Performance Evaluation of F-barES-FEM-T4 in Dynamic Analysis

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Background

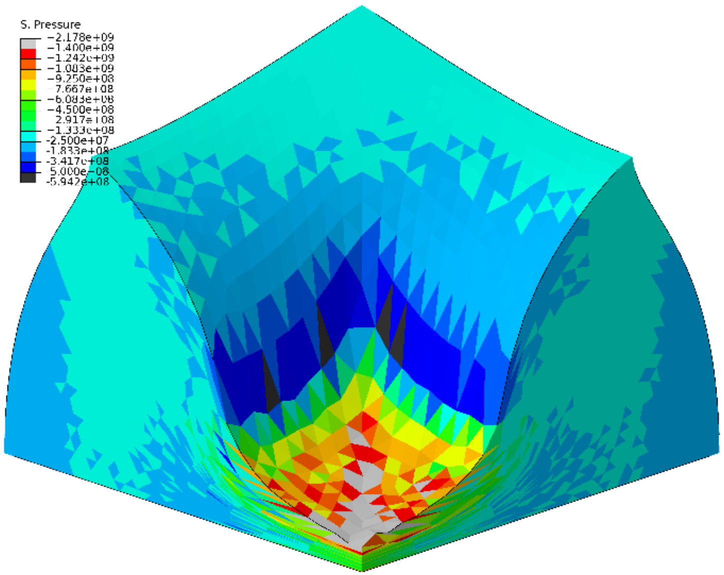
In the previous talk, F-bar aided edge-based smoothed finite element method with tetrahedral elements (**F-barES-FEM-T4**) is presented.

Characteristics of **F-barES-FEM-T4** in ***static*** analysis are as follows.

- ✔ Excellent accuracy,
- ✔ No increase in DOF,
- ✗ Increase in bandwidth of stiffness matrix.

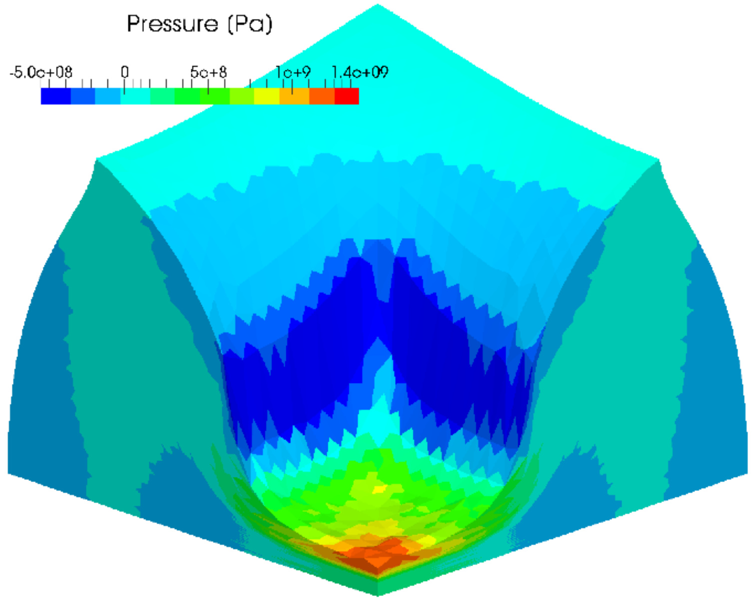
Characteristics (1 of 2)

✔ Excellent accuracy



ABAQUS C3D4H

✗ pressure oscillation



F-barES-FEM-T4(3)

↗ # of cyclic smoothings

Our method shows excellent accuracy in ***static*** problems!

Characteristics (2 of 2)

✔ No increase in DOF

- F-barES-FEM-T4 is a purely displacement-based formulation.
- In contrast to the u/p hybrid formulations, F-barES-FEM-T4 can be directly applied to explicit dynamics.

✗ Increase in bandwidth of stiffness matrix

- F-barES-FEM-T4 takes longer time to solve the equilibrium equations…
- In explicit dynamics, however, we don’t need to solve them!

F-barES-FEM-T4 would be suitable
for explicit dynamics of rubber-like materials!

Objective

Objective

Evaluate the performance of F-barES-FEM-T4 in explicit dynamics for rubber-like materials.

Table of Body Contents

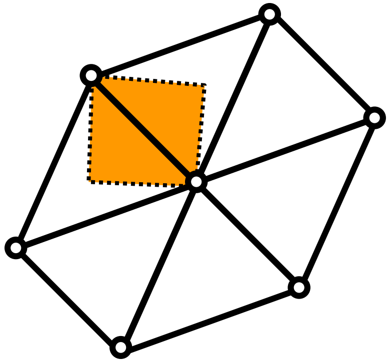
- Methods: Quick introduction of F-barES-FEM-T4
- Results & Discussion: A few verification analyses
- Summary

Methods

Procedure of F-barES-FEM (1 of 2)

Deformation gradient of each edge, $\bar{\boldsymbol{F}}$ is derived as

$$\bar{\boldsymbol{F}} = \widetilde{\boldsymbol{F}}^{\mathrm{iso}} \cdot \bar{\boldsymbol{F}}^{\mathrm{vol}}$$

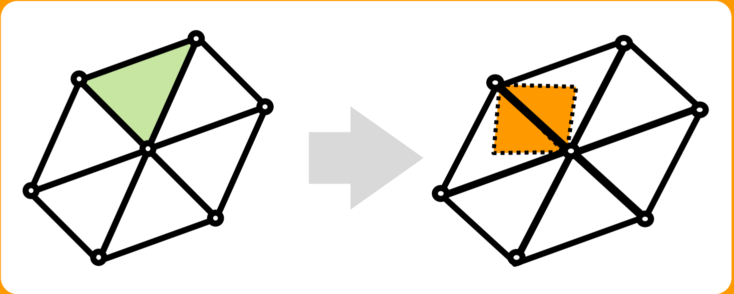


Procedure of F-barES-FEM (2 of 2)

Each part of $\bar{\boldsymbol{F}}$ is calculated as

$$\bar{\boldsymbol{F}} = \tilde{\boldsymbol{F}}^{\mathrm{iso}} \cdot \bar{\boldsymbol{F}}^{\mathrm{vol}}$$

Isovolumetric part

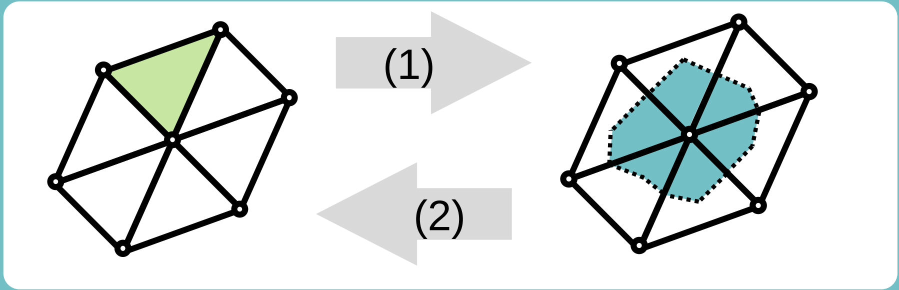


Smoothing the value of adjacent elements.

↓

The same manner as ES-FEM

Volumetric part



(1)Calculating node's value by smoothing the value of adjacent elements

(2)Calculating elements' value by smoothing the value of adjacent nodes

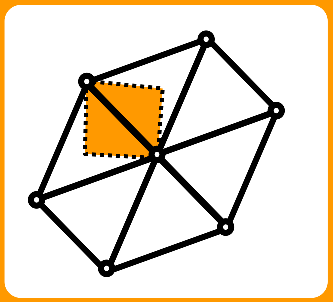
(3)Repeating (1) and (2) a few times

Advantages of F-barES-FEM

This formulation is designed to have 3 advantages.

$$\bar{\boldsymbol{F}} = \tilde{\boldsymbol{F}}^{\mathrm{iso}} \cdot \bar{\boldsymbol{F}}^{\mathrm{vol}}$$

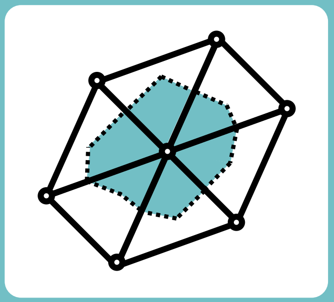
Isovolumetric part



Like a ES-FEM

1. Shear locking free

Volumetric part



Like a NS-FEM

2. Little pressure oscillation

3. Volumetric locking free
with the aid of F-bar method

Equation to solve

Equation of Motion

$$[M]\{\ddot{u}\} = \{f^{\text{ext}}\} - \{f^{\text{int}}\},$$

where

$$\{f^{\text{int}}\} = \sum [\tilde{B}]\{\bar{T}\}V$$

Defenition of $\{f^{\text{int}}\}$ in the same fashion as **F-bar method**

B-matrix of ES-FEM

Stress derived from $\bar{\boldsymbol{F}}$

Time integration

Velocity Verlet method (2nd order simplectic integrator)

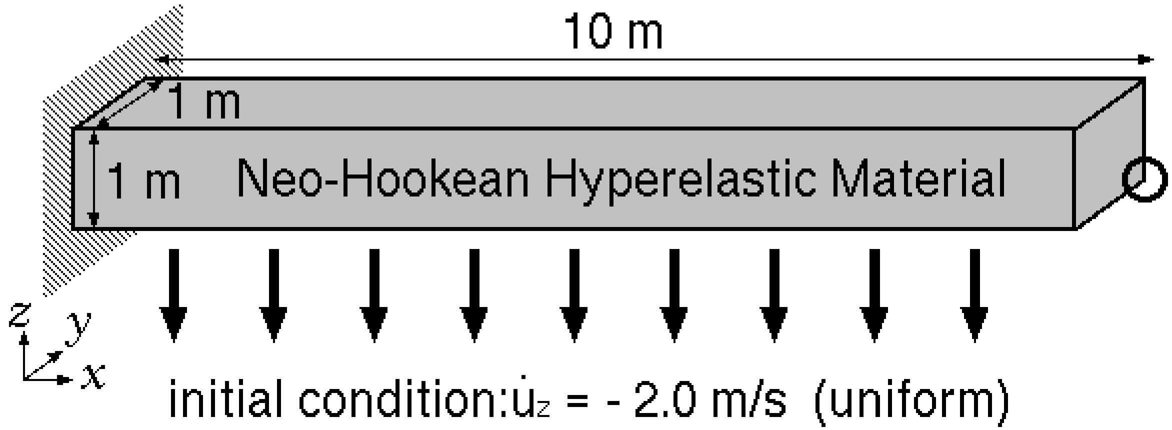
$$\{\boldsymbol{u}_{n+1}\} = \{\boldsymbol{u}_n\} + \{\dot{\boldsymbol{u}}_n\}\Delta\text{t} + \frac{1}{2}\{\ddot{\boldsymbol{u}}_n\}\Delta\text{t}^2$$

$$\{\ddot{\boldsymbol{u}}_{n+1}\} = [\boldsymbol{M}^{-1}](\{\boldsymbol{f}^{\text{ext}}\} - \{\boldsymbol{f}^{\text{int}}(\boldsymbol{u}_{\boldsymbol{n+1}})\})$$

$$\{\dot{\boldsymbol{u}}_{n+1}\} = \{\dot{\boldsymbol{u}}_n\} + (\{\ddot{\boldsymbol{u}}_n\} + \{\ddot{\boldsymbol{u}}_{n+1}\})\Delta\text{t}/2$$

Result & Discussion

#1 Bending of a cantilever

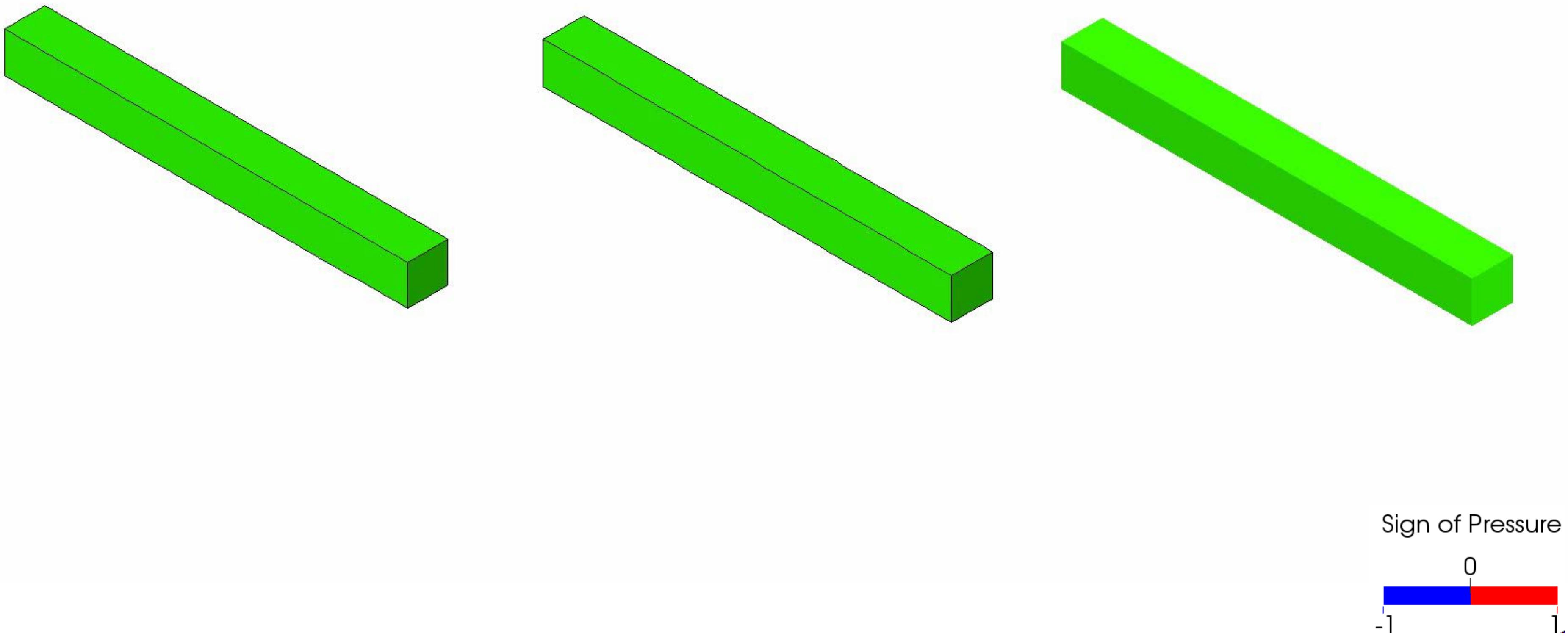


- Dynamic explicit analysis.
- Neo-Hookean material

Initial Young's modulus:	6.0 MPa,
Initial Poisson's ratio:	0.499,
Density:	10000 kg/m^3.

- Compare the results of F-barES-FEM-T4, Standard T4 (ABAQUS/Explicit C3D4) and Selective H8 (ABAQUS/Explicit C3D8) elements.

Time history of deformed shapes

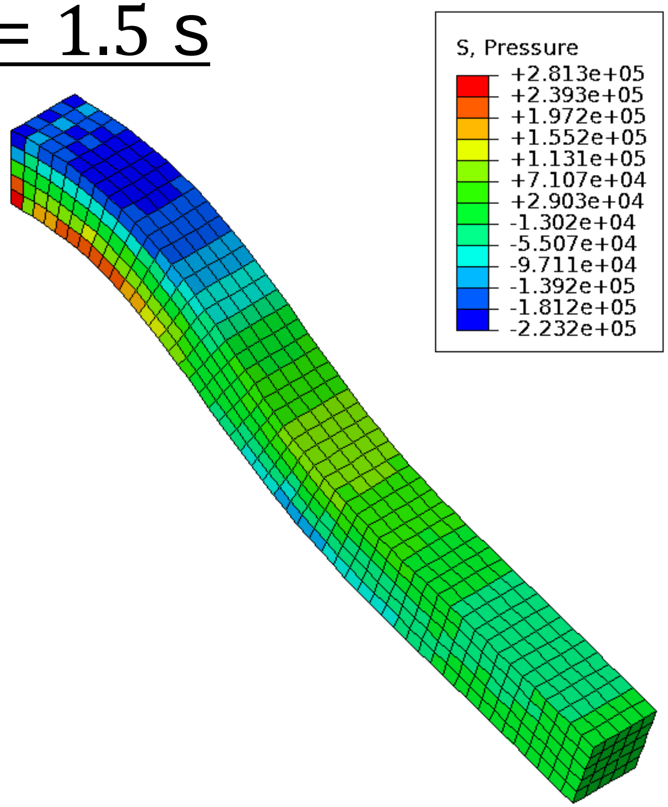


ABAQUS/Explicit C3D4	ABAQUS/Explicit C3D8	F-barES-FEM-T4(2)
(Standard T4 element)	(Selective H8 element)	(Proposed method)
✗ Pressure oscillation ✗ Locking	**Reference**	✔ No pressure oscillation ✔ No locking

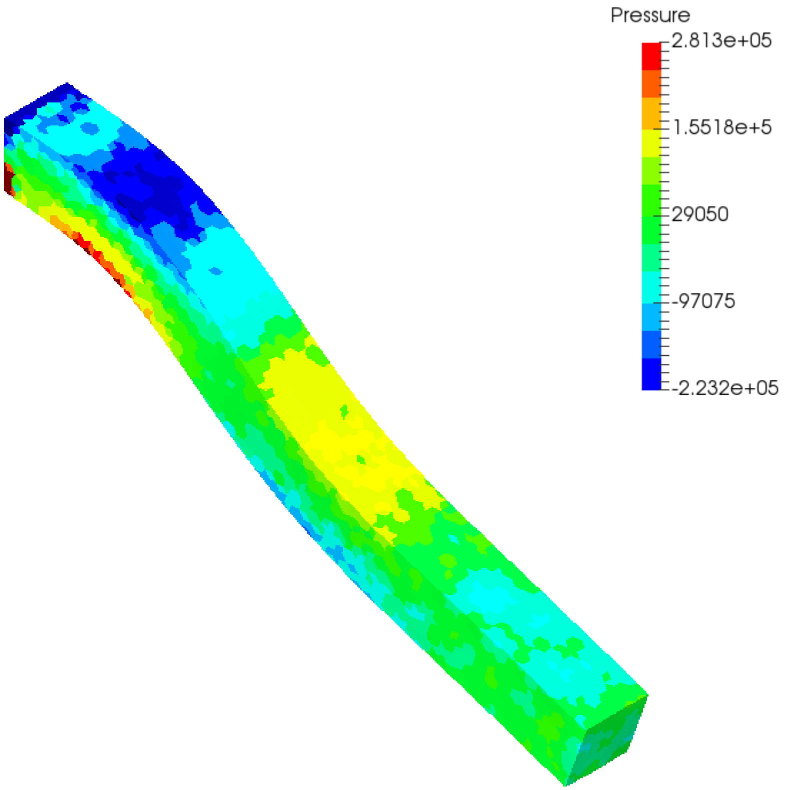
Proposed method suppresses pressure oscillation and locking!

Deformed shapes and pressure distributions

at $t = 1.5$ s



ABAQUS/Explicit C3D8
(Selective H8 element)
Reference

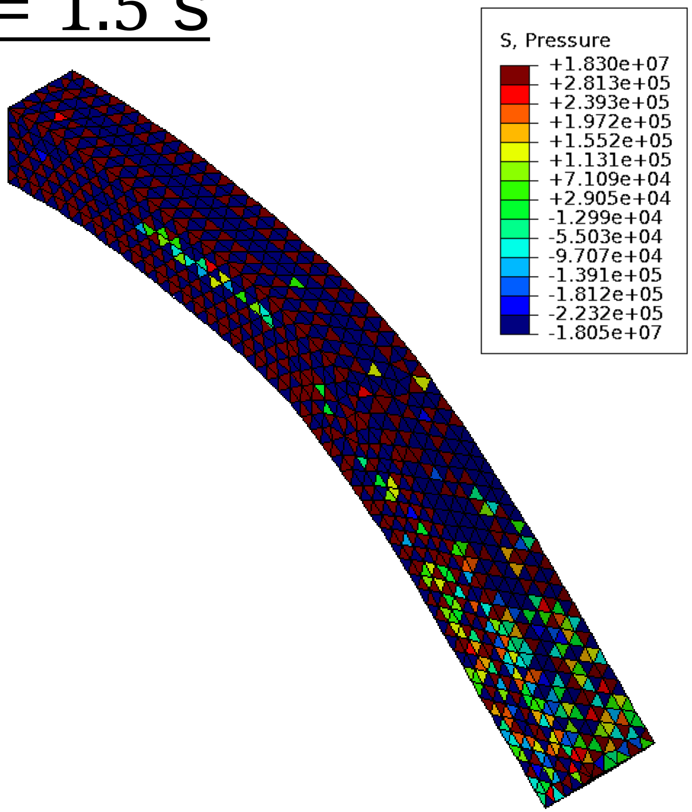


F-barES-FEM-T4(2)
(Proposed method)

Proposed method is comparable to Selective H8 element!

Deformed shapes and pressure distributions

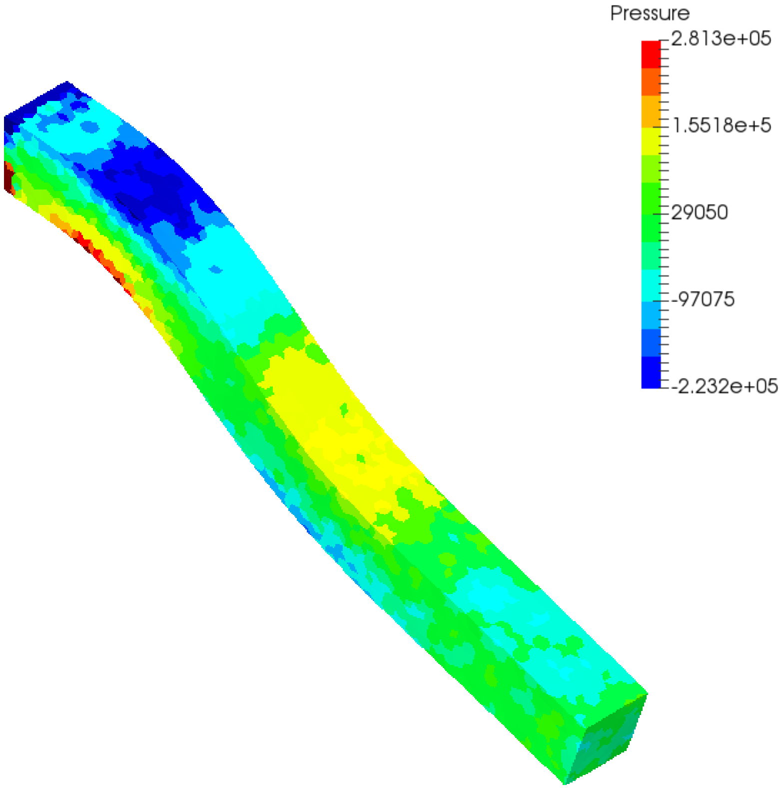
at $t = 1.5$ s



ABAQUS/Explicit C3D4

Standard T4 element

✗ Pressure oscillation and locking

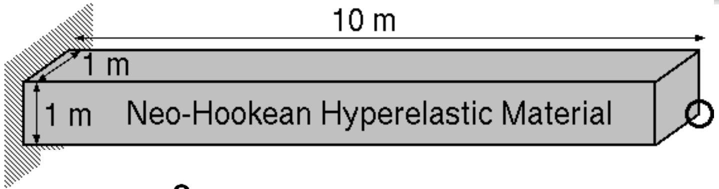


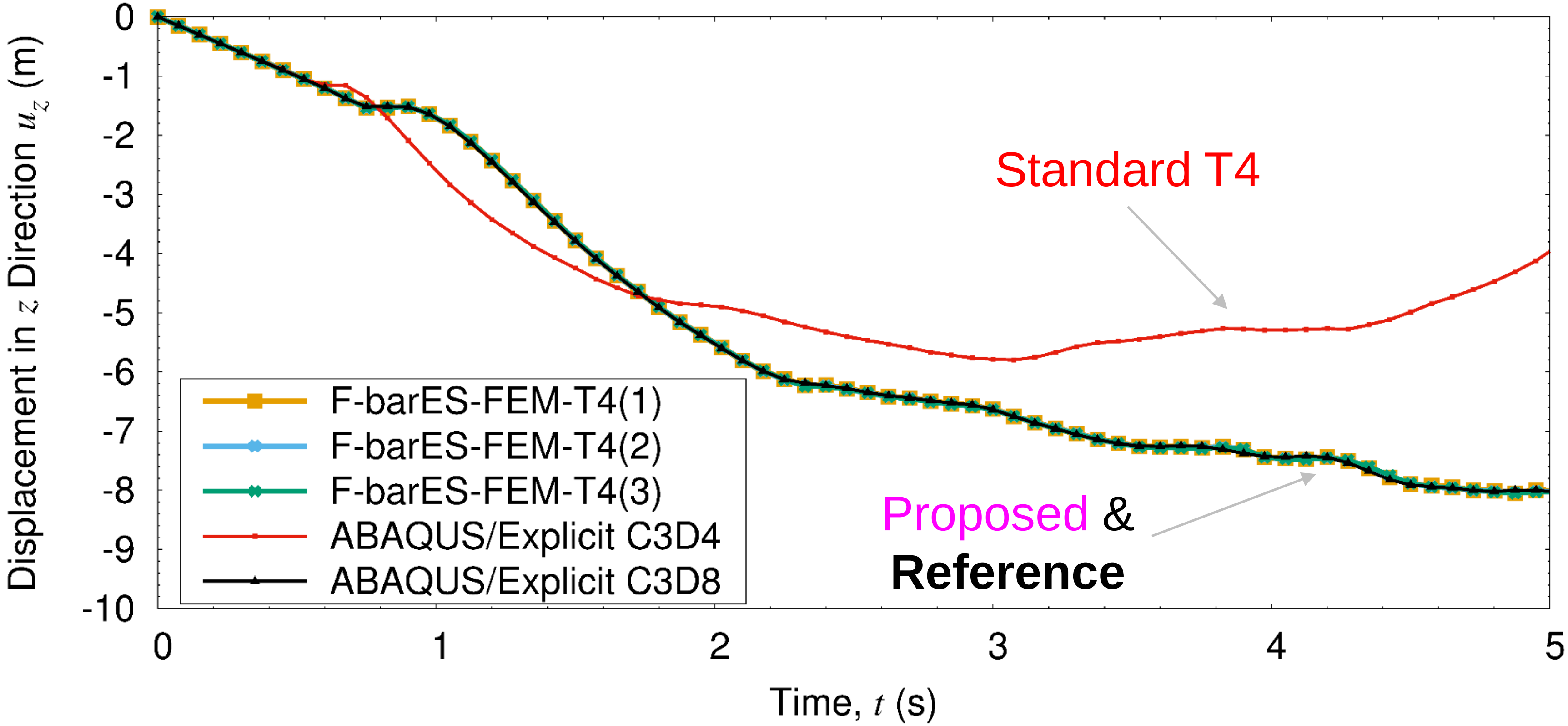
F-barES-FEM-T4(2)

(Proposed method)

Proposed method shows far better solutions than Standard T4!

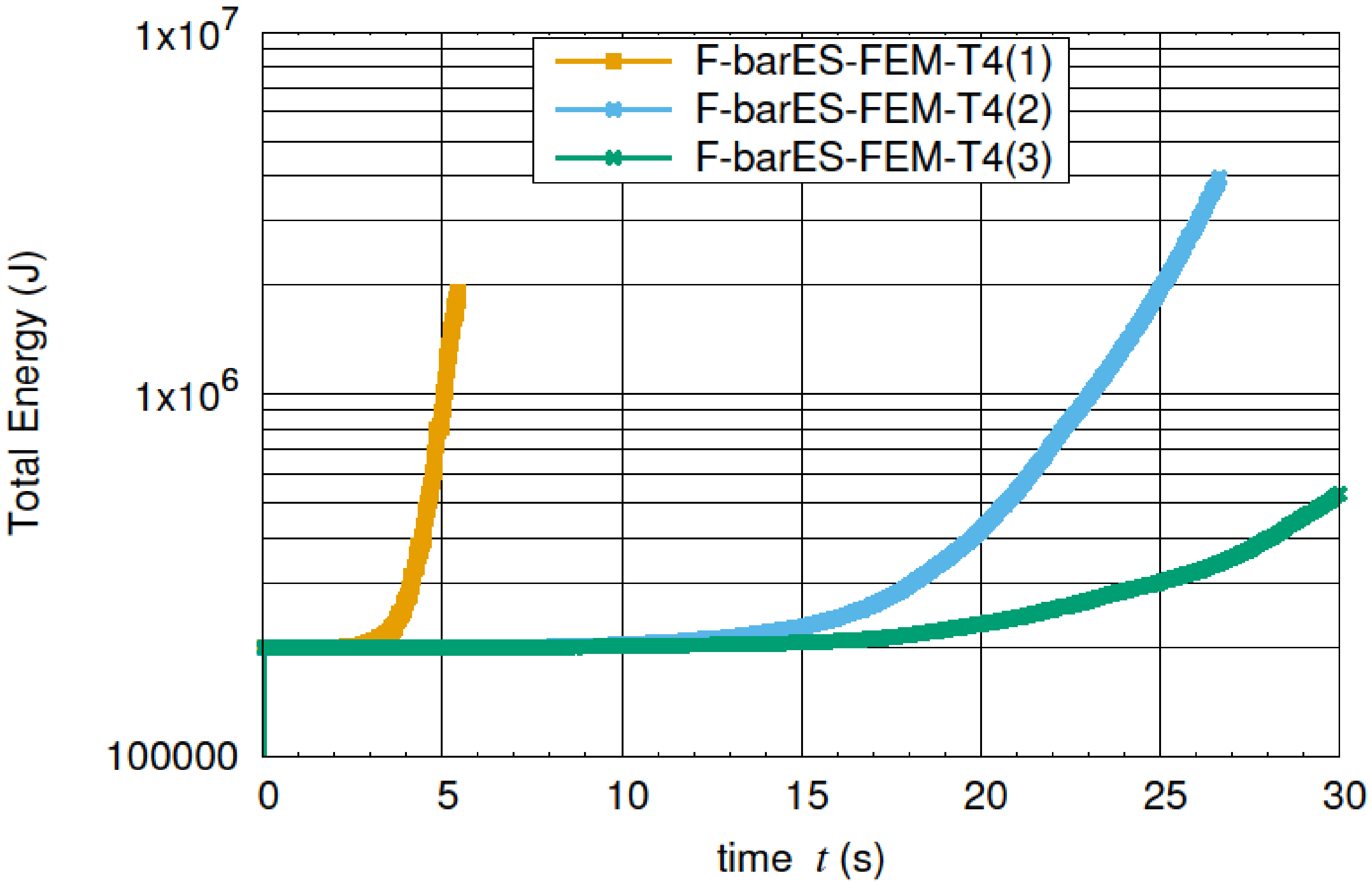
Time history of displacement





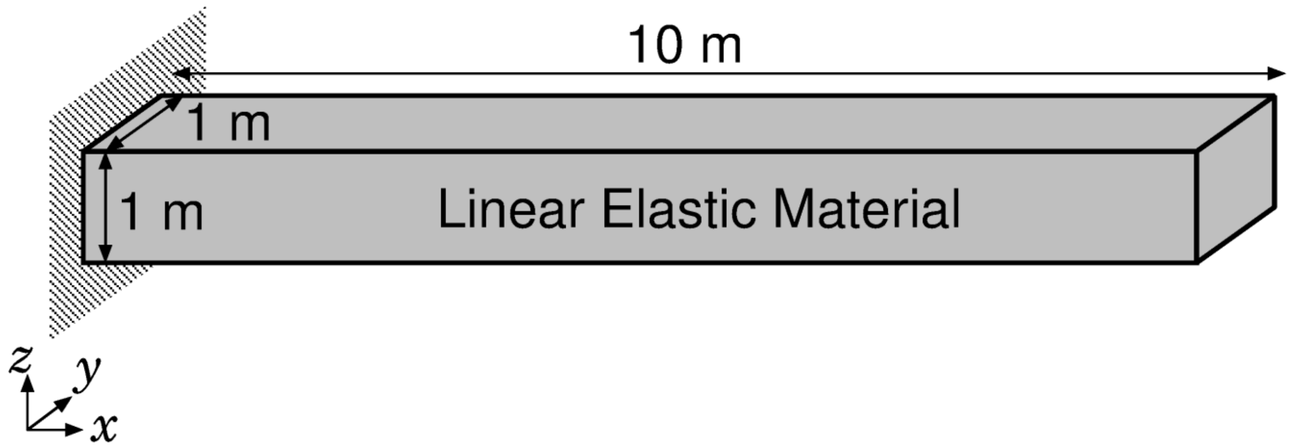
- Proposed method shows good result without locking.
- The accuracy of displacement does not depend on the number of cyclic smoothings.

Time history of total energy



- Energy divergences arise in earlier stage…
- Increasing the number of smoothings suppresses the speed of divergence.

#2 Natural mode of a cantilever

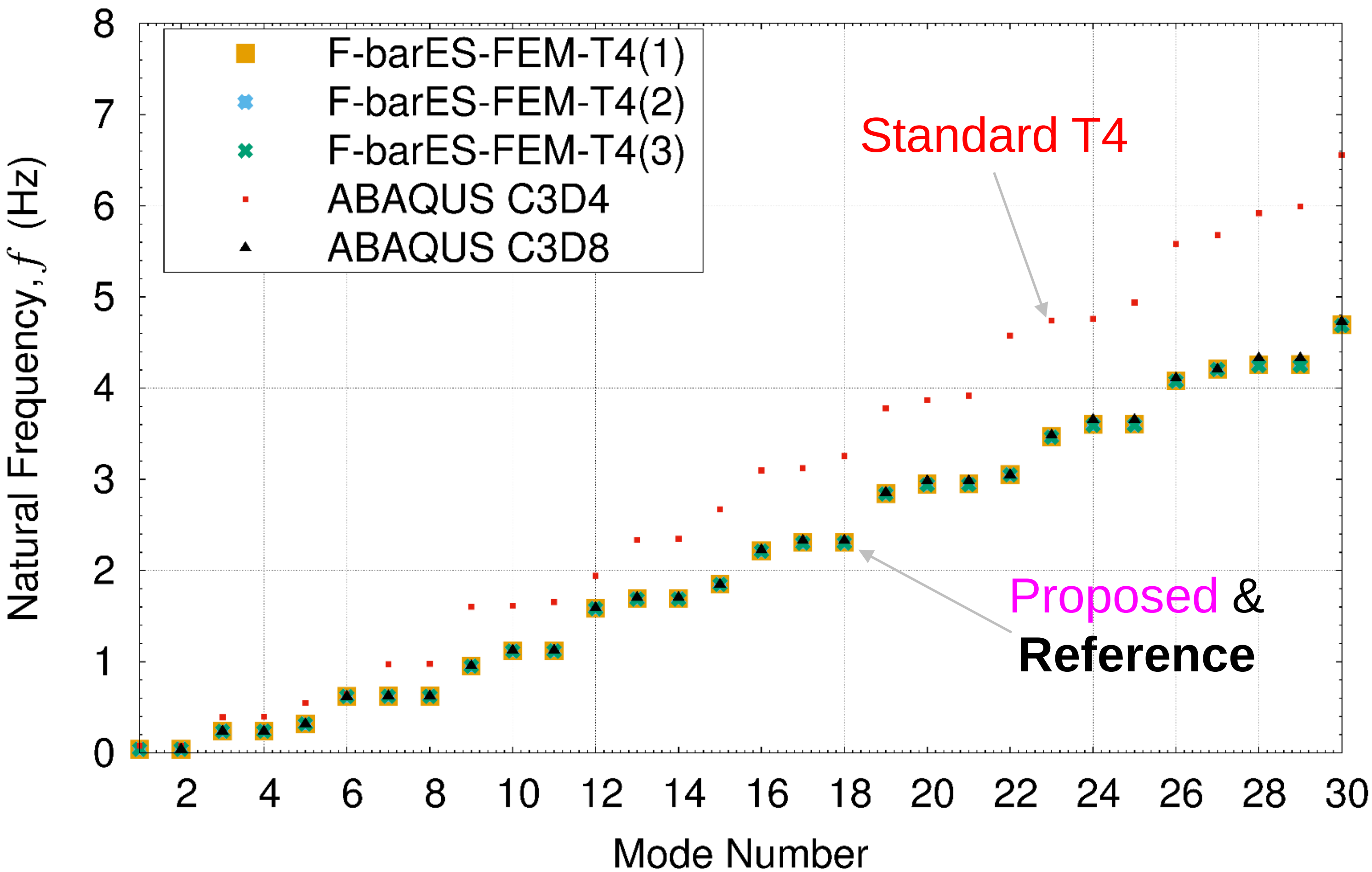


- Modal analysis.
- Linear elastic material
  - Young's modulus: 6.0 MPa,
  - Poisson's ratio: 0.499,
  - Density: 10000 kg/m^3.

Same initial elasticity as the previous example

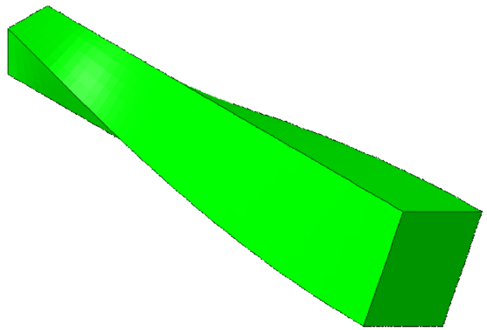
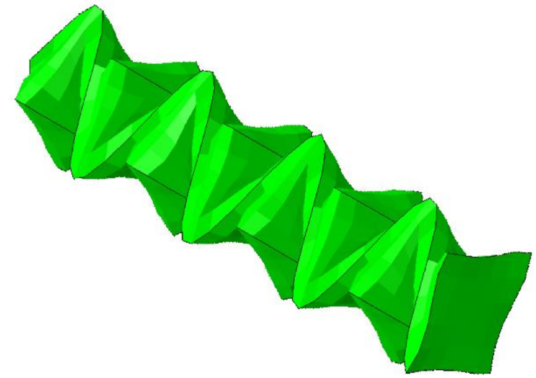
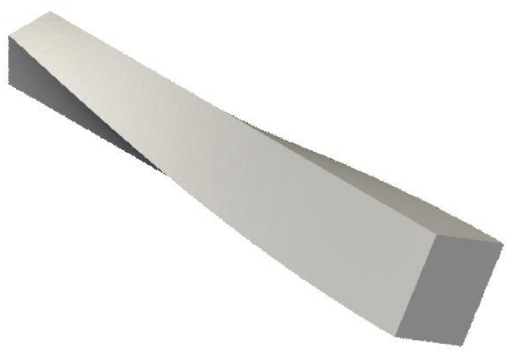
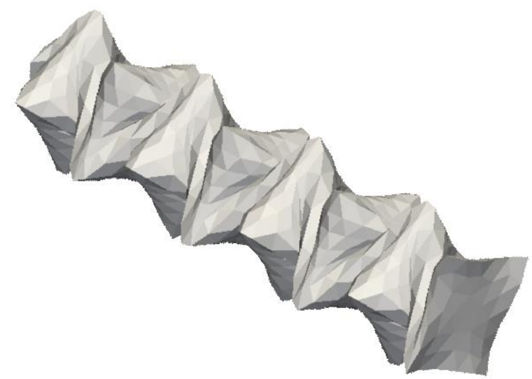
- Compare the results of F-barES-FEM-T4, Standard T4 (ABAQUS/Explicit C3D4) and Selective H8 (ABAQUS/Explicit C3D8) elements.

Natural frequencies of each mode



Natural frequencies of our method agree with those of reference !

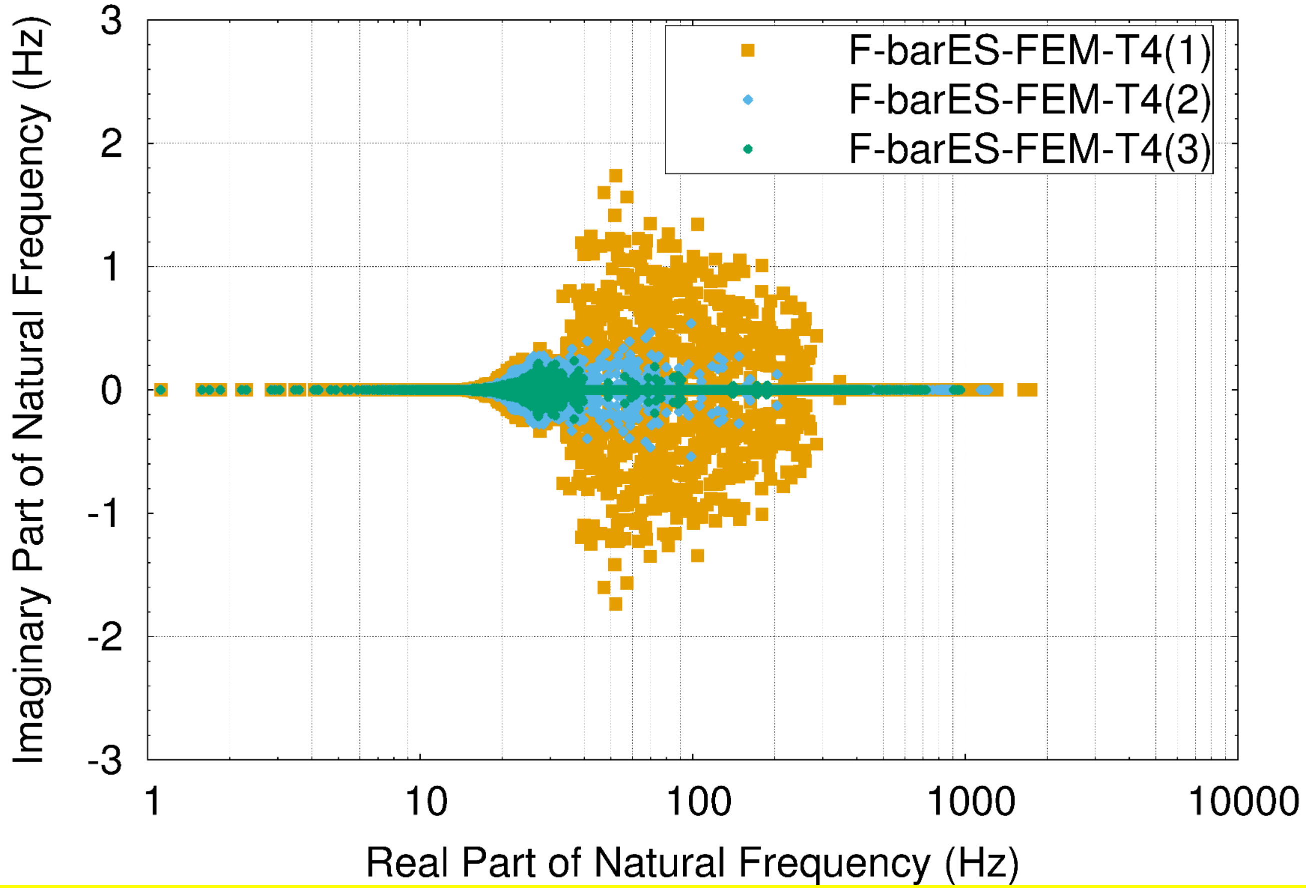
Natural mode shapes

	5th	30th
ABAQUS C3D8 **(Reference)**		
F-barES-FEM(2) (Proposed method)		

Mode shapes also agree with the reference solutions.

Distributions of natural frequencies

Imag. Part



Some natural frequencies have small imaginary part…

Increasing the number of smoothings makes the frequencies close to real numbers.

Cause of energy divergence

Due to the adoption of F-bar method,
the stiffness matrix $[K]$ becomes asymmetric.

Equation of Motion: $[M]\{\ddot{x}\} + [K]\{x\} = \{f^{\mathrm{ext}}\}$

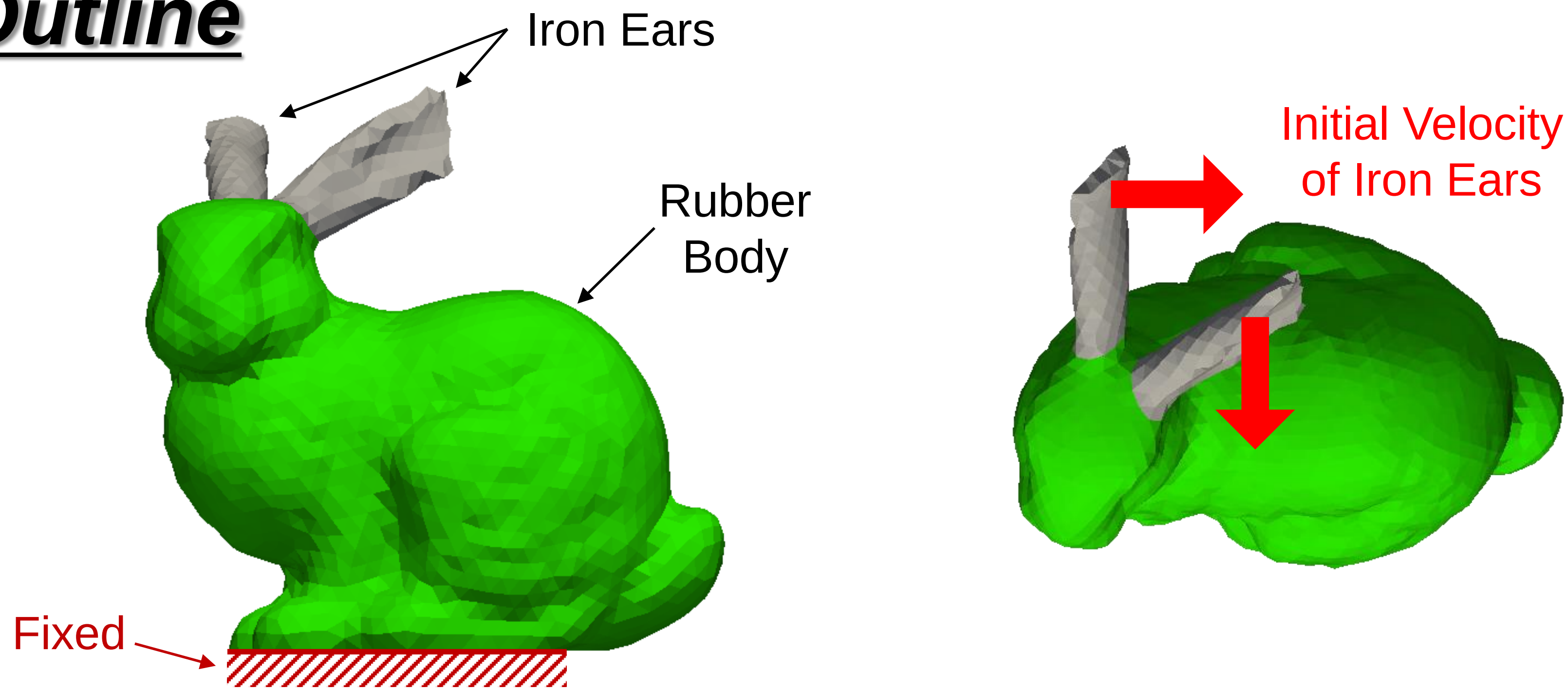
asymmetric

- Asymmetric stiffness matrix gives rise to imaginary part of natural frequencies and instability in dynamic problem.
- As shown before, increasing the number of smoothings suppress the energy divergence speed.

Our method is restricted to
short-term analysis (such as impact analysis)
with a sufficient number of smoothings.

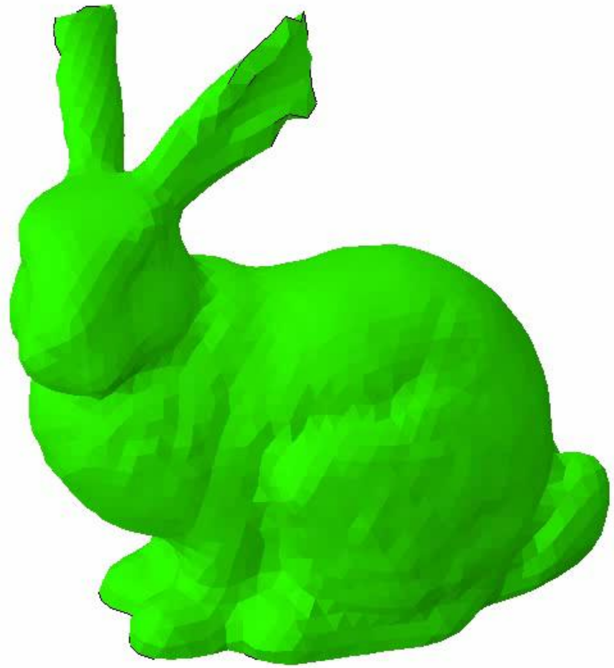
#3 Swinging of Bunny Ears

Outline

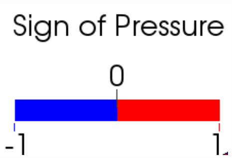


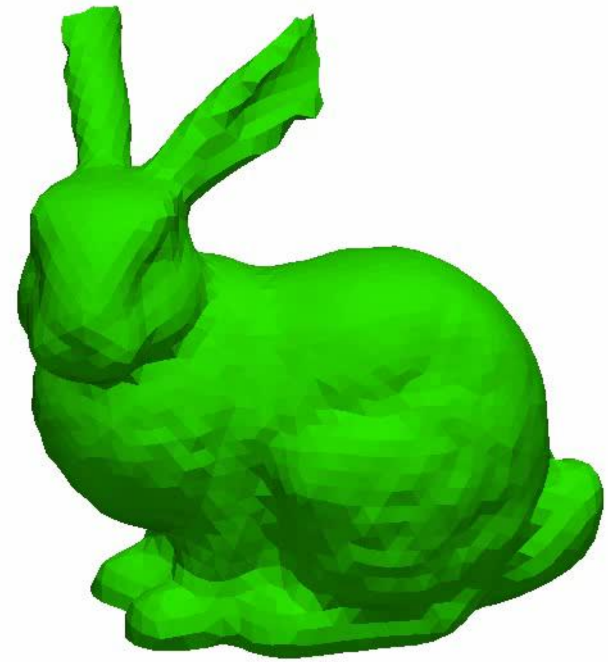
- Iron ears: $E_{\mathrm{ini}} = 200\ \mathrm{GPa},\ \nu_{\mathrm{ini}} = \mathbf{0.3},\ \rho = 7800\ \mathrm{kg/m^3}$, Neo-Hookean, **No cyclic smoothing.**
- Rubber body: $E_{\mathrm{ini}} = 6\ \mathrm{MPa},\ \nu_{\mathrm{ini}} = \mathbf{0.49},\ \rho = 920\ \mathrm{kg/m^3}$, Neo-Hookean, **1 cycle of smoothing.**
- Compared to ABAQUS/Explicit C3D4. No Hex mesh available!

Time histories of deformed shapes



ABAQUS/Explicit C3D4
(Standard T4 element)
- ✗ Pressure oscillation
- ✗ Locking



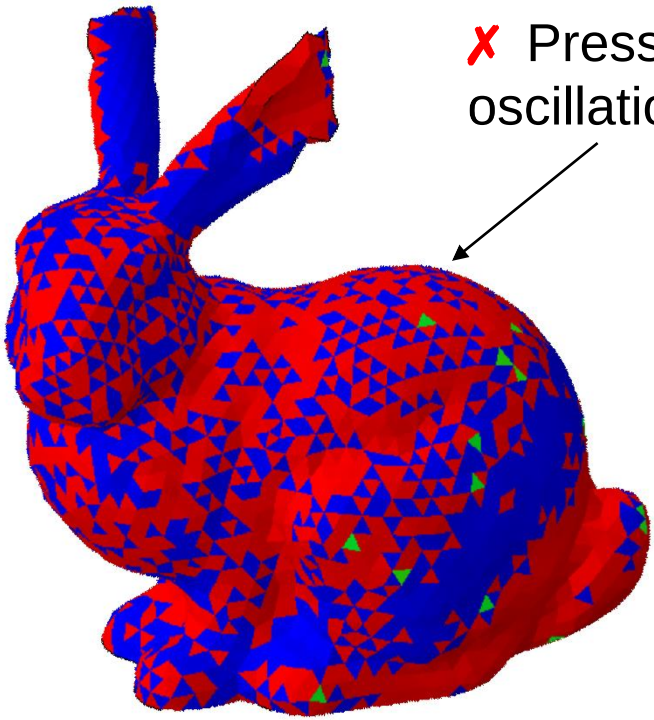


F-barES-FEM-T4
(Proposed method)
- ✔ No pressure oscillation
- ✔ No locking

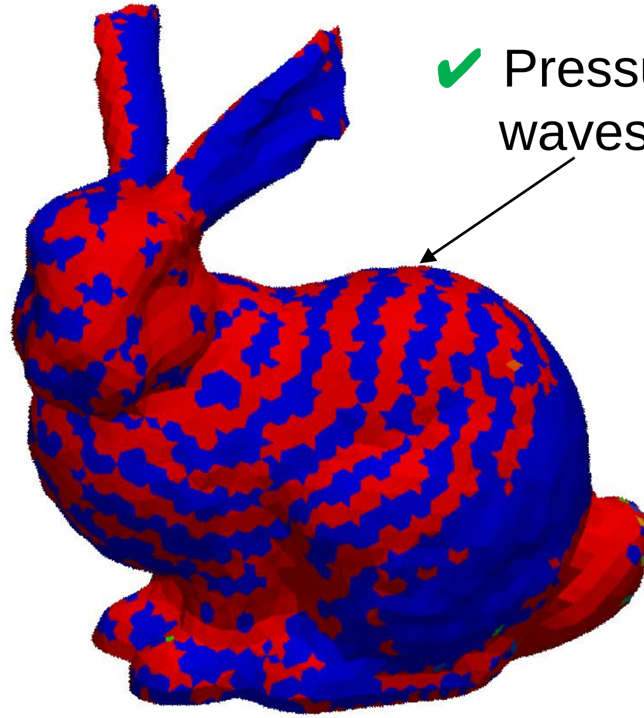
Proposed method seems be representing not pressure oscillations but pressure waves.

Deformed shapes and sign of pressure

In an early stage



ABAQUS/Explicit C3D4
(Standard T4 element)

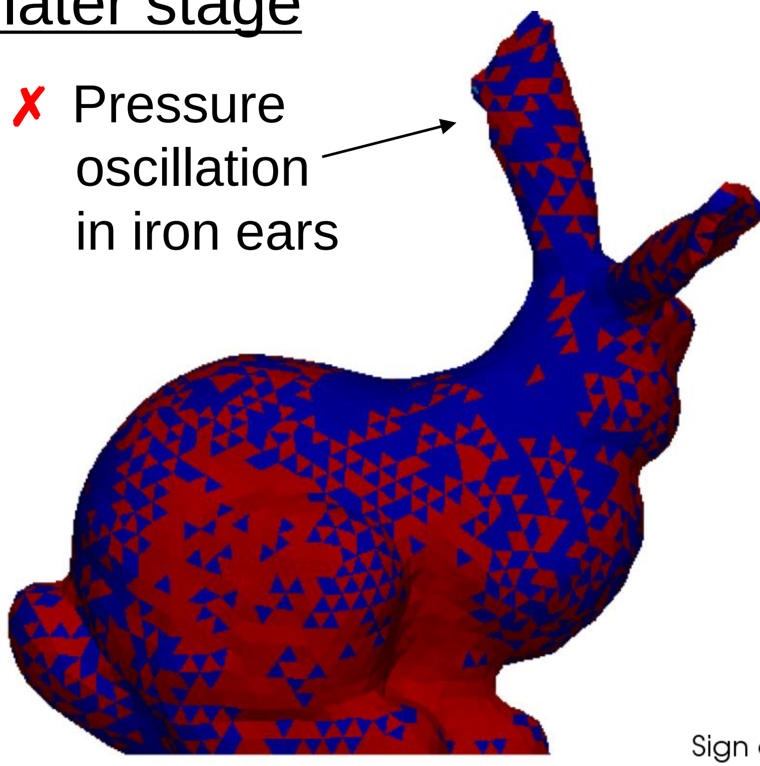


F-barES-FEM-T4
(Proposed method)

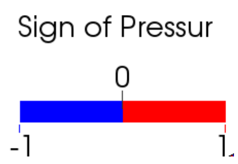
Our method represents pressure waves correctly!

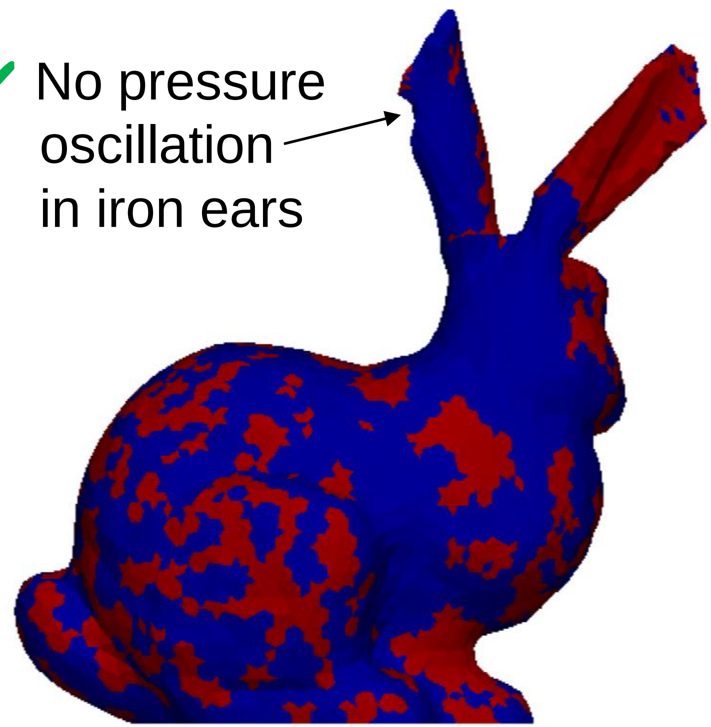
Deformed shapes and sign of pressure

In a later stage



ABAQUS/Explicit C3D4
(Standard T4 element)





F-barES-FEM-T4
(Proposed method)

It should be noted that a presence of rubber spoils over all accuracy of the analysis with Standard T4 elements.

A rubber parts is a "bad apple" when Standard T4 elements are used.

Summary

Summary

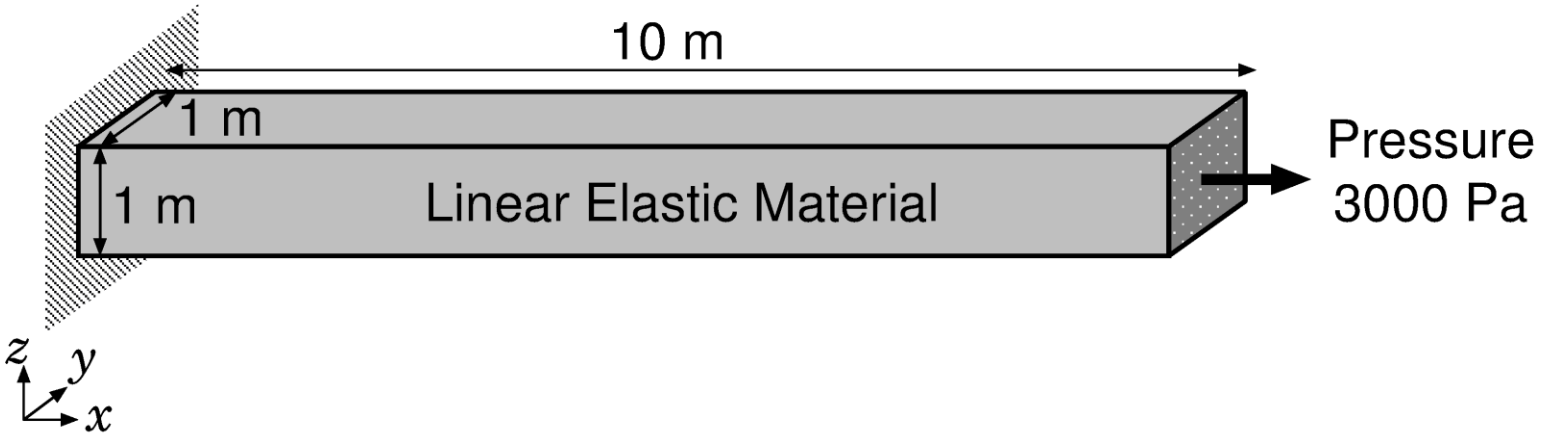
- F-barES-FEM-T4 was applied to dynamic explicit analysis.
- A few examples of analysis revealed that our method has excellent accuracy on relatively short-term problems.
- On long-term problems, however, our method is unstable because of complex natural frequencies.
- Improvement for the long-term stability is our future work.

Thank you for your kind attention.

Appendix

Propagation of 1D pressure wave

Outline

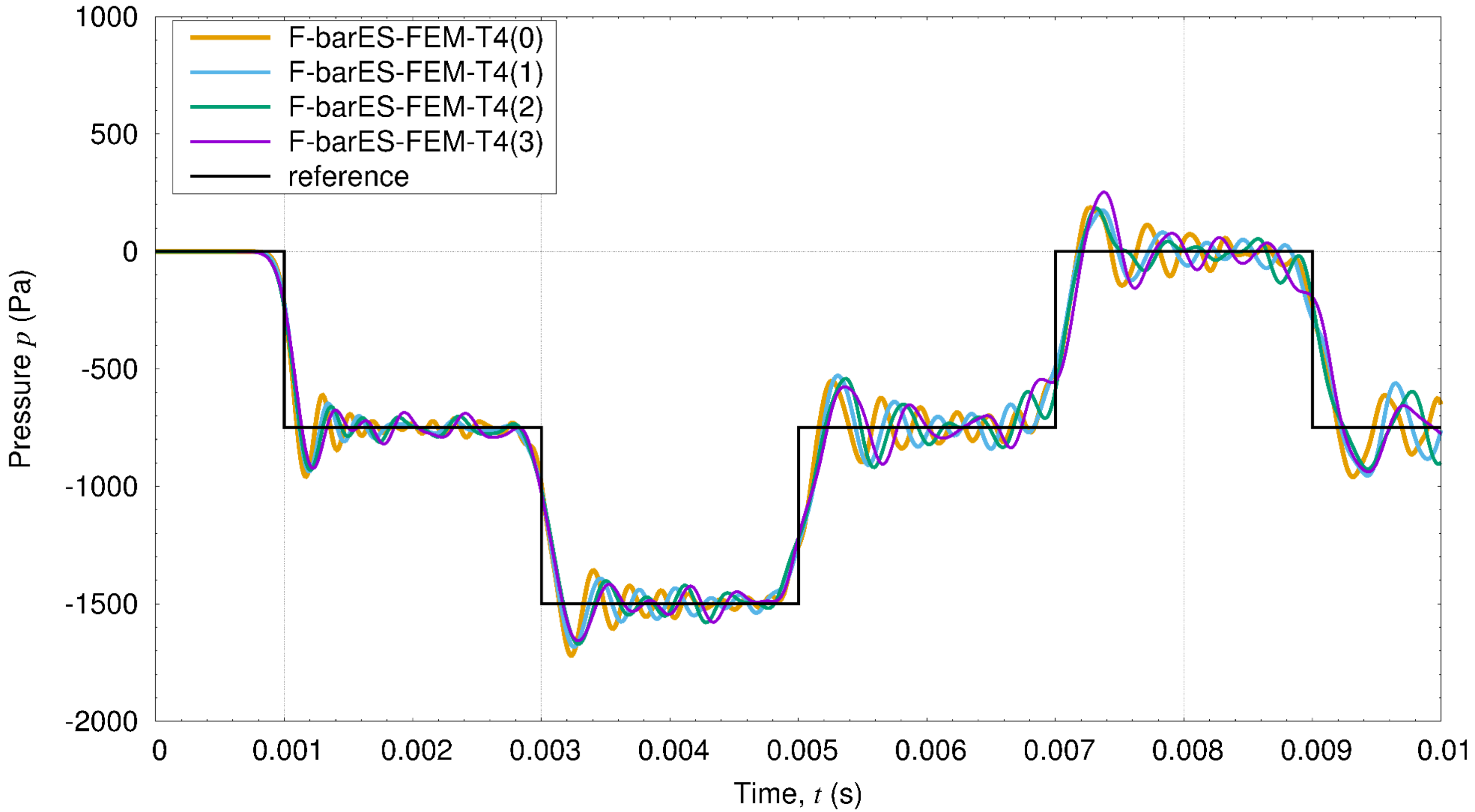


- Small deformation analysis.
- Linear elastic material,

  Young's modulus: 200 GPa,
  Poisson's ratio: 0.0,
  Density: 8000 kg/m^3.
- Results of F-barES-FEM(0), (1), (2), and (3) are compared to the analysical solution.

Propagation of 1D pressure wave

Results



Velocity Verlet Method

Algorithm

1. Calculate the next displacement $\{u_{n+1}\}$ as

$$\{u_{n+1}\} = \{u_n\} + \{\dot{u}_n\}\Delta\mathrm{t} + \frac{1}{2}\{\ddot{u}_n\}\Delta\mathrm{t}^2.$$

2. Calculate the next acceleration $\{\ddot{u}_{n+1}\}$ as

$$\{\ddot{u}_{n+1}\} = [M^{-1}](\{f^{\mathrm{ext}}\} - \{f^{\mathrm{int}}(u_{n+1})\}).$$

3. Calculate the next velocity $\{\dot{u}_{n+1}\}$ as

$$\{\dot{u}_{n+1}\} = \{\dot{u}_n\} + \{\ddot{u}_{n+1}\}\Delta t$$

Characteristics

- 2nd order symplectic scheme in time.
- Less energy divergence.

Cause of energy divergence

Due to the adoption of F-bar method, the stiffness matrix $[K]$ becomes asymmetric and thus the dynamic system turns to unstable.

Equation of natural vibration, $[M]\{\ddot{u}\} + [K]\{u\} = \{0\}$, derives an eigen equation, $([M]^{-1}[K])\{u\} = \omega^2\{u\}$, which has asymmetric left-hand side matrix.

⇒Some of eigen frequencies could be complex numbers.

⇒When an angular frequency $\omega_k = a + \mathrm{i}b$ $(b > 0)$, the time variation of the kth mode is

$$\{u(t)\} = \mathrm{Re}[\{u_k\}\exp(-\mathrm{i}\omega_k t)]$$
$$= \mathrm{Re}[\{u_k\}\exp(-\mathrm{i}at)\exp(bt)]$$

Divergent term!