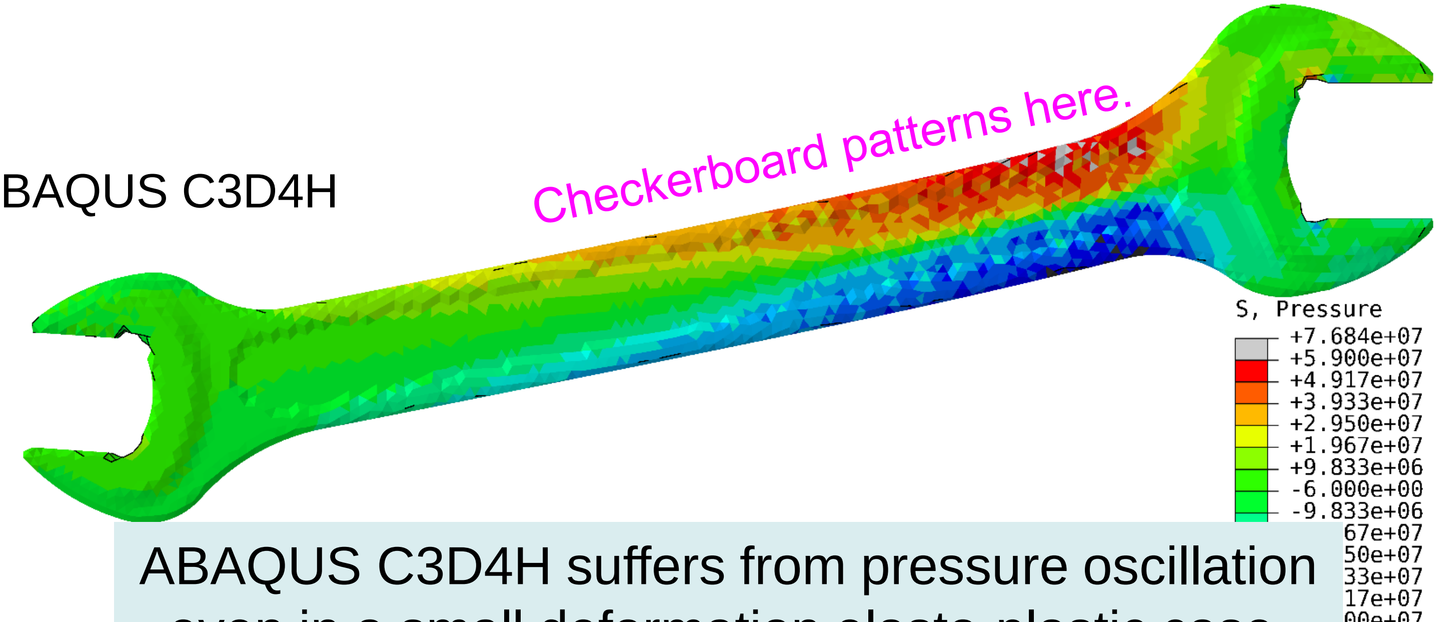
Bending of Elasto-Plastic Spanner

Pressure

F-barES-FEM-T4



ABAQUS C3D4H

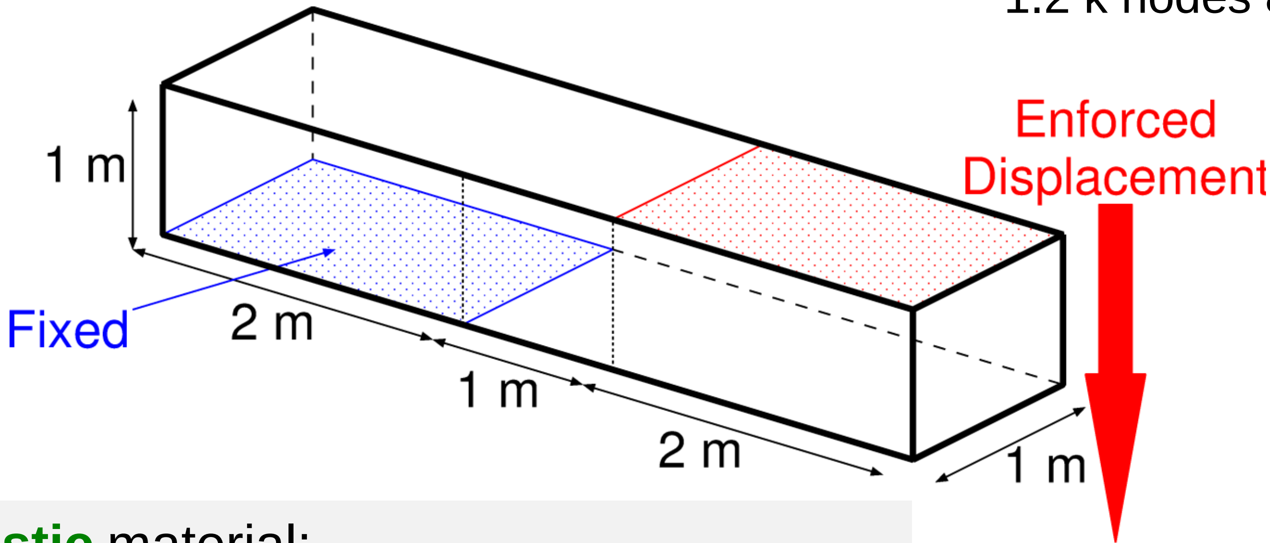


ABAQUS C3D4H suffers from pressure oscillation even in a small deformation elasto-plastic case.

Shearing & Tensioning of Elasto-Plastic Bar

Outline

1.2 k nodes & 4.8 k elems.



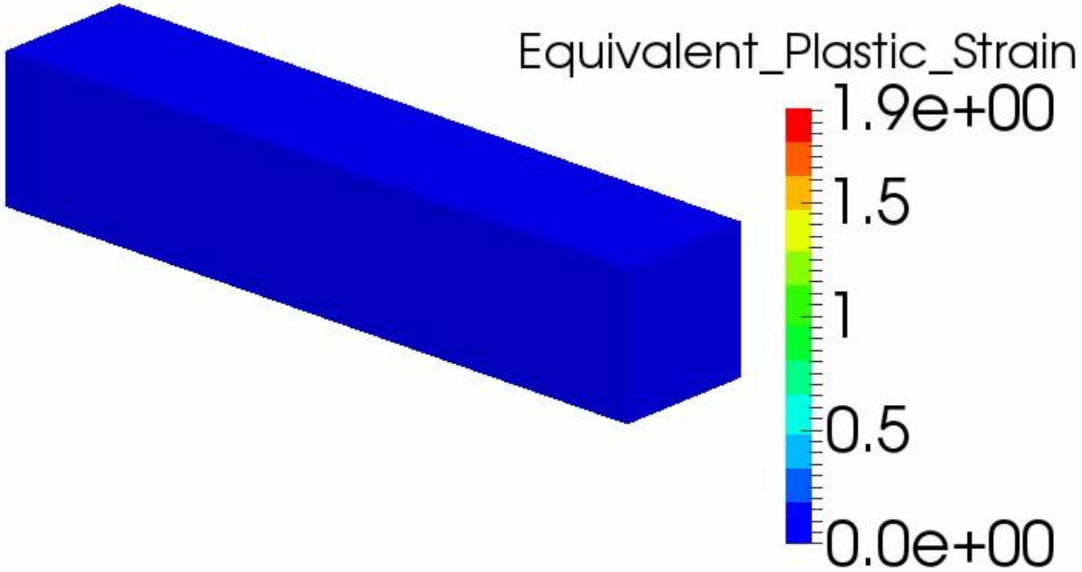
Elasto-plastic material:

- Hencky elasticity with $E = 1\ \mathrm{GPa}$ and $\nu = 0.3$.
- Isotropic von Mises yield criterion with $\sigma_\mathrm{Y} = 1\ \mathrm{MPa}$ and $H = 0.1\ \mathrm{GPa}$ (constant).

- Blue face is perfectly constrained.
- Red face is constrained in plane and pressed down.
- Compared to ABAQUS C3D4H with the same unstructured T4 mesh.

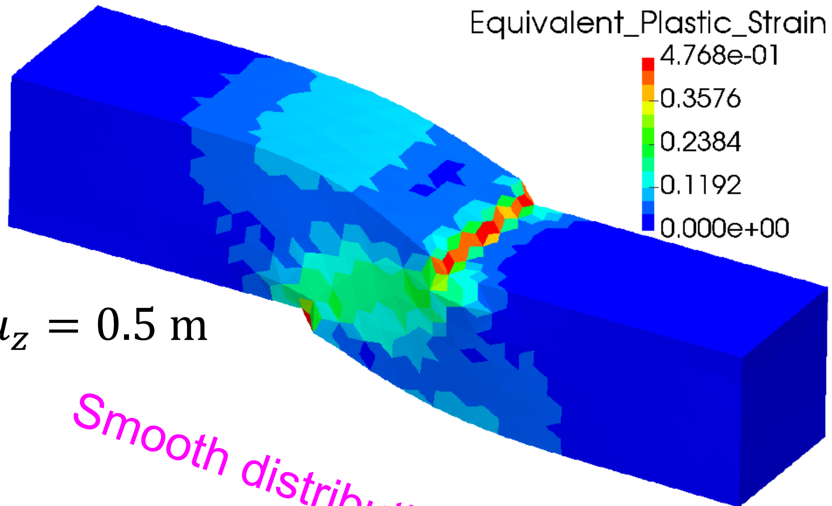
Shearing & Tensioning of Elasto-Plastic Bar

Result of F-bar ES-FEM (Equiv. Plastic Strain)

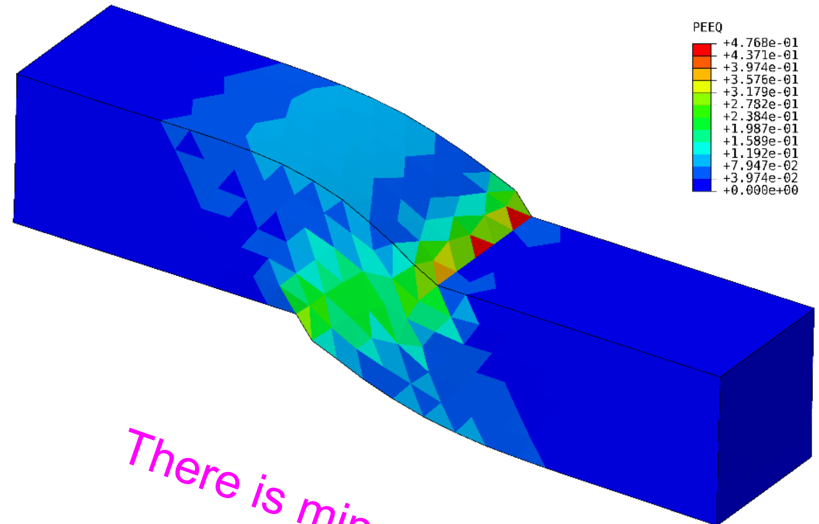


Shearing & Tensioning of Elasto-Plastic Bar

Equivalent Plastic Strain

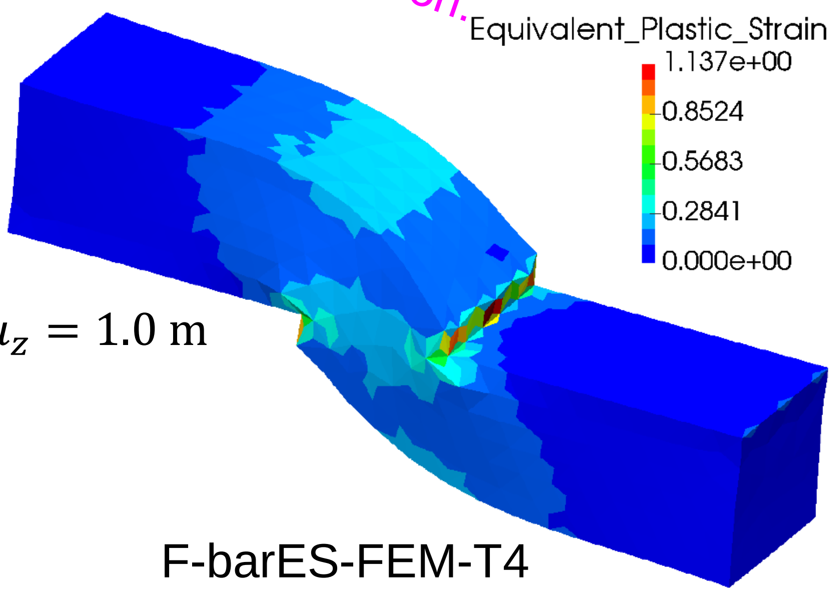


$u_z = 0.5$ m

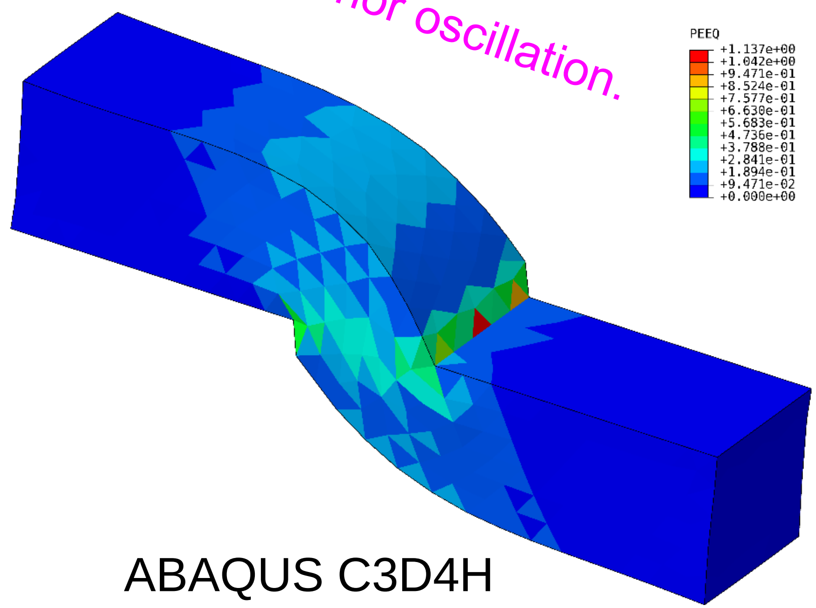


Smooth distribution.

There is minor oscillation.



$u_z = 1.0$ m

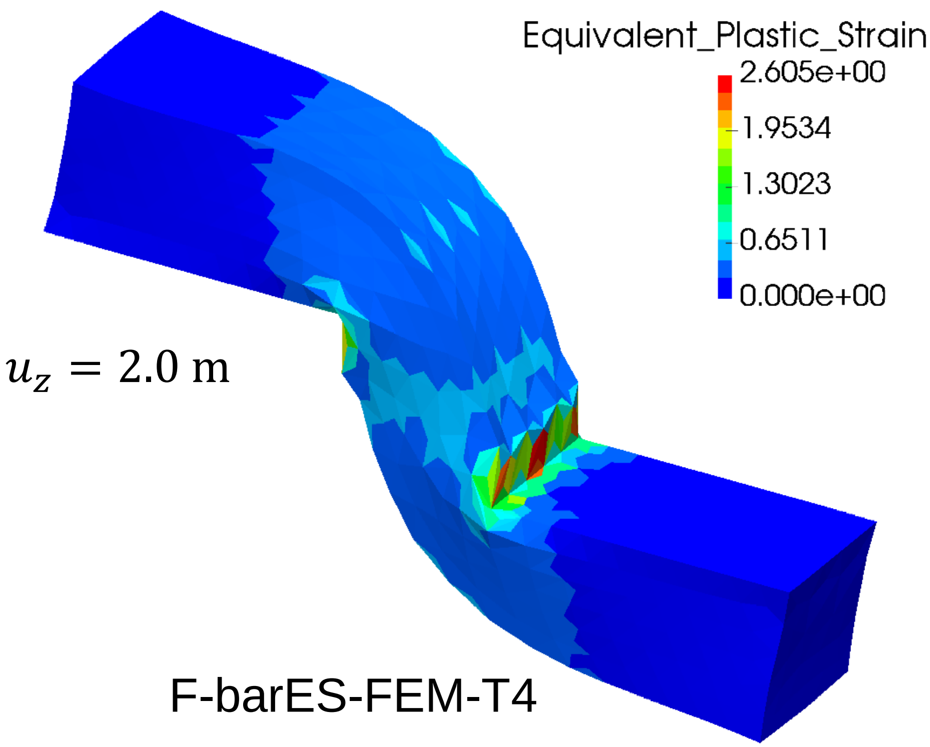


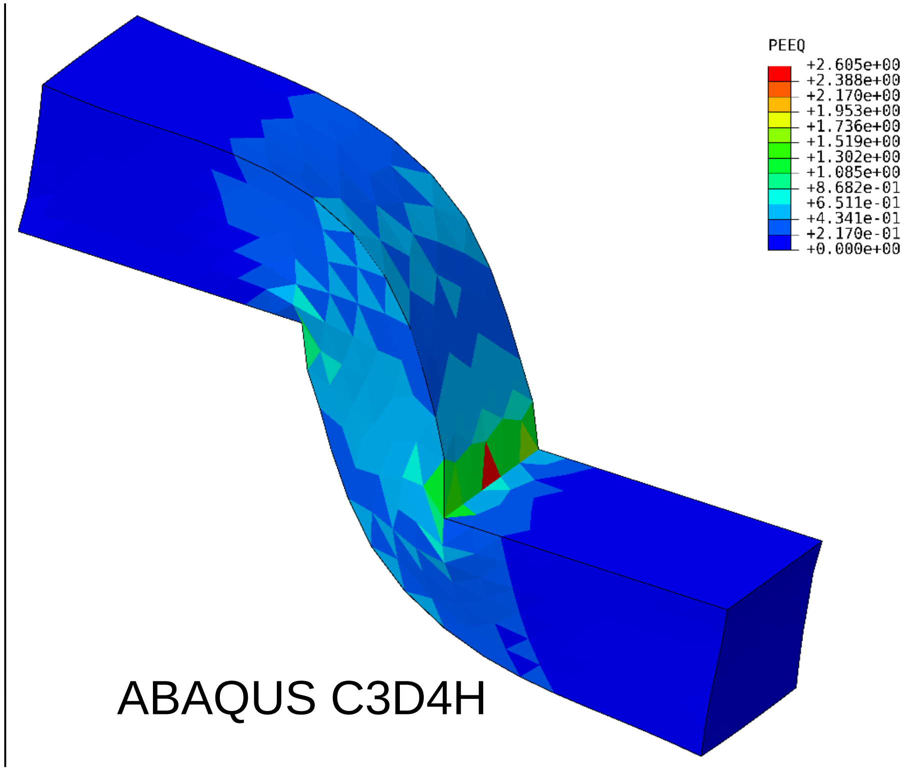
F-barES-FEM-T4

ABAQUS C3D4H

Shearing & Tensioning of Elasto-Plastic Bar

Equivalent Plastic Strain

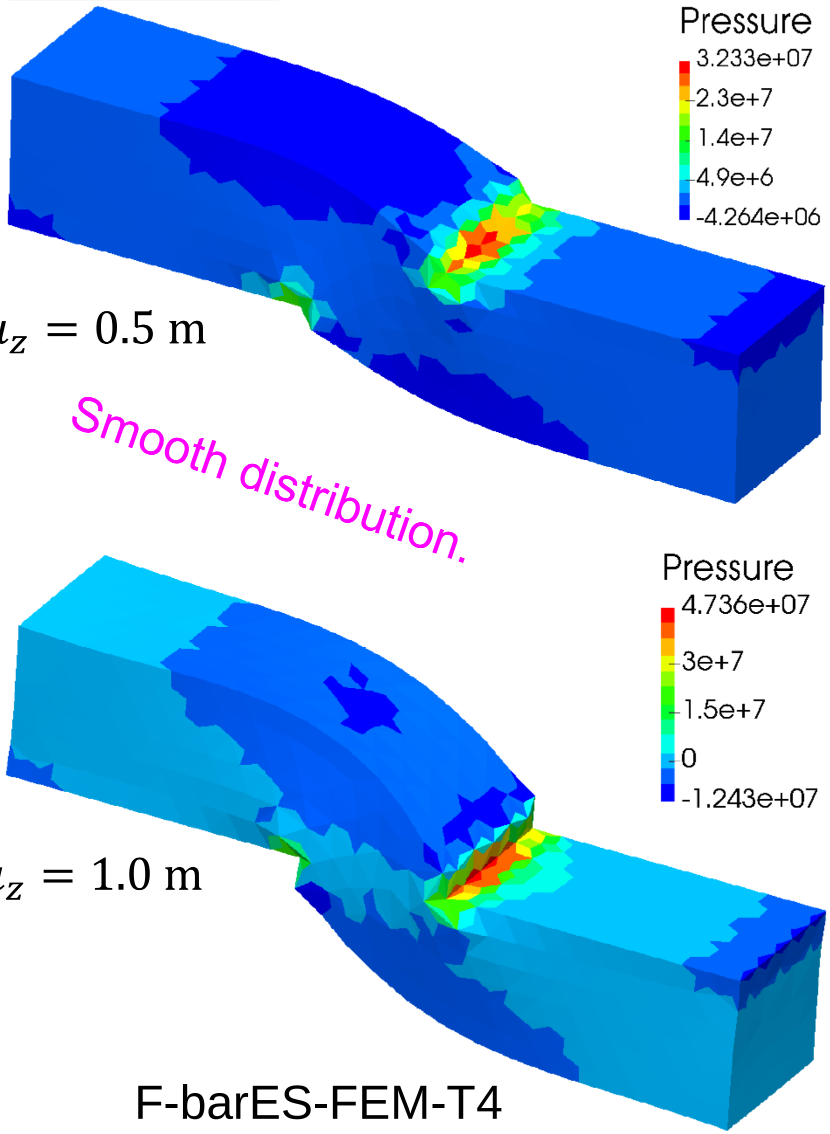




Accuracy of equivalent plastic strain seems no much different.

Shearing & Tensioning of Elasto-Plastic Bar

Pressure

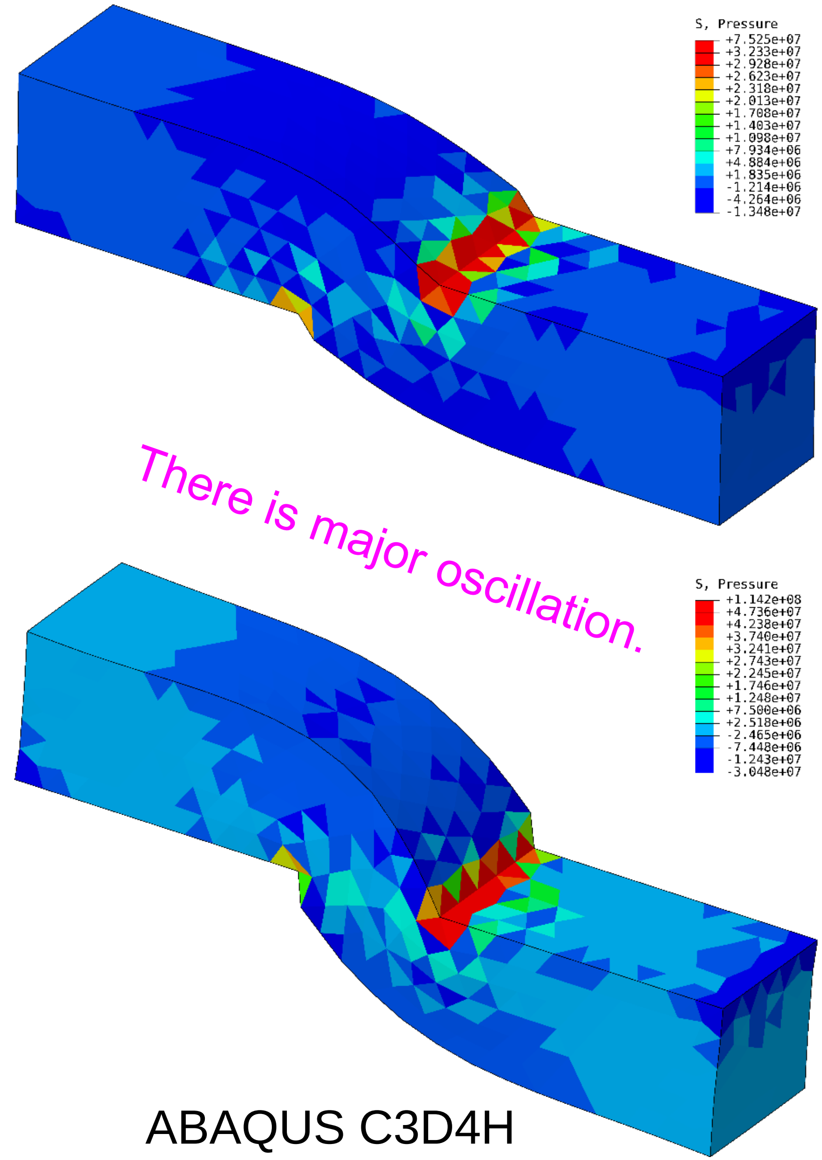


$u_z = 0.5$ m

$u_z = 1.0$ m

Smooth distribution.

F-barES-FEM-T4



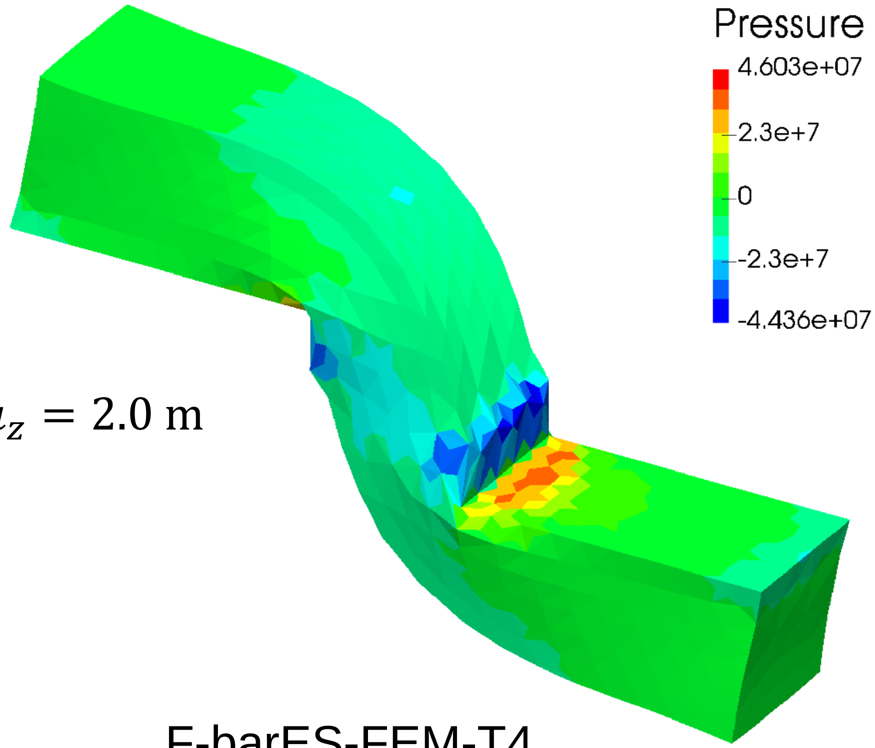
There is major oscillation.

ABAQUS C3D4H

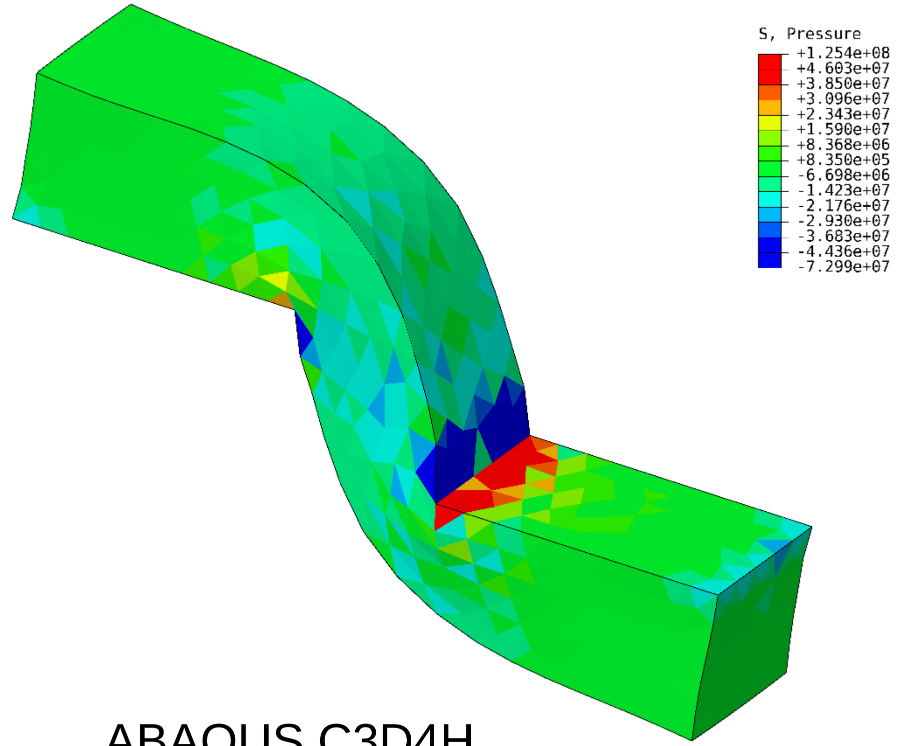
Shearing & Tensioning of Elasto-Plastic Bar

Pressure

$u_z = 2.0$ m



F-barES-FEM-T4



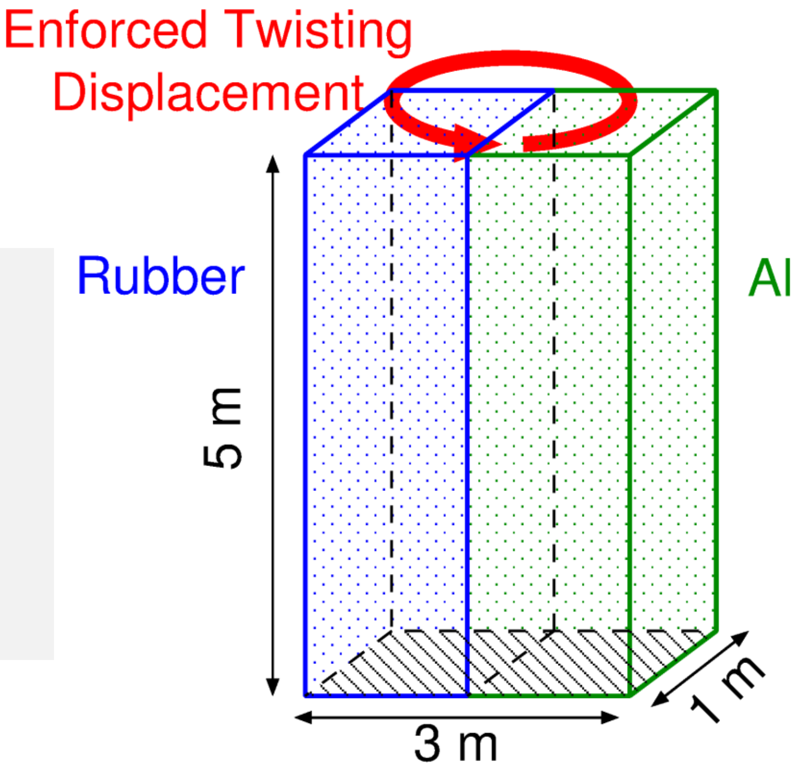
ABAQUS C3D4H

Accuracy of pressure is quite different due to the pressure oscillation of ABAQUS C3D4H.

Twist of Rubber/Aluminium Composite Plate

Outline

3 k nodes & 14 k elems.



[Rubber]
Neo-Hook
Hyperelasticity:
$E^{\mathrm{ini}} = 5$ MPa,
$\nu^{\mathrm{ini}} = 0.49$,
$(c = 1)$

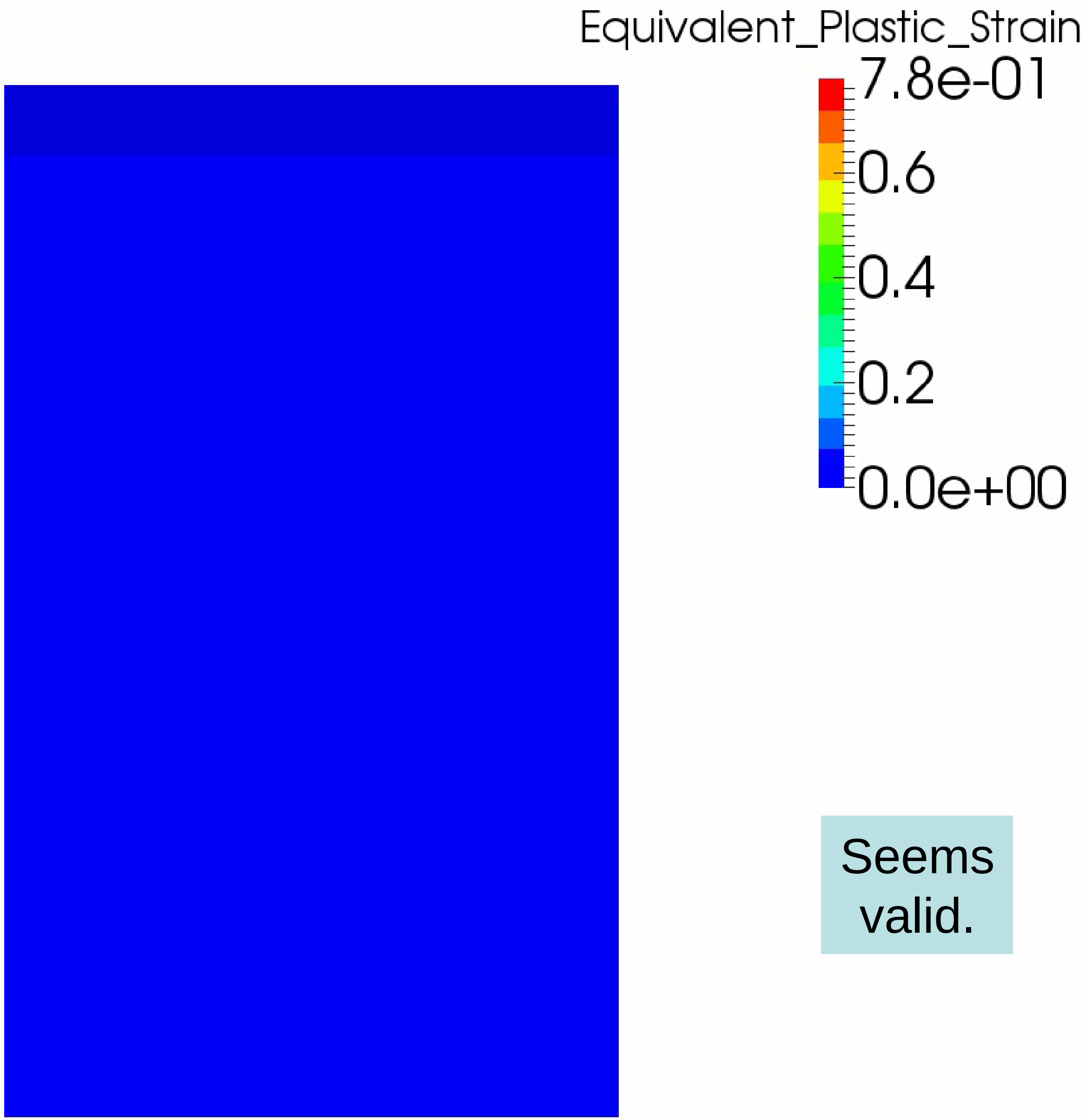
[Aluminium]
Hencky elasticity:
$E = 70$ GPa,
$\nu = 0.3$.
Isotropic von Mises
plasticity:
$\sigma_{\mathrm{Y}} = 100$ MPa ,
$H = 0.7$ GPa (const.),
$(c = 2)$

- Bottom face is perfectly constrained.
- Top face is constrained in the plane and twisted 360 deg. around the vertical axis.
- Calculated by F-barES-FEM-T4 only.
- Multiple Fs at edges on the material interface.

Twist of Rubber/Aluminium Composite Plate

Result of F-bar ES-FEM-T4

Equivalent Plastic Strain

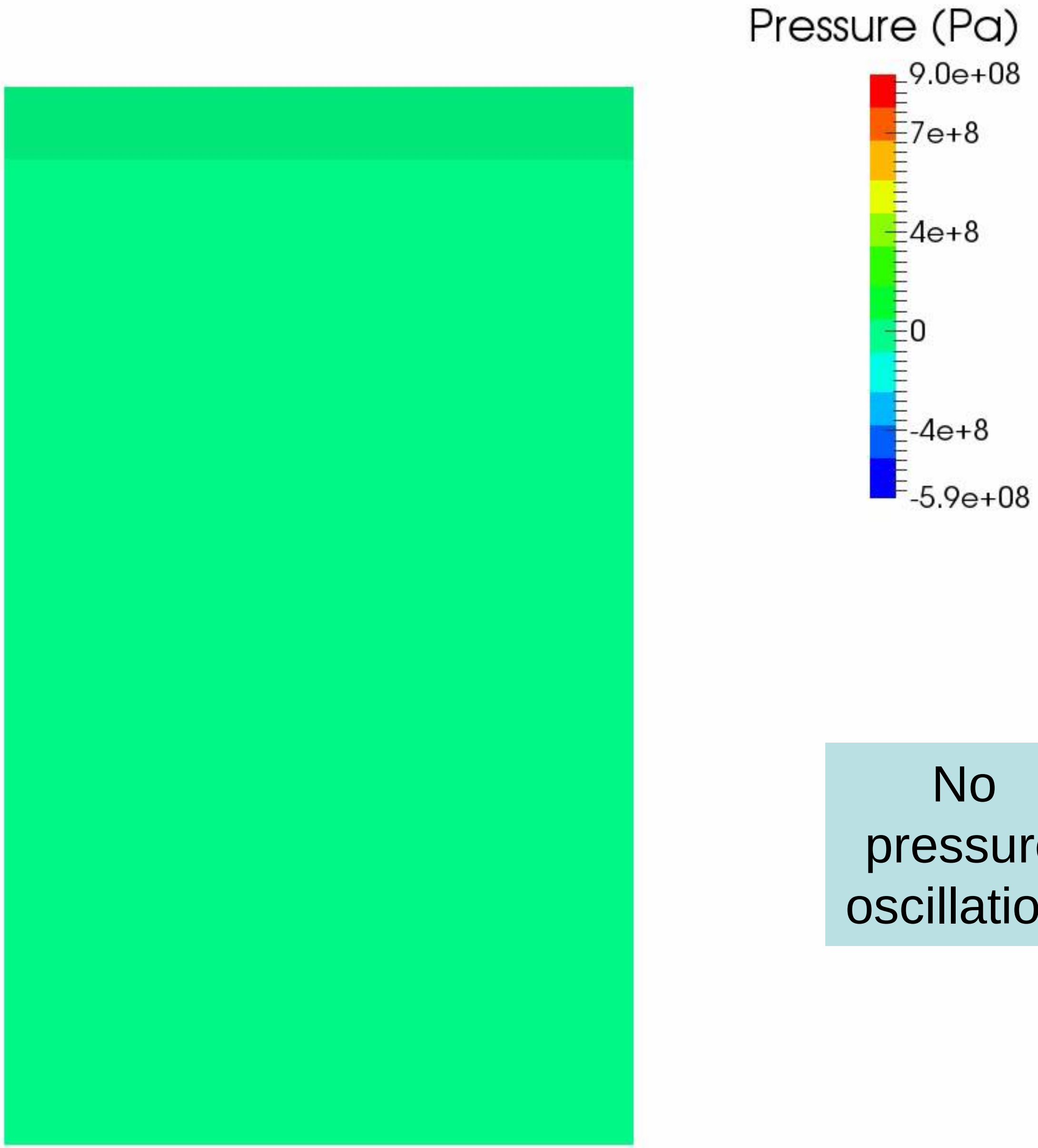


Seems valid.

Twist of Rubber/Aluminium Composite Plate

Result of F-bar ES-FEM-T4

Pressure

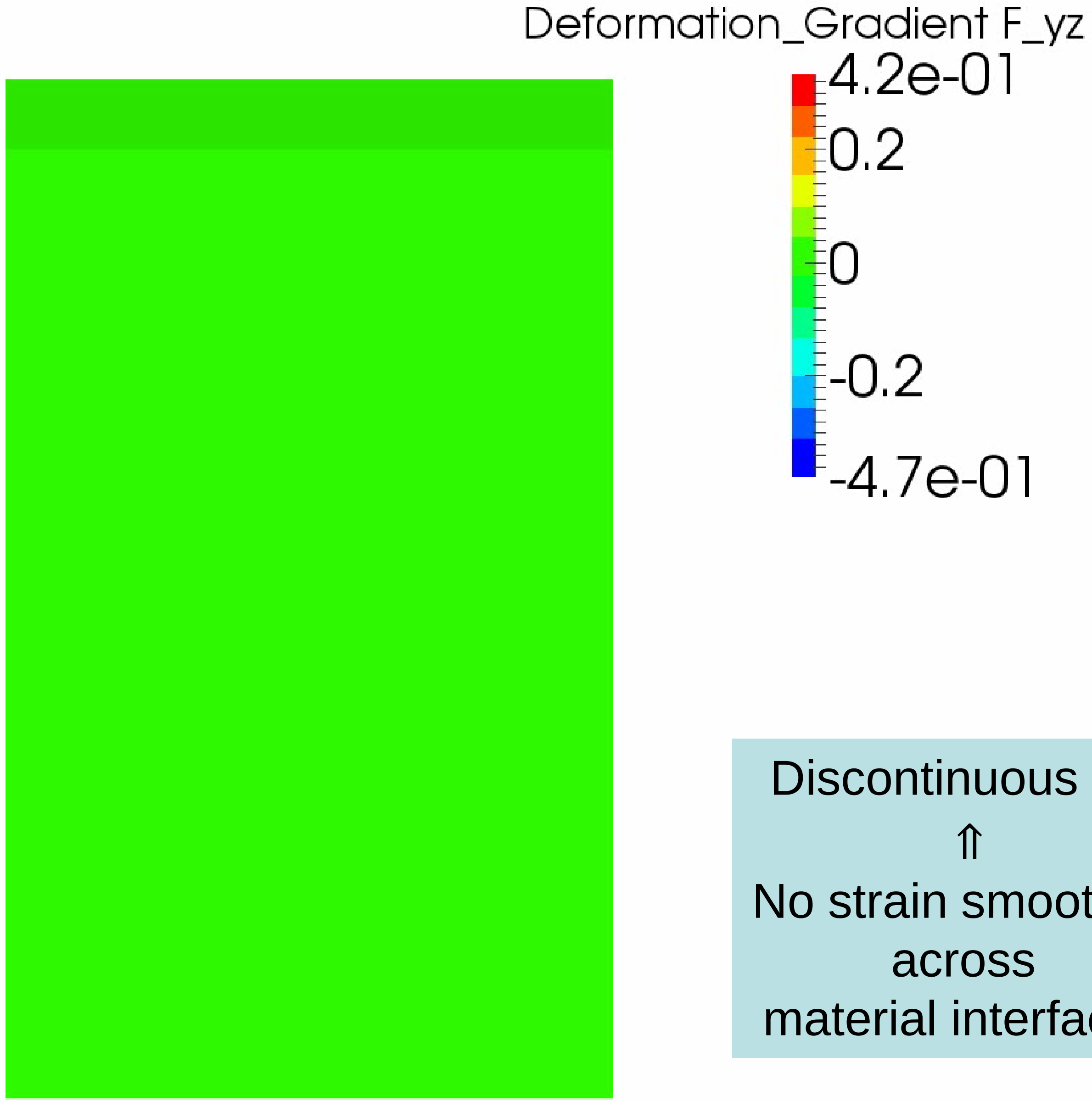


No pressure oscillation.

Twist of Rubber/Aluminium Composite Plate

Result of
F-bar
ES-FEM-T4

Deformation Gradient F_{yz}



Discontinuous F_{yz}.
⇑
No strain smoothing across material interfaces.

Summary

Benefits and Drawbacks of F-barES-FEM-T4

Benefits

- ✓ Locking-free with 1st order tetra meshes.
  No difficulty in severe strain or contact analysis.
- ✓ No increase in DOF.
  Purely displacement-based formulation.
- ✓ No restriction of material constitutive model.
  Pressure dependent models are acceptable.
- ✓ Less corner locking and pressure oscillation.

Drawbacks

- ✗ The more cyclic smoothing necessitates the more CPU time due to the wider bandwidth.

FYI

If you are interested in F-barES-FEM-T4,
please refer to the following paper:

“F-bar aided edge-based smoothed finite element method using tetrahedral elements for finite deformation analysis of nearly incompressible solids, *International Journal for Numerical Methods in Engineering* (***IJNME***), **Jul. 2016**.

Thank you for your kind attention!