F-bar aided edge-based smoothed finite element method with 4-node tetrahedral elements (F-barES-FEM-T4) for viscoelastic large deformation problems

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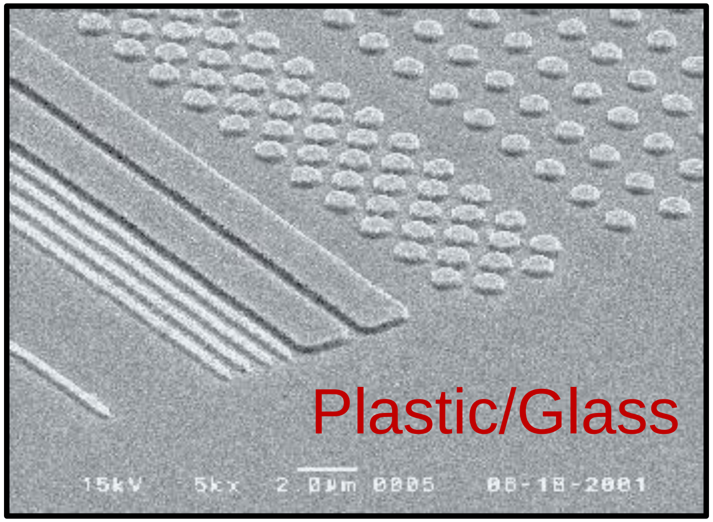
Motivation

What we want to do:

- Solve **hyper large deformation** analyses accurately and stably.
- Treat complex geometries with **tetrahedral meshes**.
- Consider **nearly incompressible materials** $(\nu \simeq 0.5)$.
- Support **contact** problems.
- Handle **auto re-meshing**.



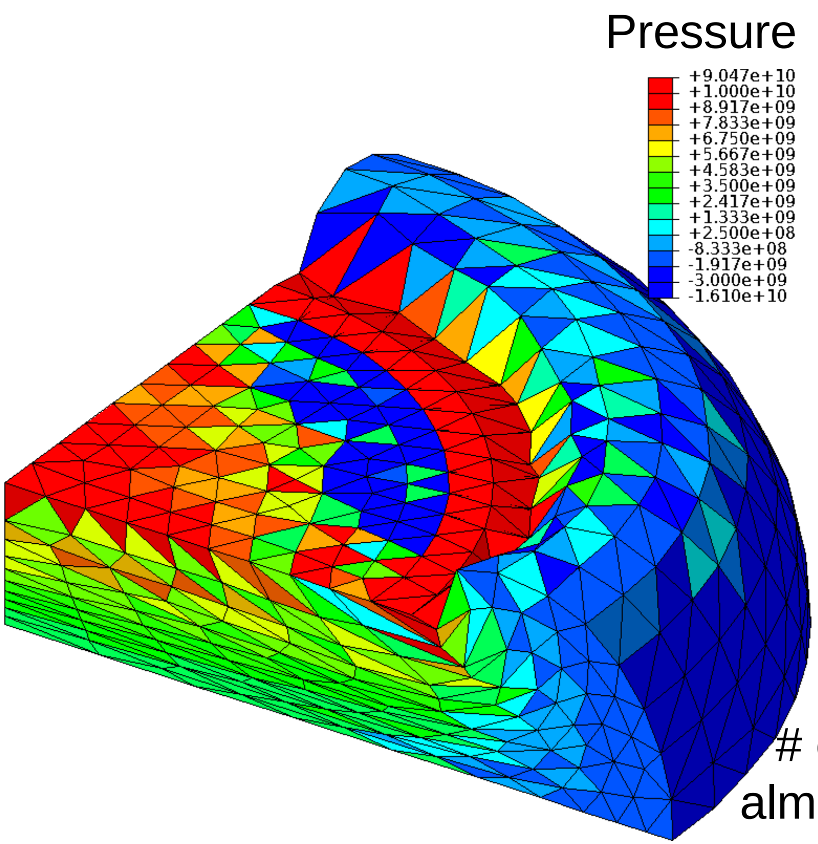
Issues

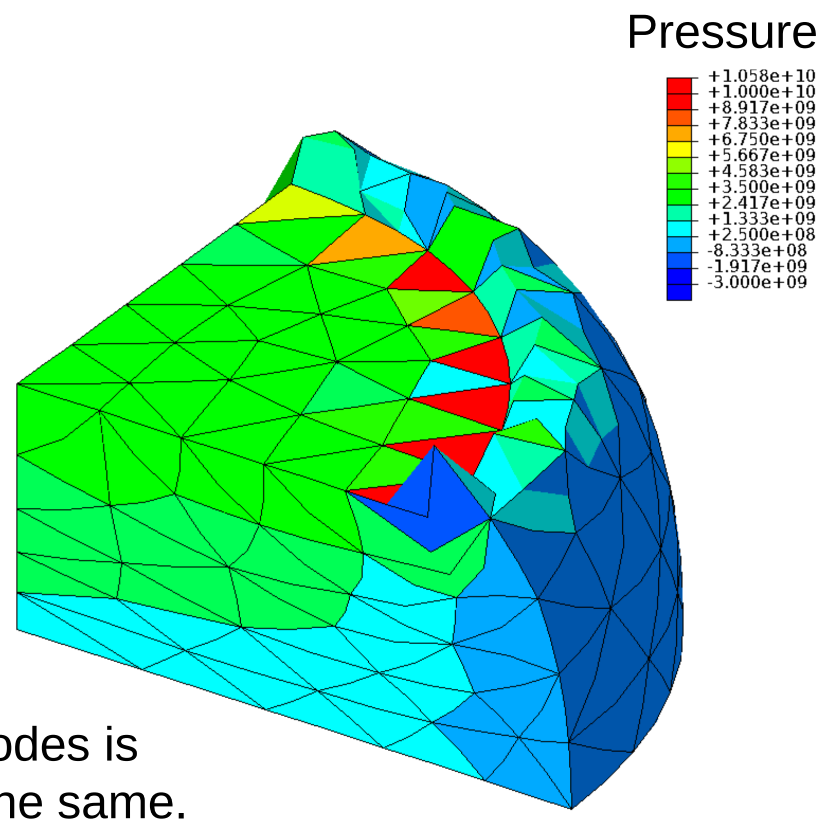
Conventional tetrahedral (T4/T10) FE formulations still have issues in accuracy or stability especially in nearly incompressible cases.

- 2nd or higher order elements:
  - ✗ Volumetric locking.
    Accuracy loss in large strain due to intermediate nodes.
- Enhanced assumed strain method (EAS):
  - ✗ Spurious low-energy modes.
- B-bar method, F-bar method, Selective reduced integration:
  - ✗ Not applicable to tetrahedral element directly.
- F-bar-Patch method:
  - ✗ Difficulty in building good-quality patches.
- **u/p mixed (hybrid) method:**
  (e.g., ABAQUS/Standard **C3D4H** and **C3D10MH**)
  - ✗ Pressure checkerboarding, Early convergence failure etc..

Issues (cont.)

E.g.) Compression of neo-Hookean ***hyperelastic*** body with $\nu_{\mathrm{ini}} = 0.49$





# of Nodes is almost the same.

1st order hybrid T4 (C3D4H)

- ✓ No volumetric locking
- ✗ Pressure checkerboarding
- ✗ Shear & corner locking

2nd order modified hybrid T10 (C3D10MH)

- ✓ No shear/volumetric locking
- ✗ Early convergence failure
- ✗ Low interpolation accuracy

A Recent Solution

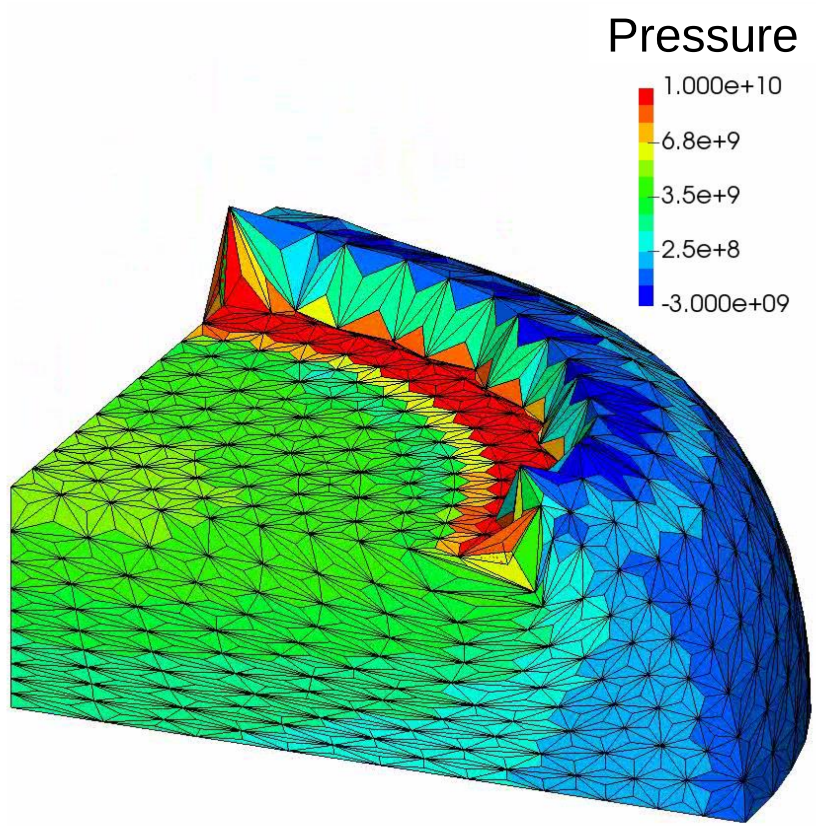
A new idea of FE formulation called "Smoothed Finite Element Method (**S-FEM**)" was recently proposed and is in researching today widely.

Our group has proposed a latest S-FEM named "**F-barES-FEM-T4**" (detailed later):

- No intermediate node & No additional DOF, (i.e., Purely displacement-based 4-node tetrahedral (T4) element),
- Free from shear, volumetric and corner locking,
- No pressure checkerboarding,
- Long lasting in large deformation.

A Recent Solution (cont.)

E.g.1) Compression of ***hyperelastic*** body with $\nu_{\mathrm{ini}} = 0.49$



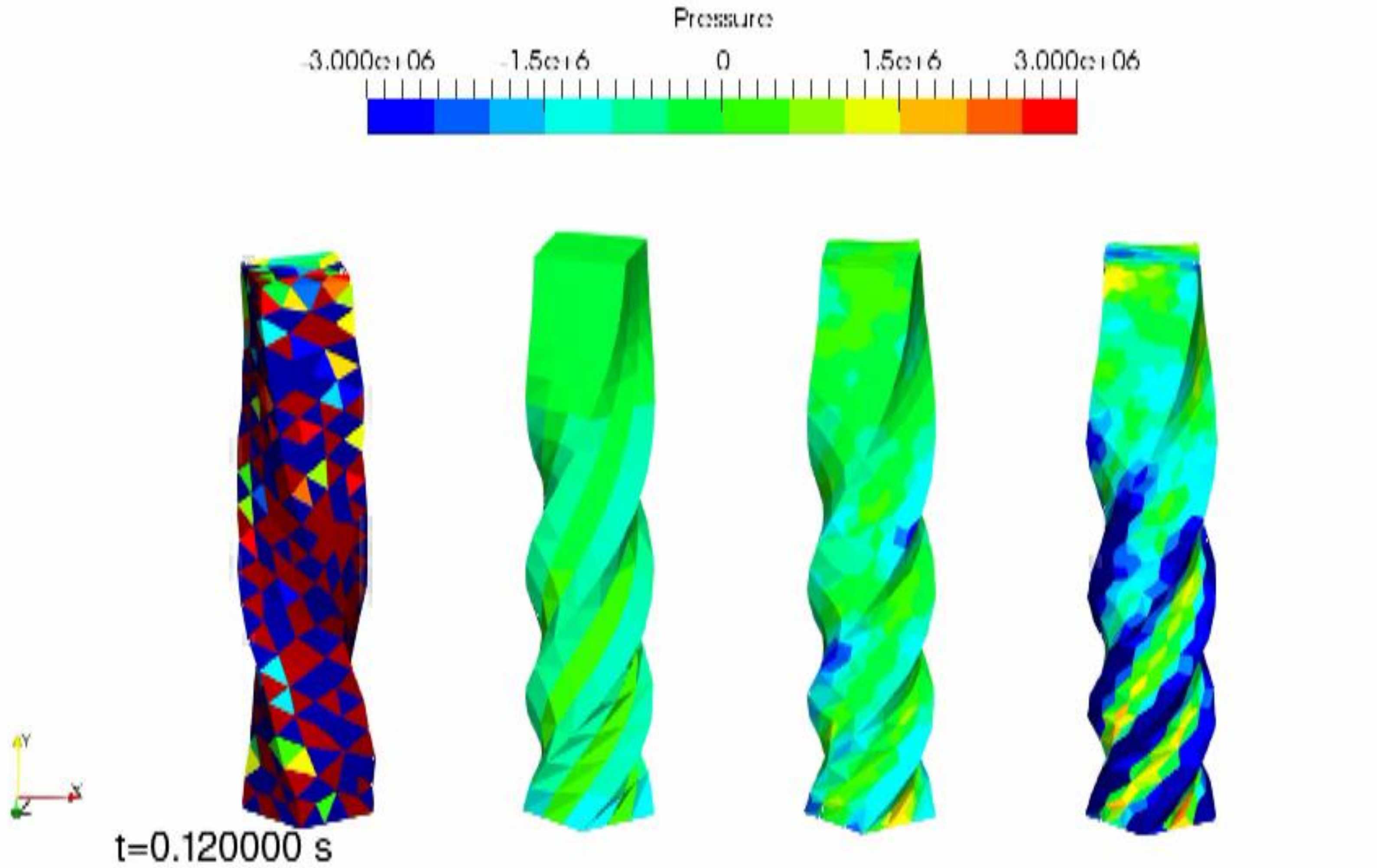
Same mesh as C3D4H case.

F-barES-FEM-T4 (One of the latest S-FEM)

- ✓ No shear/volumetric locking
- ✓ No corner locking
- ✓ No pressure checkerboarding

A Recent Solution (cont.)

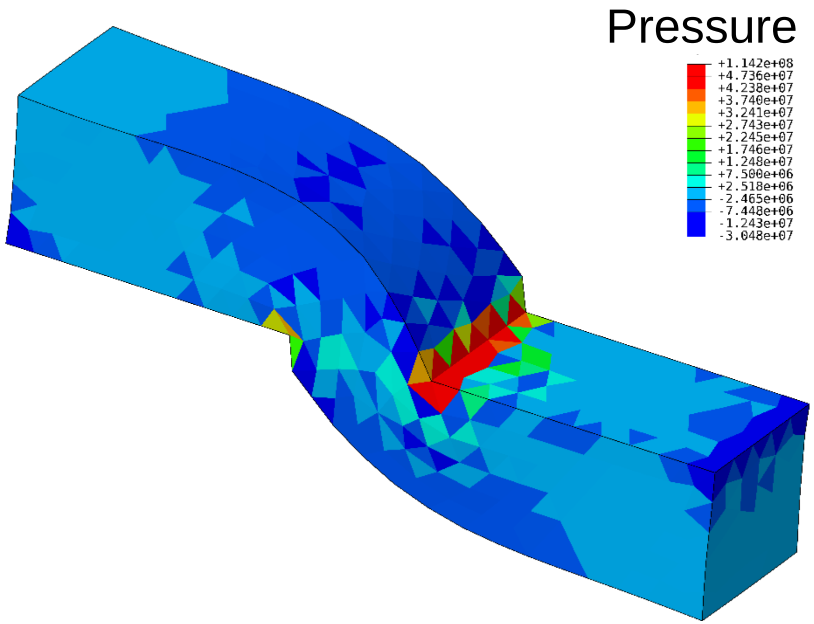
E.g.2) Explicit dynamic twist of ***hyperelastic*** body with $\nu_{\text{ini}} = 0.49$

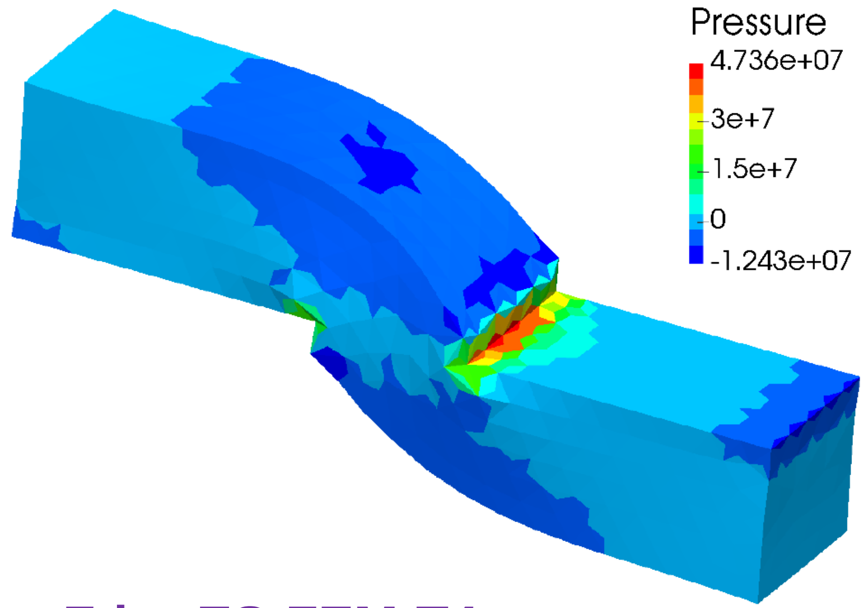


C3D4 C3D8 F-barES-FEM-T4 SymF-barES-FEM-T4

A Recent Solution (cont.)

E.g.3) Shear of ***elastoplastic*** body with soft hardening coeff.





1st order hybrid T4 (C3D4H)

- ✓ No volumetric locking
- ✗ Shear locking
- ✗ Pressure checkerboarding

F-barES-FEM-T4

- ✓ No volumetric locking
- ✓ No shear locking
- ✓ No pressure checkerboarding

We have evaluated F-barES-FEM-T4 in elastic and elastoplastic cases but NOT in viscoelastic cases yet.

Objective

To applying and demonstrate
the latest S-FEM called **F-barES-FEM-T4**
to viscoelastic large deformation problems.

Table of Body Contents

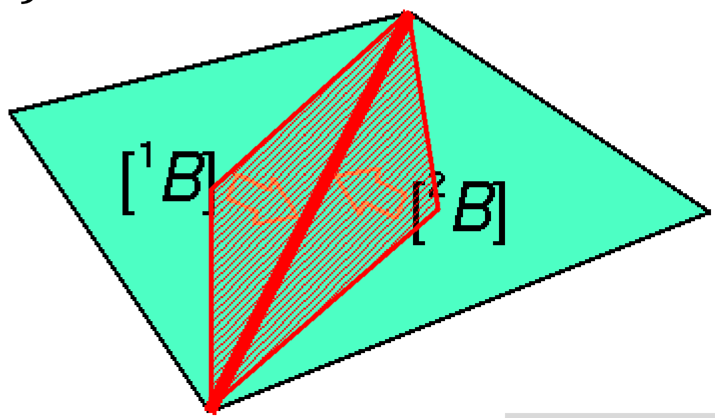
- Introduction of **F-barES-FEM-T4**'s formulation
- Demonstration of **F-barES-FEM-T4**
  in viscoelastic problems
- Summary

Introduction of F-barES-FEM-T4's formulation

1. Brief of Edge-based S-FEM (ES-FEM)

- Calculate $[B]$ at each element as usual.
- Distribute $[B]$ to the connecting edges with area weight and build $[\,^{\text{Edge}}B]$.
- Calculate $\boldsymbol{F}, \boldsymbol{T}, \{f^{\text{int}}\}$ etc. in each edge smoothing domain.

As if putting
an integration point
on each edge center



$[^{\text{Edge}}B]$

↓

$^{\text{Edge}}T$

↓

$\{f^{\text{int}}\}$

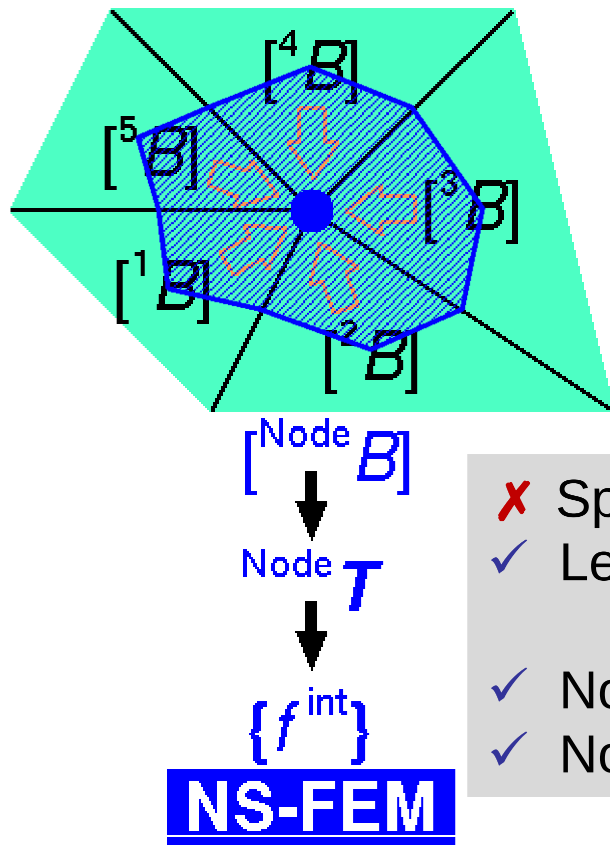
ES-FEM

- ✗ Volumetric locking
- ✗ Pressure checkerboarding
- ✓ No shear locking
- ✓ No spurious modes

2. Brief of Node-based S-FEM (NS-FEM)

- Calculate $[B]$ at each element as usual.
- Distribute $[B]$ to the connecting nodes with area weight and build $[{}^{\mathrm{Node}}B]$.
- Calculate $\boldsymbol{F}, \boldsymbol{T}, \{f^{\mathrm{int}}\}$ etc. in each node smoothing domain.

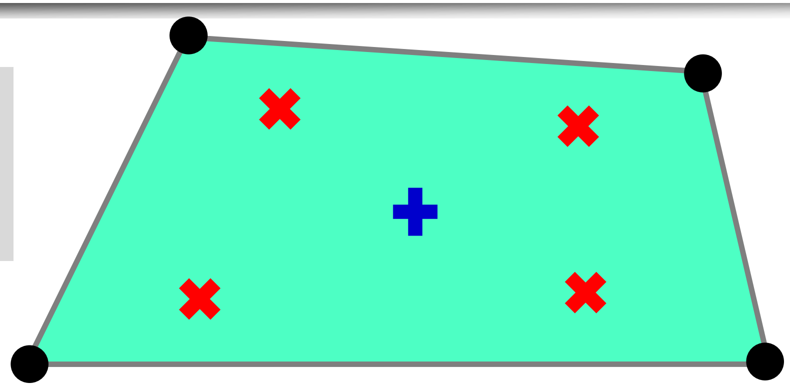
As if putting
an integration point
on each node



- ✗ Spurious low-energy mode
- ✓ Less pressure checkerboarding
- ✓ No shear locking
- ✓ No volumetric locking

3. Brief of F-bar Method

For quadrilateral (Q4) or hexahedral (H8) elements



Algorithm

1. Calculate deformation gradient $\boldsymbol{F}$ at the element center, and then make the relative volume change $\bar{J}$ $(= \det(\boldsymbol{F}))$.
2. Calculate deformation gradient $\boldsymbol{F}$ at each gauss point as usual, and then make $\boldsymbol{F}^{\mathrm{iso}}$ $(= \boldsymbol{F} \,/\, J^{1/3})$.
3. Modify $\boldsymbol{F}$ at each gauss point to obtain $\bar{\boldsymbol{F}}$ as
$$\bar{\boldsymbol{F}} = \bar{J}^{1/3} \boldsymbol{F}^{\mathrm{iso}}.$$
4. Use $\bar{\boldsymbol{F}}$ to calculate the stress $\boldsymbol{T}$, nodal force $\{f^{\mathrm{int}}\}$ etc..

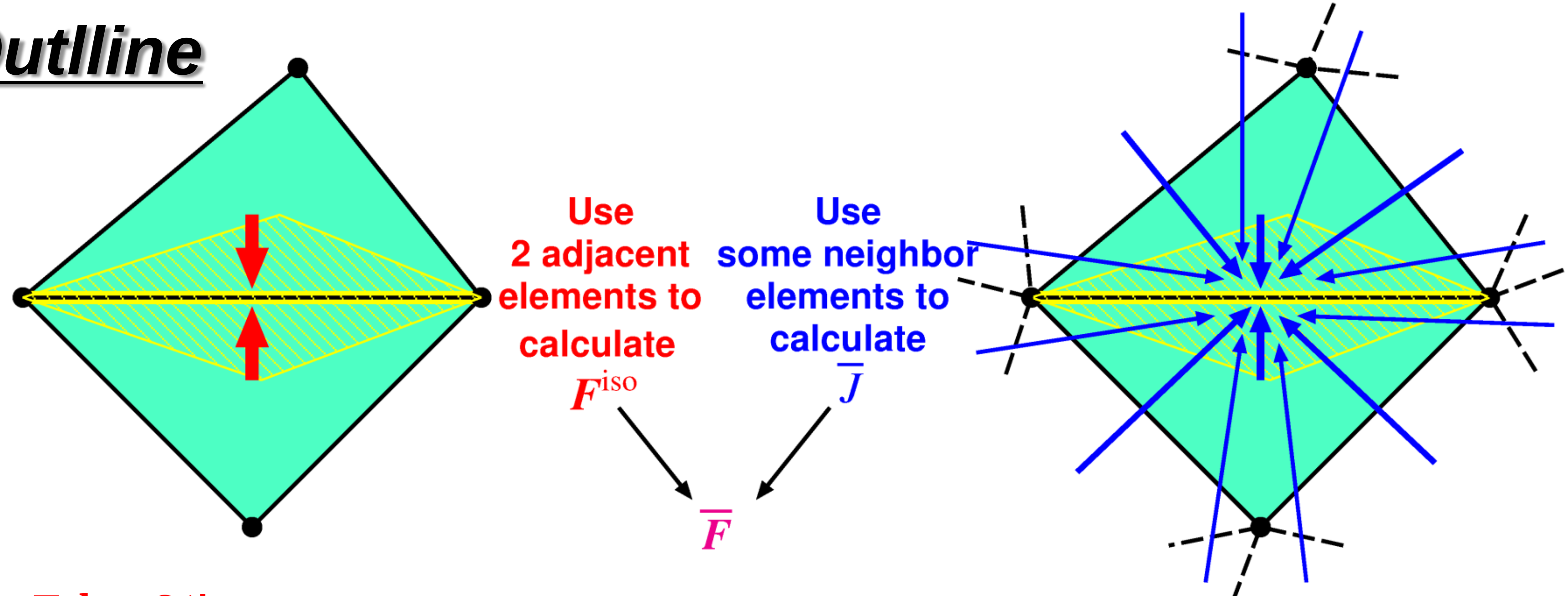
A kind of low-pass filter for J

F-bar method is used to **avoid volumetric locking** in Q4 or H8 elements. Yet, it **cannot avoid shear locking**.

Outline of F-barES-FEM

Concept: combine ES-FEM and NS-FEM using F-bar method

<u>Outlline</u>



- ${}^{\mathrm{Edge}}\tilde{\boldsymbol{F}}^{\mathrm{iso}}$ is given by ES-FEM.
- ${}^{\mathrm{Edge}}\bar{J}$ is given by cyclically applied NS-FEM.
- ${}^{\mathrm{Edge}}\bar{\boldsymbol{F}}$ is calculated in the manner of F-bar method:

$${}^{\mathrm{Edge}}\bar{\boldsymbol{F}} = {}^{\mathrm{Edge}}\bar{J}^{\,1/3}\ {}^{\mathrm{Edge}}\tilde{\boldsymbol{F}}^{\mathrm{iso}}.$$

Outline of F-barES-FEM (cont.)

Brief Formulation

1. Make ${}^{\mathrm{Elem}}\boldsymbol{F}$ as usual and calculate ${}^{\mathrm{Elem}}J$.

2. Smooth ${}^{\mathrm{Elem}}J$ at nodes and get ${}^{\mathrm{Node}}\tilde{J}$.
3. Smooth ${}^{\mathrm{Node}}\tilde{J}$ at elements and get ${}^{\mathrm{Elem}}\tilde{J}$.
4. Repeat 2. and 3. as necessary (c times).

Cyclic Smoothing of J

A kind of low-pass filter

5. Smooth ${}^{\mathrm{Elem}}\tilde{\tilde{J}}$ (c layers of ~) at edges and get ${}^{\mathrm{Edge}}\bar{J}$.
6. Combine ${}^{\mathrm{Edge}}\bar{J}$ and ${}^{\mathrm{Edge}}\boldsymbol{F}^{\mathrm{iso}}$ of ES-FEM as

$$ {}^{\mathrm{Edge}}\bar{\boldsymbol{F}} = {}^{\mathrm{Edge}}\bar{J}^{1/3}\ {}^{\mathrm{Edge}}\boldsymbol{F}^{\mathrm{iso}}. $$

Hereafter, F-barES-FEM-T4 with c cycles of smoothing is called "F-barES-FEM-T4(c)".

How to Treat Viscoelastic Model

The target constitutive model to treat is the Hencky's viscoelastic model based on the generalized Maxwell model.

The most standard one.

■ Stress

Bulk modulus

Hencky (Logarithmic) strain

$$\begin{cases} \boldsymbol{T}^{\mathrm{hyd}} = K \,\mathrm{tr}(\boldsymbol{H})\, \boldsymbol{I}, \\ \boldsymbol{T}^{\mathrm{dev}} = 2G_0 \left(\boldsymbol{H}^{\mathrm{dev}} - \sum g_i \boldsymbol{H}_i^{\mathrm{v}} \right). \end{cases}$$

Viscosity only in deviatoric stress.

Instantaneous shear modulus

Prony coeff

Viscous strain

■ Time advance of viscous strain

$$\boldsymbol{H}_i^{\mathrm{v}+} = \boldsymbol{R} \cdot \boldsymbol{H}_i^{\mathrm{v}} \cdot \boldsymbol{R}^{\mathrm{T}} + \Delta \boldsymbol{H}_i^{\mathrm{v}}.$$

Rigid rotation in an increment

Viscous strain increment

■ Equation to solve

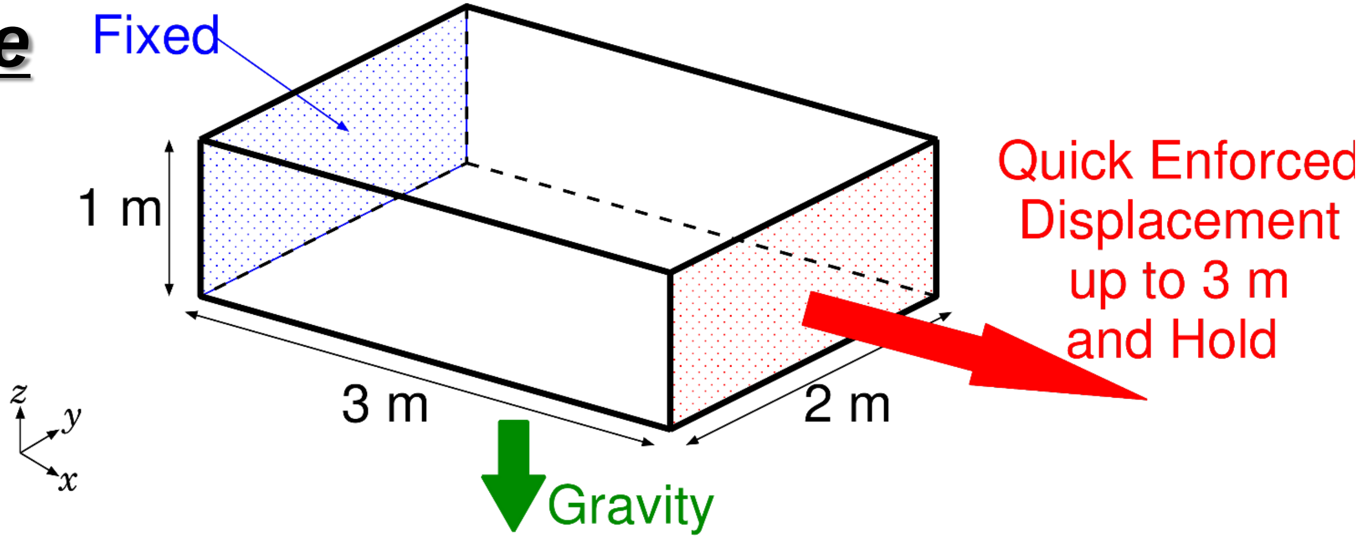
$$[\boldsymbol{K}]\{\boldsymbol{u}\} = \{\boldsymbol{f}\}$$

Same as static problems due to the absence of inertia

Demonstration of
F-barES-FEM-T4
in viscoelastic problems

Tensile Suspension of Viscoelastic Block

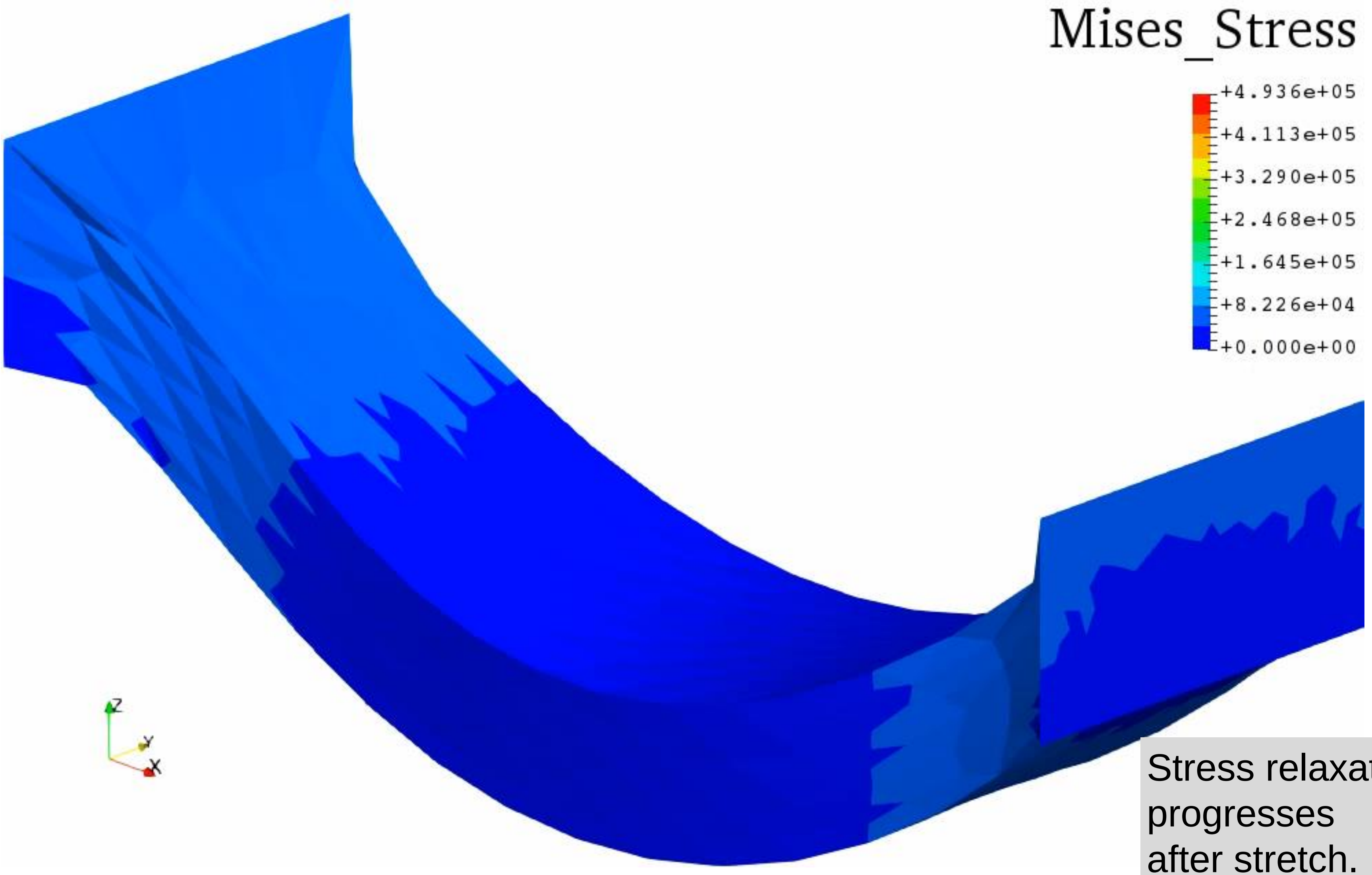
Outline



- $1\ \mathrm{m} \times 2\ \mathrm{m} \times 3\ \mathrm{m}$ block subjected to 100% stretch in 10 s, and hold the enforced displacement for 1000 s under the gravity.
- Hencky's viscoelastic body based on the generalized Maxwell model with 1 maxwell element & 1 long-term spring.
  - Poisson's ratio: $\nu_0 = 0.3$, and $\nu_\infty = 0.49$.
  - Relaxation time: $\tau = 10\ \mathrm{s}$.
- Compare the results of F-barES-FEM-T4(2), ABAQUS C3D4, C3D4H, and C3D8.

Tensile Suspension of Viscoelastic Block

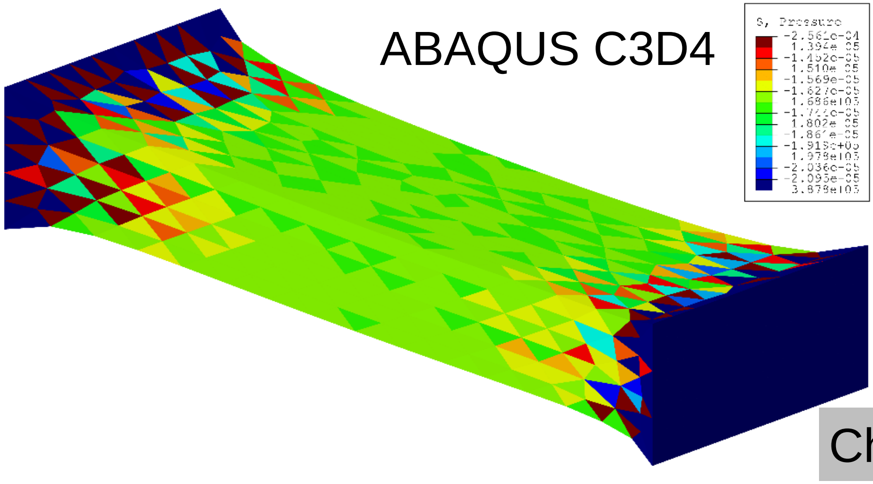
Animation of Mises Stress (F-barES-FEM-T4(2))

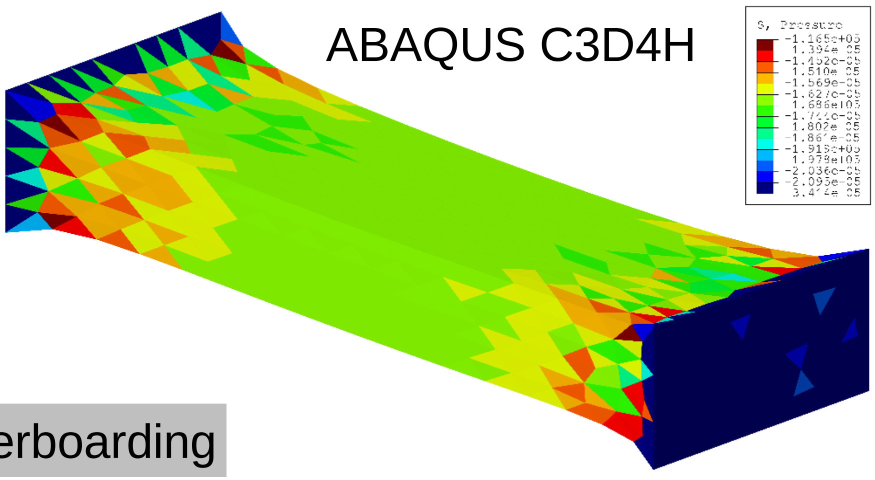


Stress relaxation progresses after stretch.

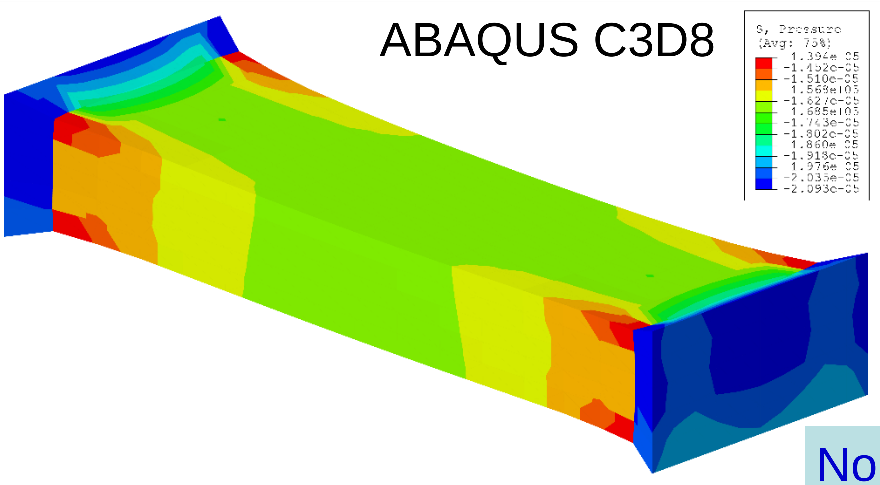
Tensile Suspension of Viscoelastic Block

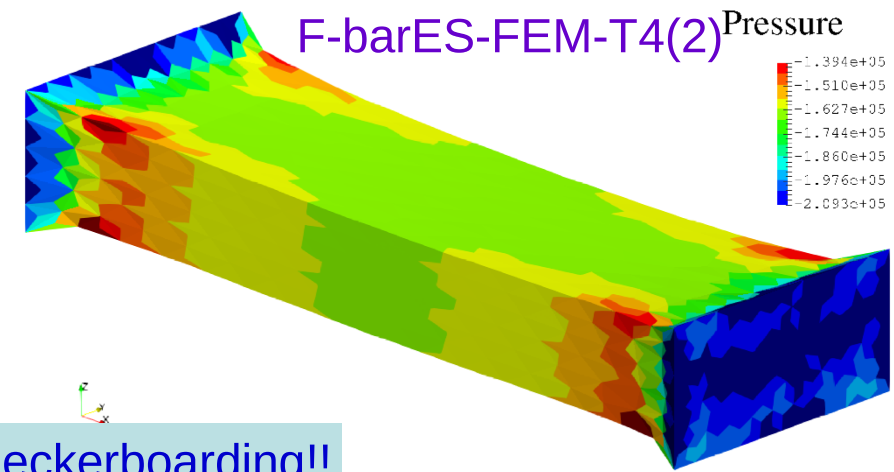
***Pressure* at the end of stretch (common contour range)**





Checkerboarding

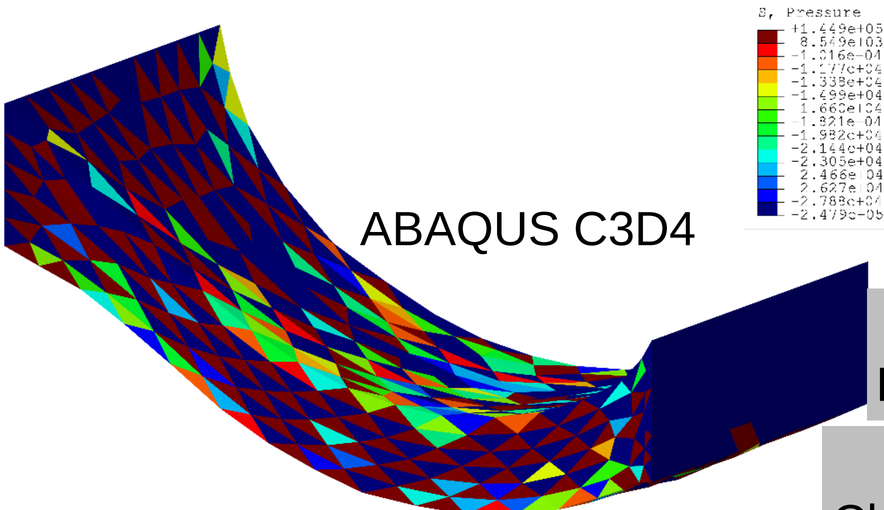


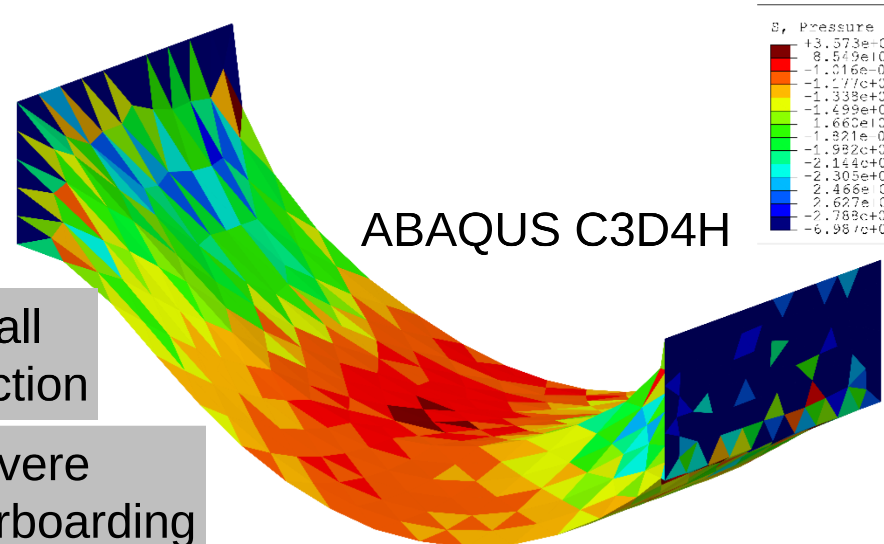


No Checkerboarding!!

Tensile Suspension of Viscoelastic Block

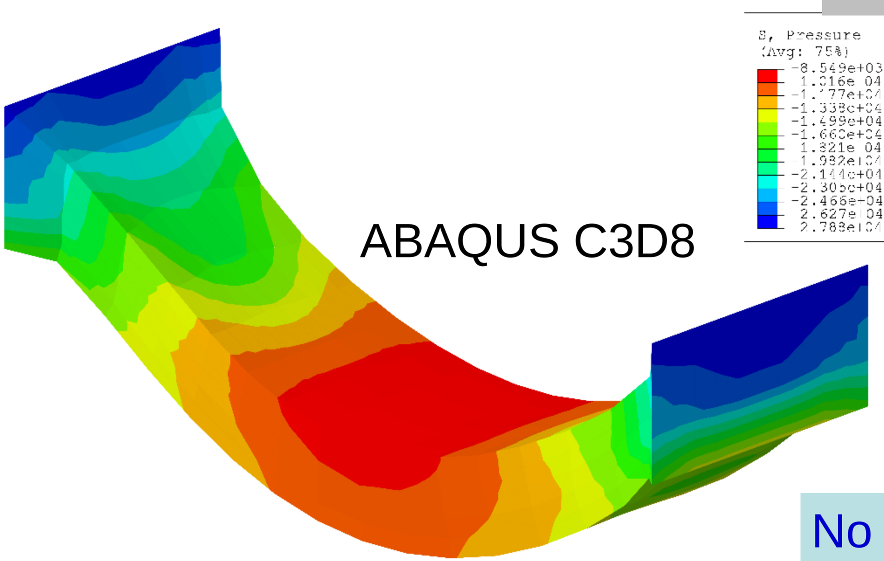
Pressure *at the final state* (common contour range)

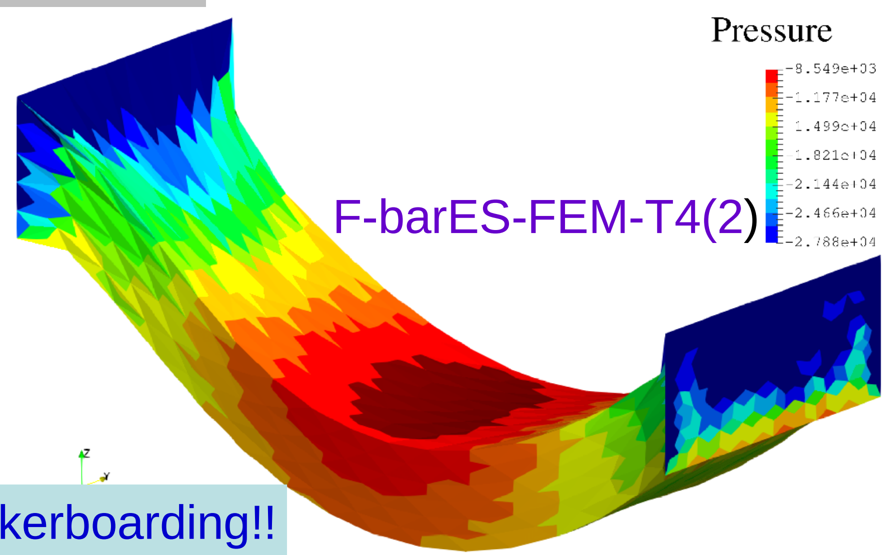




Small Deflection

Severe Checkerboarding

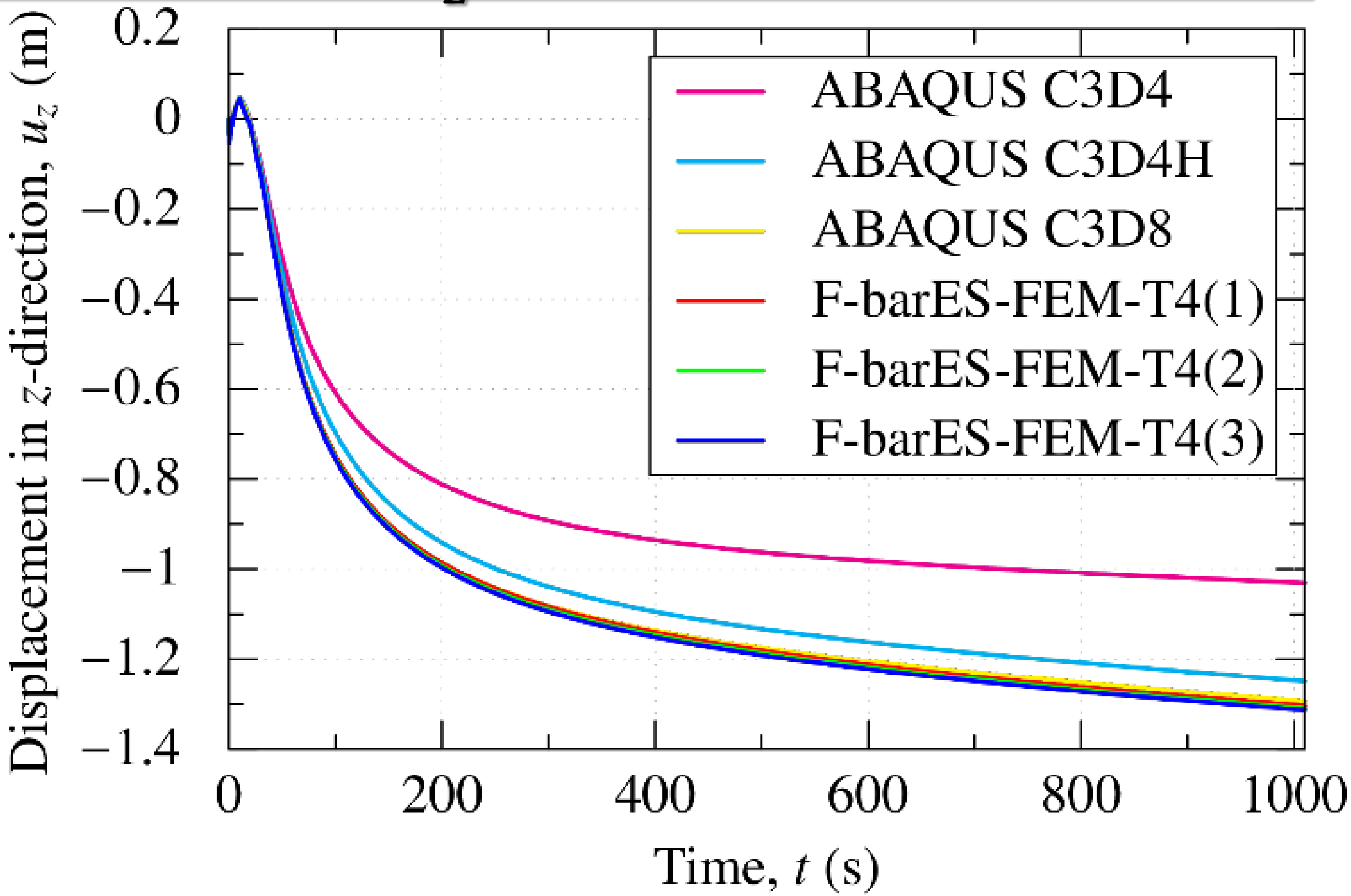




No Checkerboarding!!

Tensile Suspension of Viscoelastic Block

Time histories of u_z at the center of bottom face



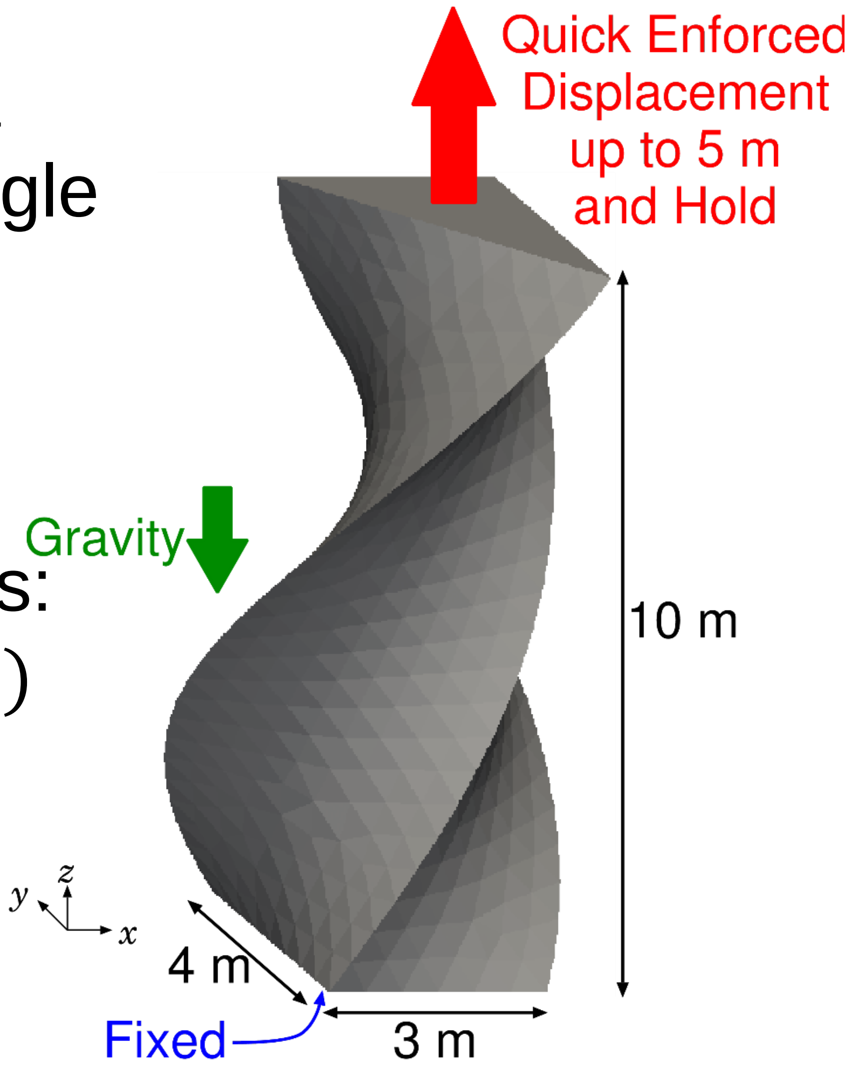
ABAQUS C3D4 and C3D4H have shear locking.

ABAQUS's T4 elements cannot avoid shear locking, whereas F-barES-FEM-T4 has good accuracy as H8-SRI element.

Tensile Drooping of Viscoelastic Twisted Prism

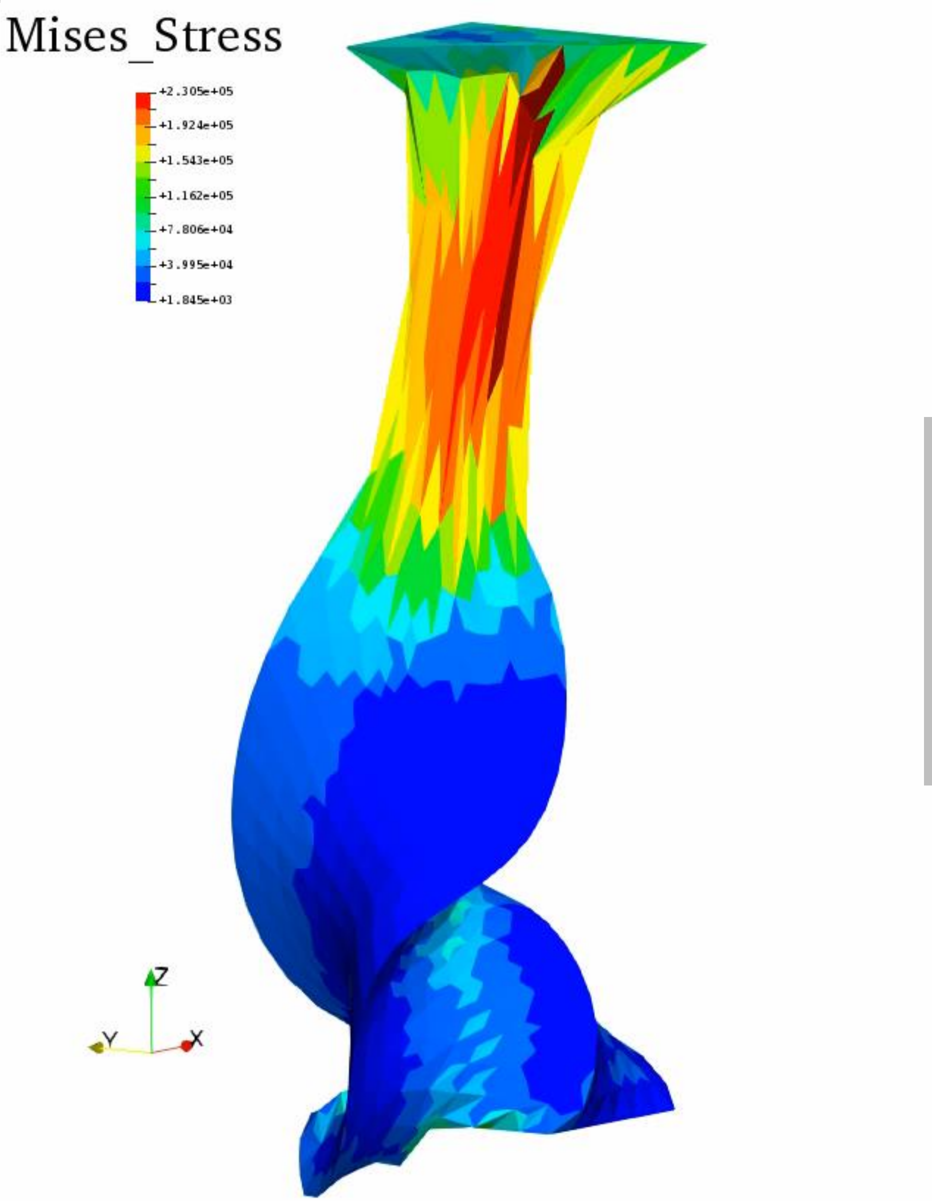
Outline

- 180^{o} twisted prism having a cross-section of a right triangle with 3, 4 and 5 m edges.
- 50% vertical stretch in 10 s and drooping under gravity.
- Same viscoelastic properties: $(\nu_0 = 0.3, \nu_\infty = 0.49, \tau = 10\ s)$
- Solved by ABAQUS C3D4H and F-barES-FEM-T4(2).



Tensile Drooping of Viscoelastic Twisted Prism

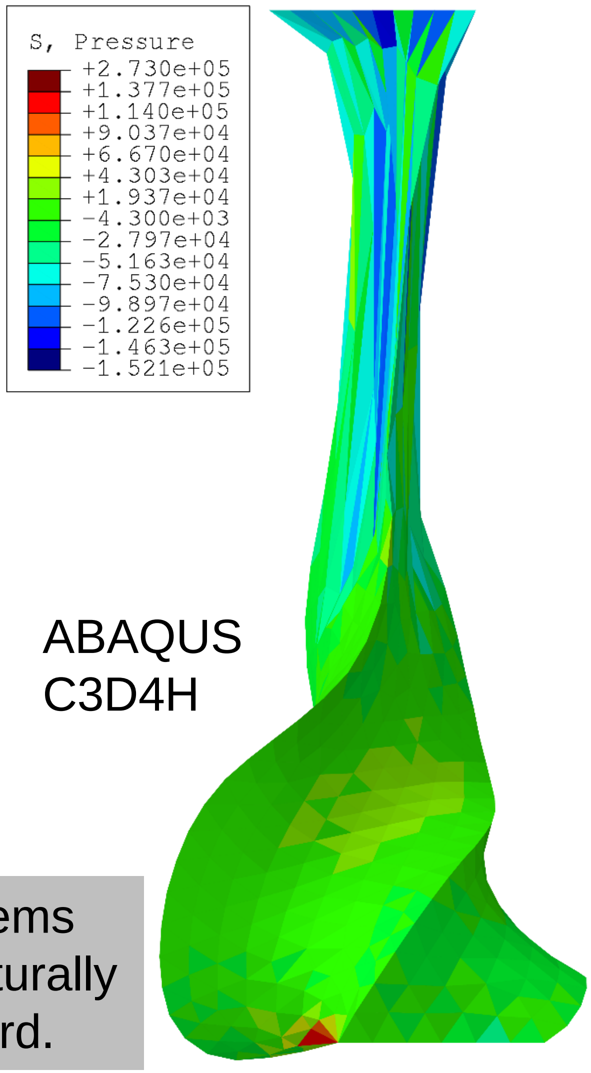
Mises Stress Dist. (F-barES-FEM-T4(2))



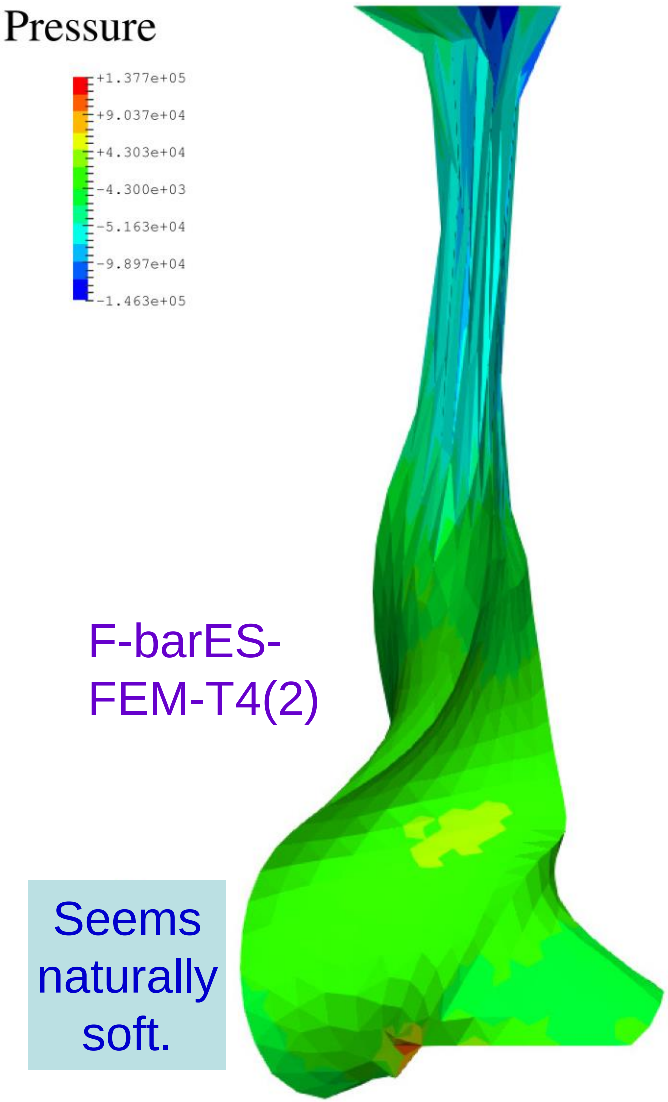
- Continuous stretch in the upper part after the enforced stretch.
- Stress relaxation in the lower part.

Tensile Drooping of Viscoelastic Twisted Prism

Pressure dist. at the final state (common contour range)



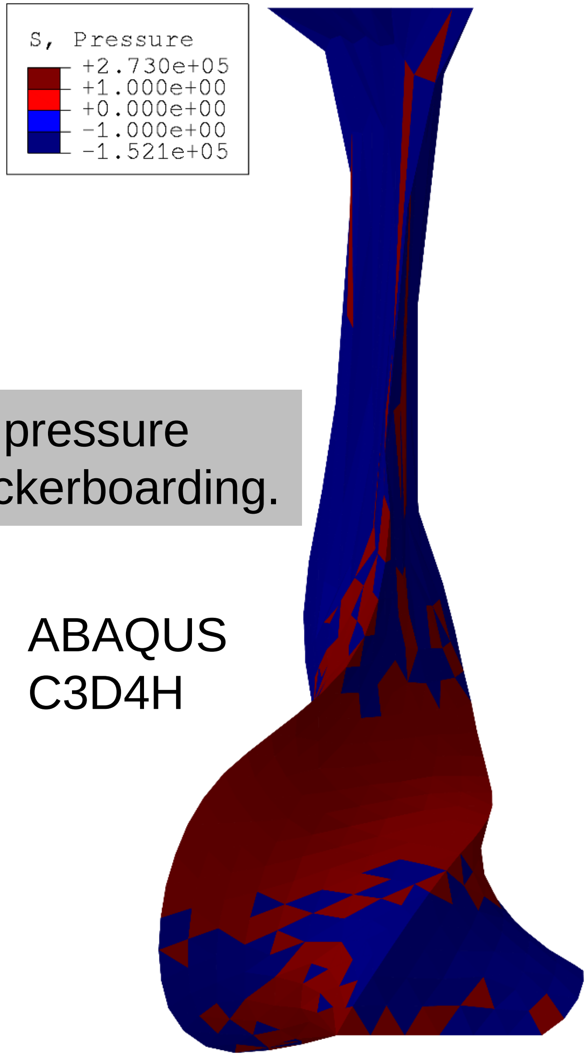
Seems unnaturally hard.



Seems naturally soft.

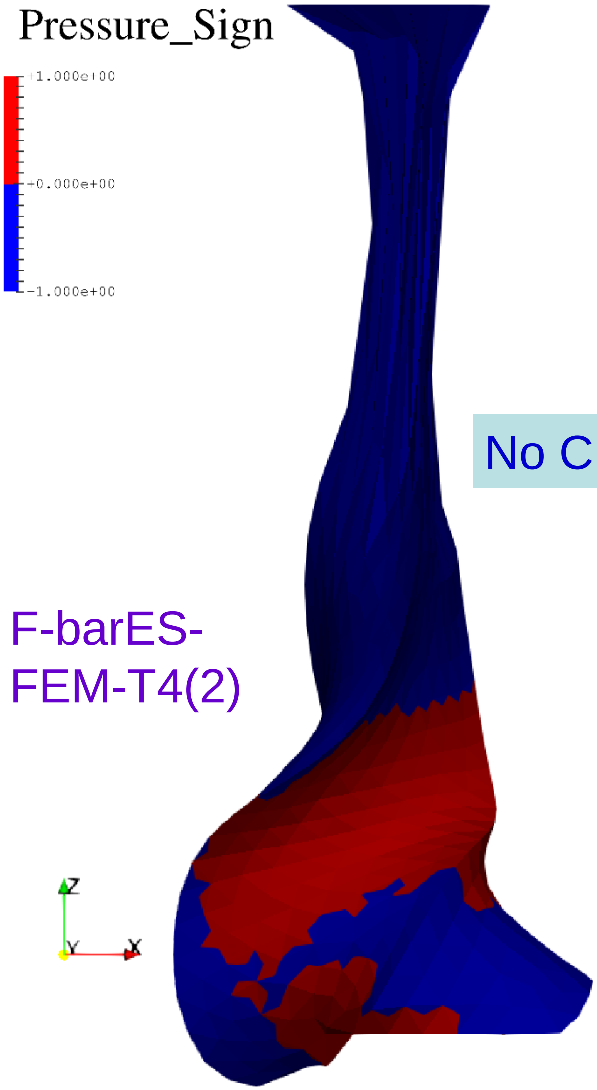
Tensile Drooping of Viscoelastic Twisted Prism

Pressure sign dist. at the final state



pressure checkerboarding.

ABAQUS
C3D4H



No Checkerboarding!!

F-barES-
FEM-T4(2)

Summary

Benefits and Drawbacks of F-barES-FEM-T4

Benefits

- ✓ Locking-free with 1st order tetra meshes.

  No difficulty in severe strain or contact analysis.

- ✓ No increase in DOF.

  Purely displacement-based formulation.

- ✓ Long lasting.
- ✓ Less pressure checkerboarding.

More stable & accurate than other T4 elements!!!

Drawbacks

- ✗ The more cyclic smoothing necessitates the more CPU time due to the wider bandwidth.

Slower than other T4 elements…

Take-Home Messages

F-barES-FEM-T4 is the current best T4 FE formulation for the large deformation with near incompressibility:

- Rubber-like materials,
- Elastoplastic materials, and
- Viscoelastic materials.

Thank you for your kind attention!

Appendix

Characteristics of [K] in F-barES-FEM-T4

✓ No increase in DOF.
(No Lagrange multiplier. No static condensation.)

✓ Positive definite.

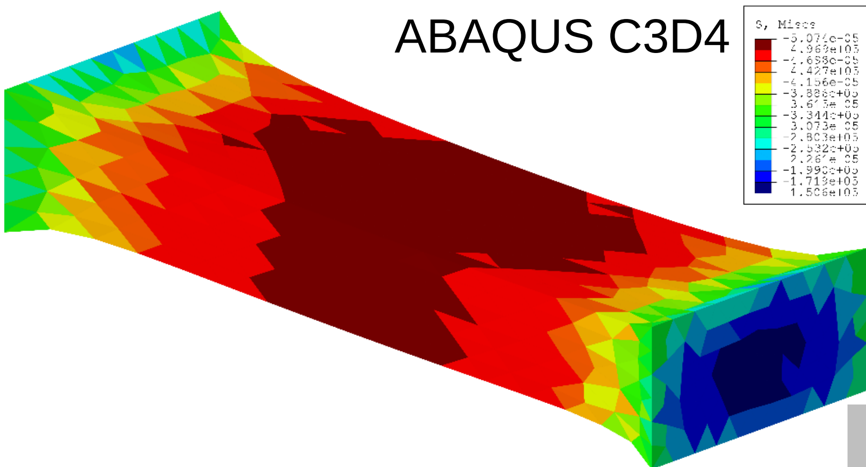
✗ Wider bandwidth.
In case of standard unstructured T4 meshes,

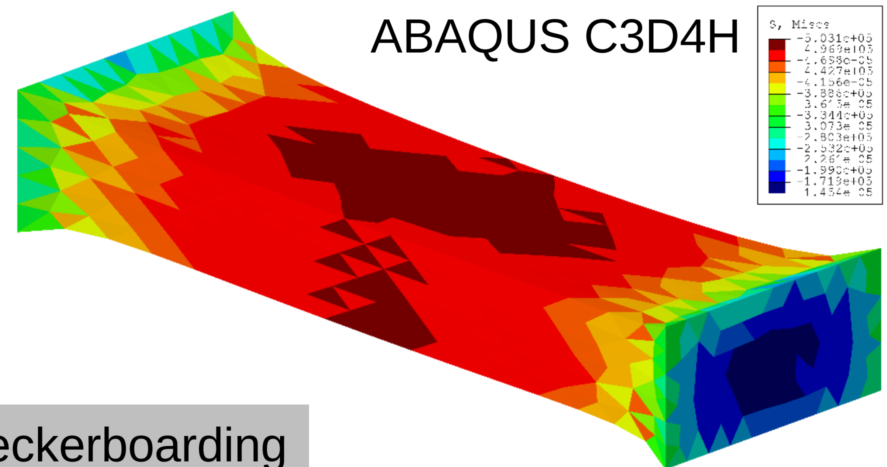
Method	Approx. Bandwidth	Approx. Ratio
Standard FEM-T4	40	1
F-barES-FEM-T4(1)	390	x10
F-barES-FEM-T4(2)	860	x20
F-barES-FEM-T4(3)	1580	x40
F-barES-FEM-T4(4)	2600	x65

✗ Ill-posedness in nearly incompressible cases.
(No improvement in condition number.)

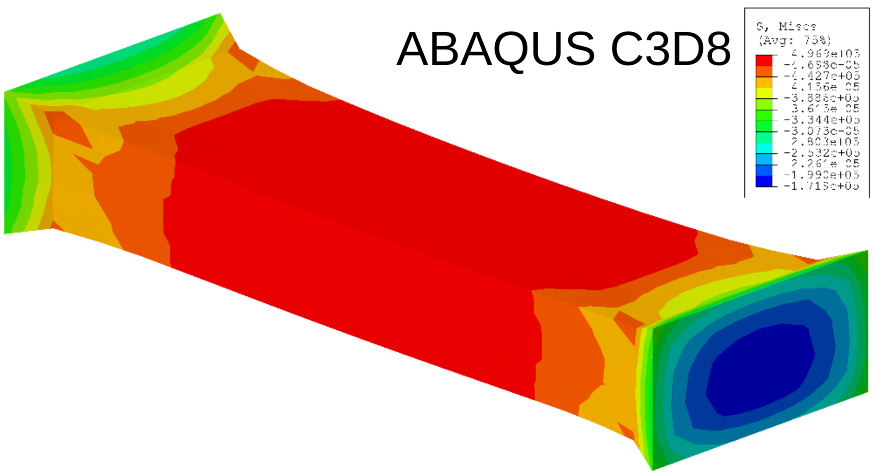
Tensile Suspension of Viscoelastic Block

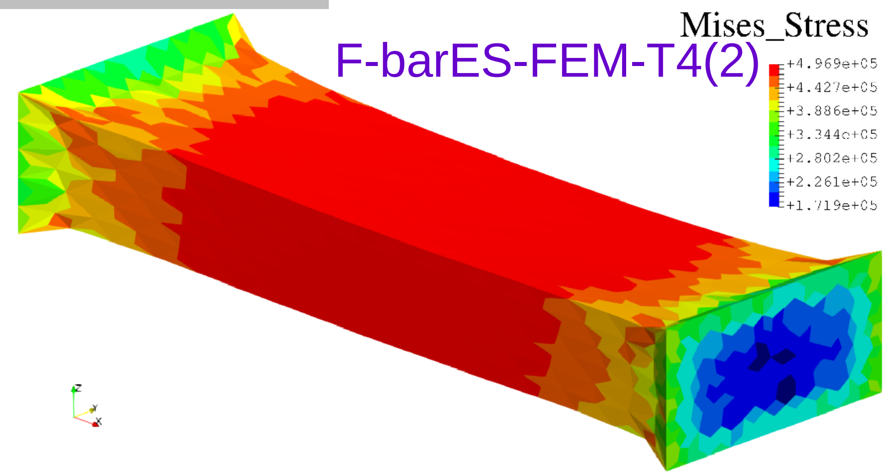
Mises stress at the end of stretch (common contour range)





Checkerboarding





Tensile Suspension of Viscoelastic Block

***Mises stress** at the final state* (common contour range)