A Novel Smoothed Finite Element Formulation Based on the Implicit Incremental Equilibrium Equation for Large Deformation Analysis with Mesh Rezoning

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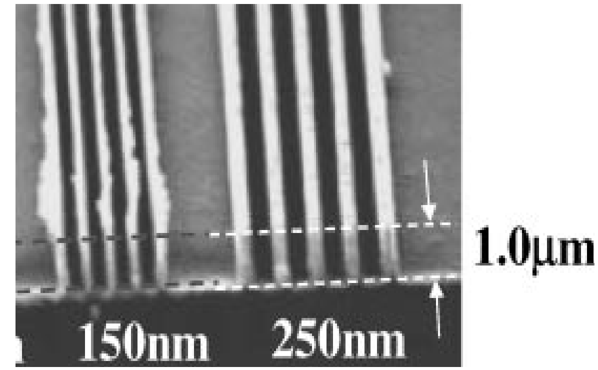
Motivation and Background

Motivation

We want to solve **severely large deformation** problems **accurately and stably**!
(Final target: thermal nanoimprinting)

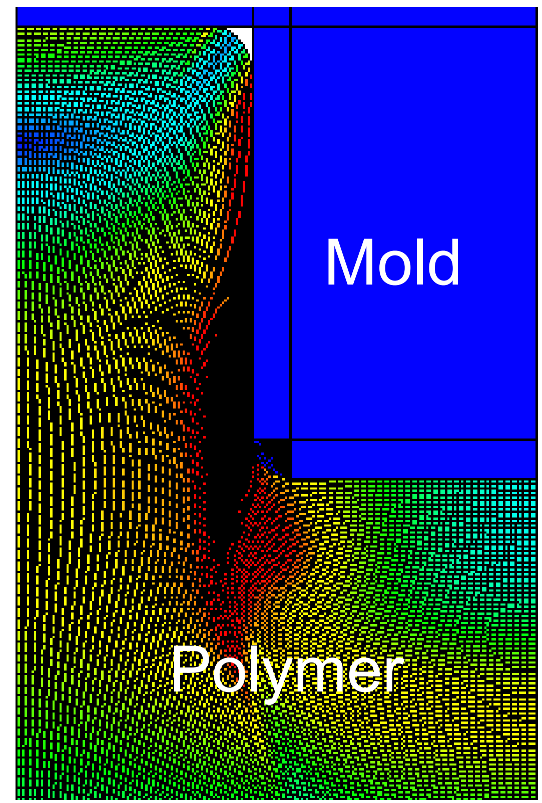


Background

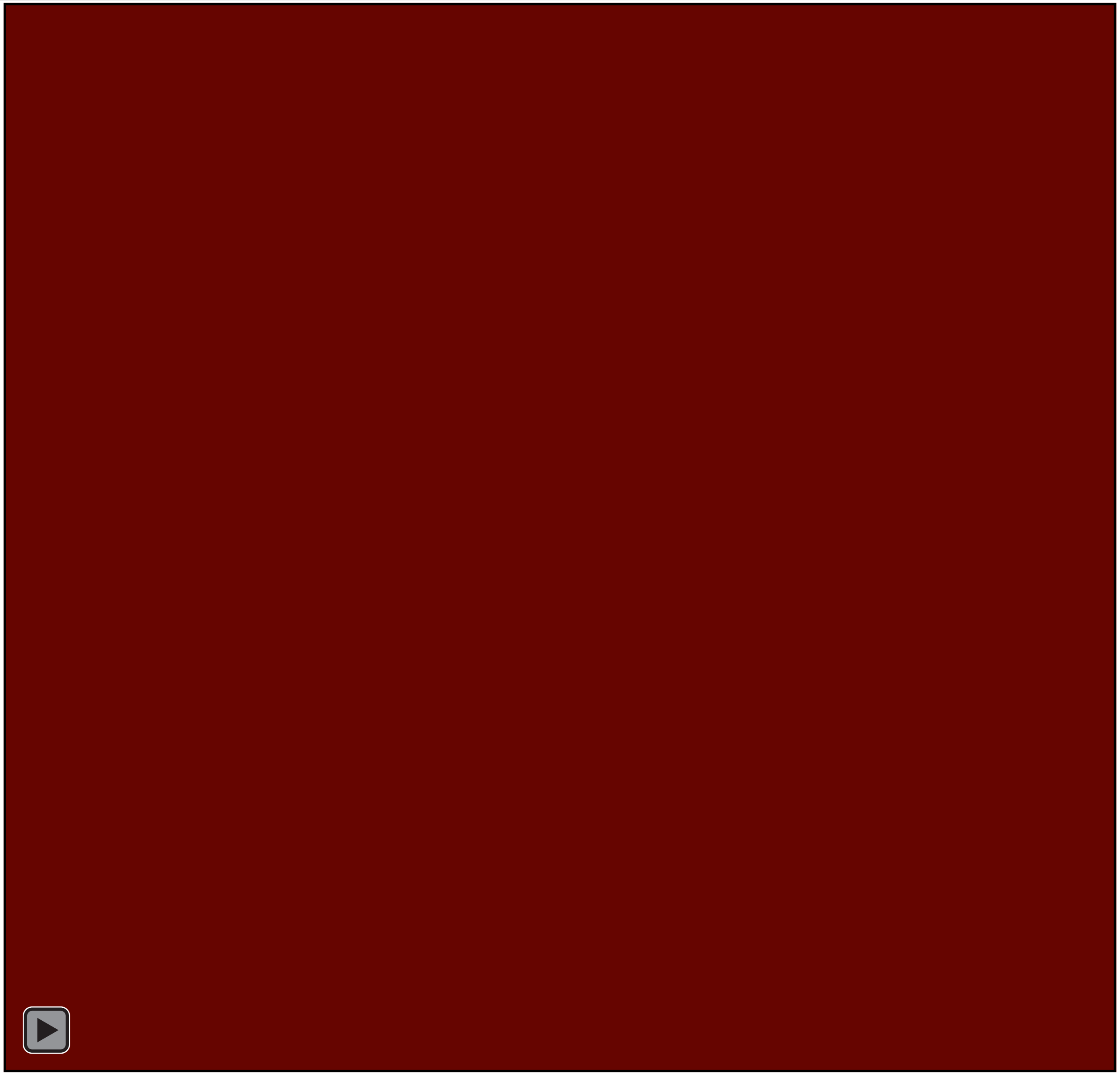
Finite elements are **distorted** in a short time, thereby resulting in convergence failure.

⇩

Mesh rezoning method (*h*-adaptive mesh-to-mesh solution mapping) is indispensable.



Our First Result in Advance

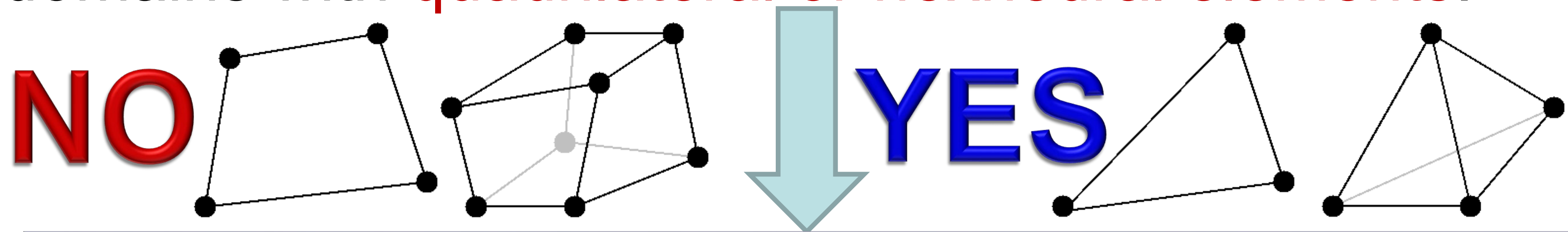


static-implicit
large deformation
analysis
with
mesh rezoning

2 Major Problems in Mesh Rezoning

Problem 1: accuracy

It is impossible to remesh arbitrary deformed 2D or 3D domains with quadrilateral or hexhedral elements.



We have to use triangular or tetrahedral elements...

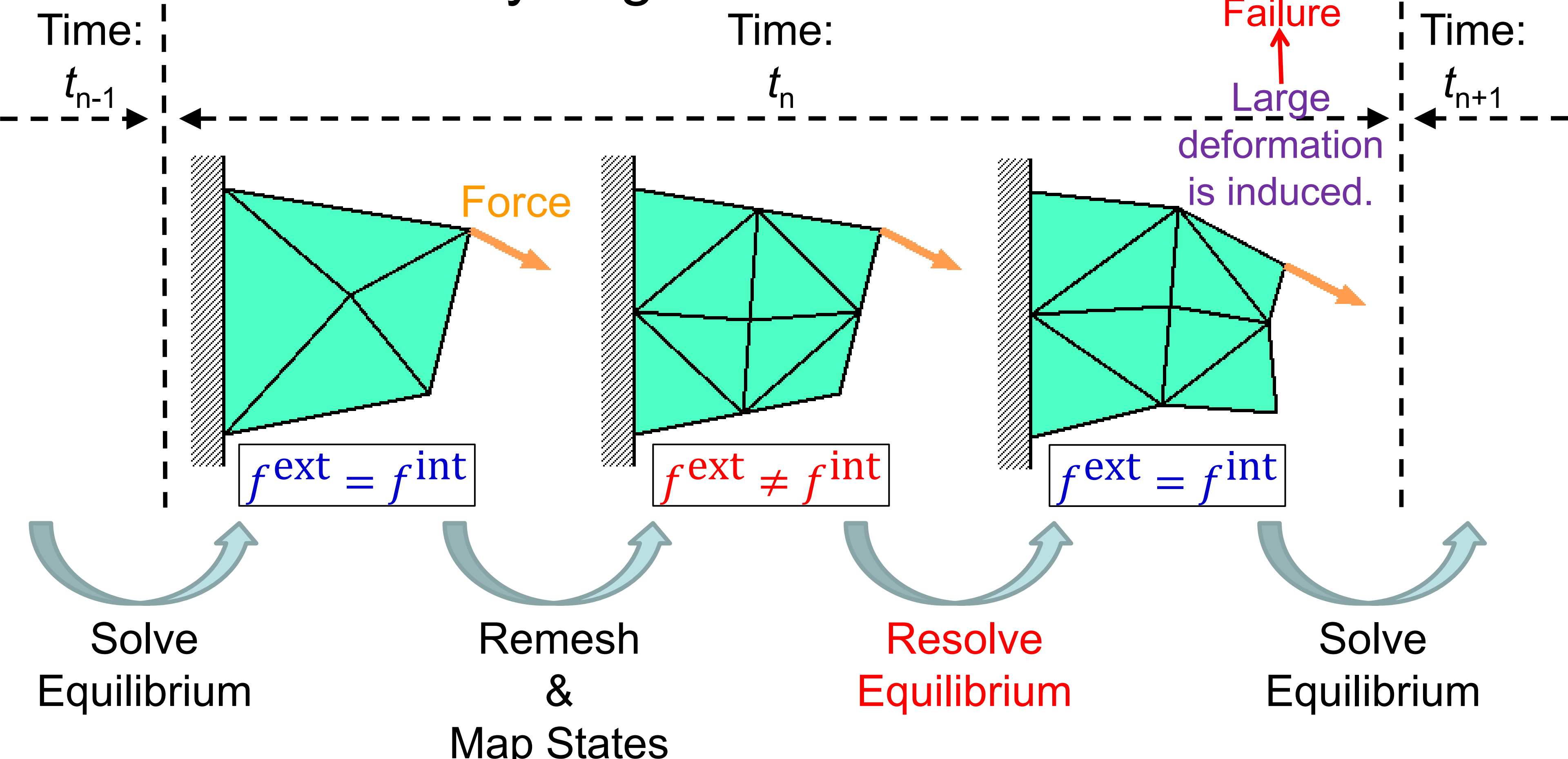
However, the *standard* (constant strain) triangular or tetrahedral elements induce shear and volumetric locking easily, which leads to inaccurate results.

- Higher order elements are not effective in large deformation.
- Neither B-bar nor F-bar is applicable to triangular/tetrahedral.
- EAS is unstable. Mixed/Hybrid formulation is complicated.

2 Major Problems in Mesh Rezoning

Problem 2: stability

The resolving process in mesh rezoning sometimes induces extremely large deformation.



Our Ideas

Idea for accuracy improvement

We adopt smoothed finite element method (S-FEM) to avoid shear and volumetric locking even with use of triangular or tetrahedral elements.

Idea for stability improvement

We adopt the incremental implicit equilibrium equation (IIEE) as the equation to solve.

In this talk today,
I focus on Problem 1 and the idea of S-FEM.

Objective

Develop an accurate
mesh rezoning method
for large deformation problems
with our modified S-FEM formulation

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- Part 2: Procedure of our mesh rezoning method
- Part 3: Examples analysis
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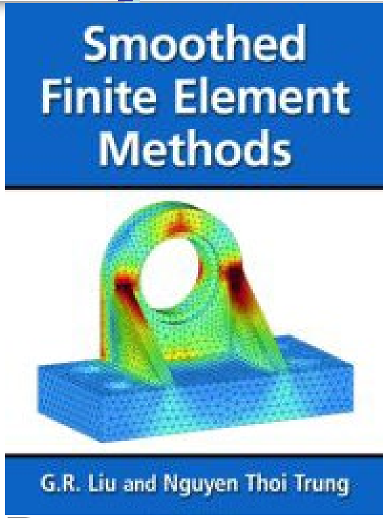
Part 1:

Introduction of Our Modified S-FEM Formulation

What is Smoothed FEM (S-FEM)?



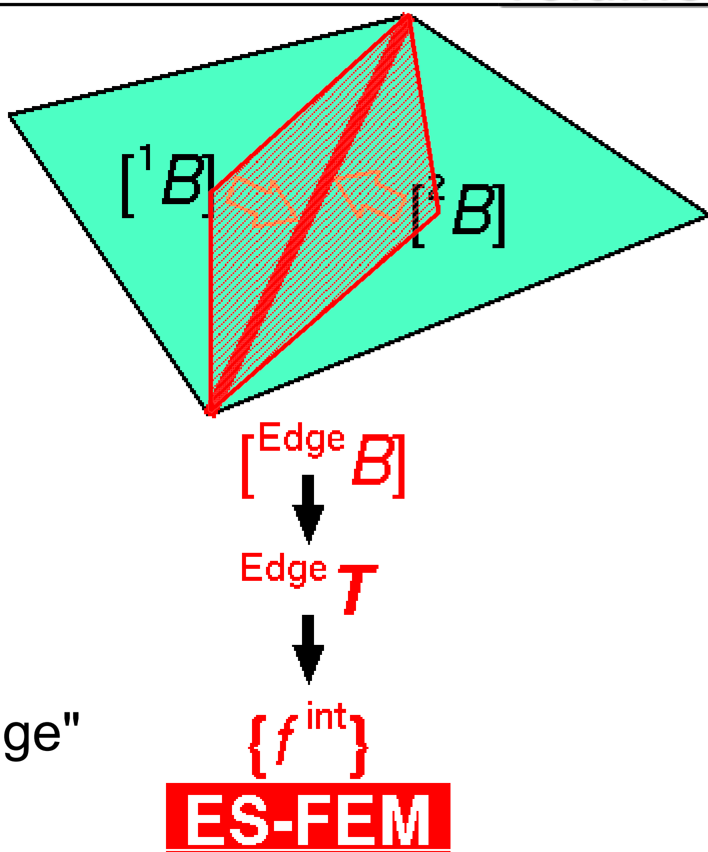
- One of strain smoothing techniques.
- There are several types of S-FEM.
  - Edge-based (ES-FEM) for 2D
  - Face-based (FS-FEM) for 3D
  - Node-based (NS-FEM) for both 2D and 3D
  - Selective edge/node-based (ES/NS-FEM) for 2D
  - Selective face/node-based (FS/NS-FEM) for 3D, etc..
- Selective S-FEMs are thought to be the best choice because they can avoid both shear and volumetric locking even with use of triangular or tetrahedral elements.

I will explain ES-FEM, NS-FEM,
and selective ES/NS-FEM one by one.

Edge-based S-FEM (ES-FEM)

- Calculate $[B]$ at element as usual.
- Distribute $[B]$ to the connecting edges and make $[\,^{\mathrm{Edge}}B]$.
- ***F*, *T*** etc and $\{f^{\mathrm{int}}\}$ are calculated on smoothed edge domains.

<u>Generally accurate but induces volumetric locking.</u>



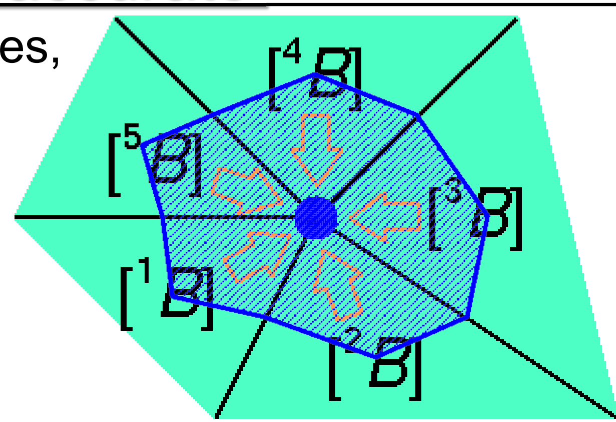
Substituting "face" for "edge" gives FS-FEM for 3D

Node-based S-FEM (NS-FEM)

- Calculate $[B]$ at element as usual.
- Distribute $[B]$ to the connecting nodes and make $[{}^{\mathrm{Node}}B]$
- ***F*, *T*** etc and $\{f^{\mathrm{int}}\}$ are calculated on smoothed node domains.

Generally not accurate but volumetric locking free.

(due to zero-energy modes, which are arisen in reduced integration finite elements as hour-glass modes)



close to FVM with vertex-based control volume

$[{}^{\mathrm{Node}}B]$

↓

${}^{\mathrm{Node}}T$

↓

$\{f^{\mathrm{int}}\}$

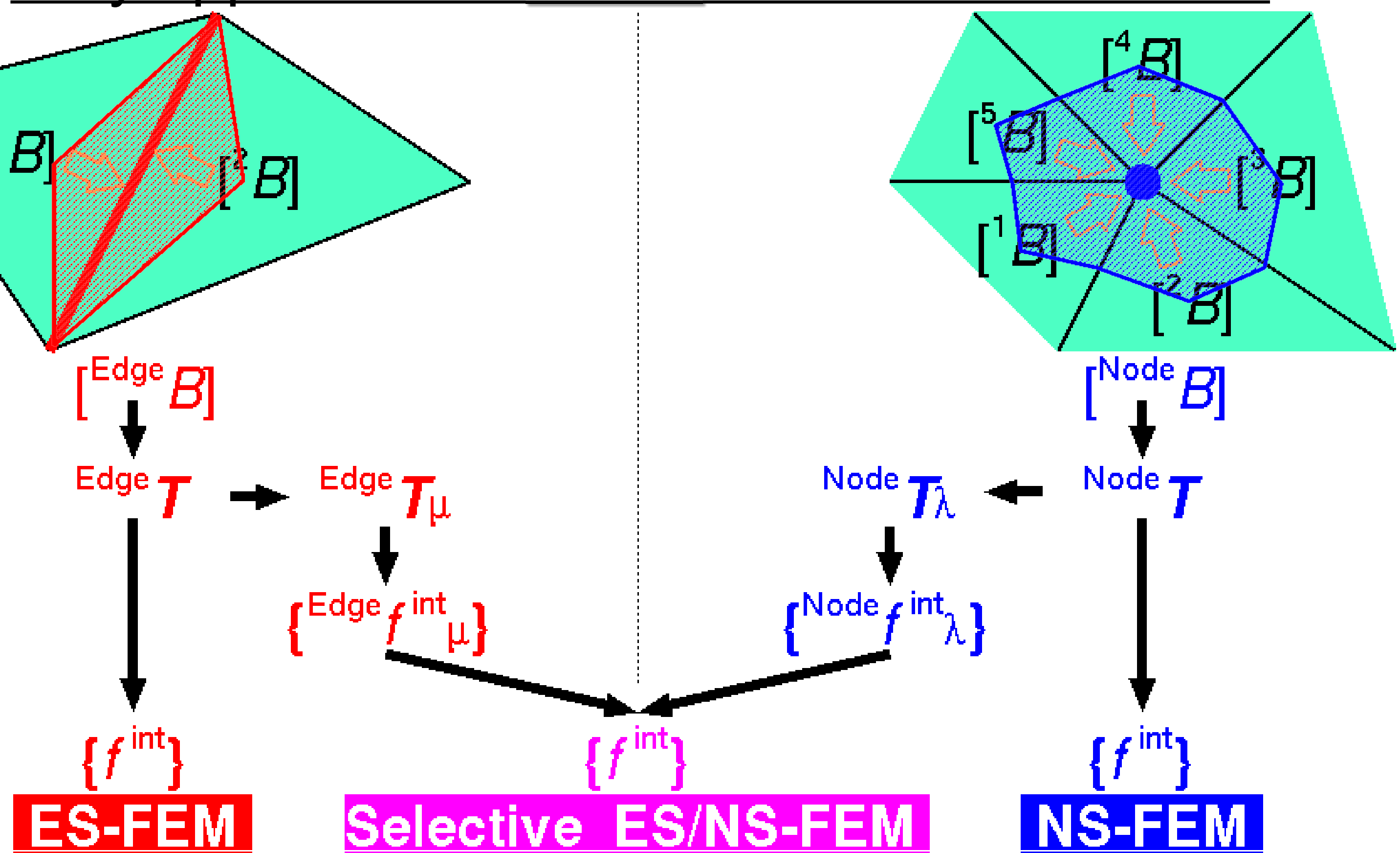
NS-FEM

Original Selective ES/NS-FEM

- Separate stress into "μ part" and "λ part", where μ and λ are the Lame's parameters.
- ***F*, *T*** etc and $\{f^{\text{int}}\}$ are calculated on both smoothed domains.

<u>Only applicable to elastic constitutive models.</u>

Substituting "face" for "edge" gives FS/NS-FEM for 3D

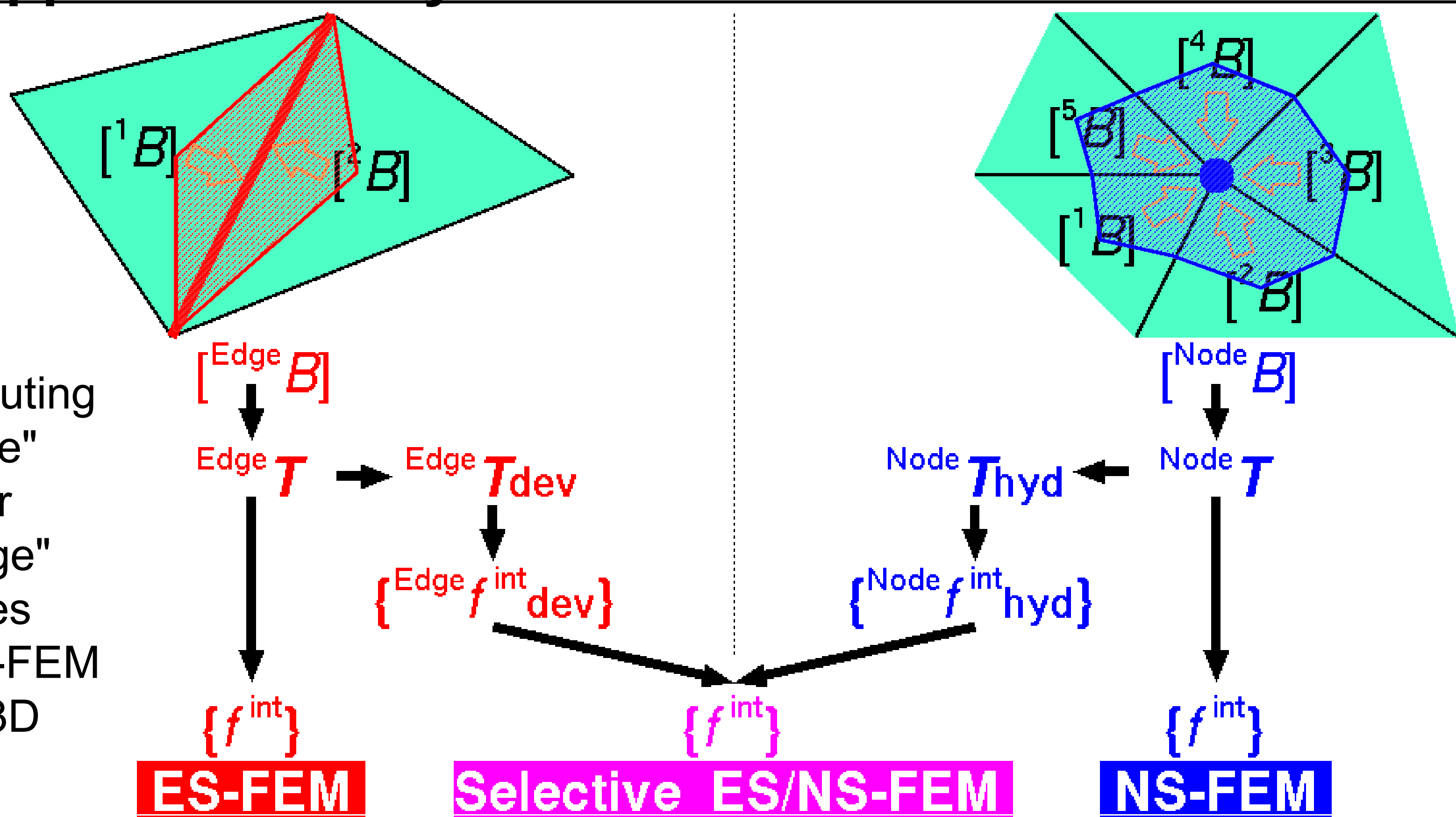


Our Selective ES/NS-FEM

- Separate stress into "deviatoric part" and "hydrostatic part" instead of "μ part" and "λ part".
- ***F*, *T*** etc and $\{f^{\text{int}}\}$ are calculated on both smoothed domains.

Applicable to any kind of material constitutive models.

Substituting "face" for "edge" gives FS/NS-FEM for 3D



Verification of Our Selective S-FEM

Cantilever Bending Test

- 10m x 1m x 1m cantilever with 20 kN dead load
- Neo-Hookean hyperelastic material

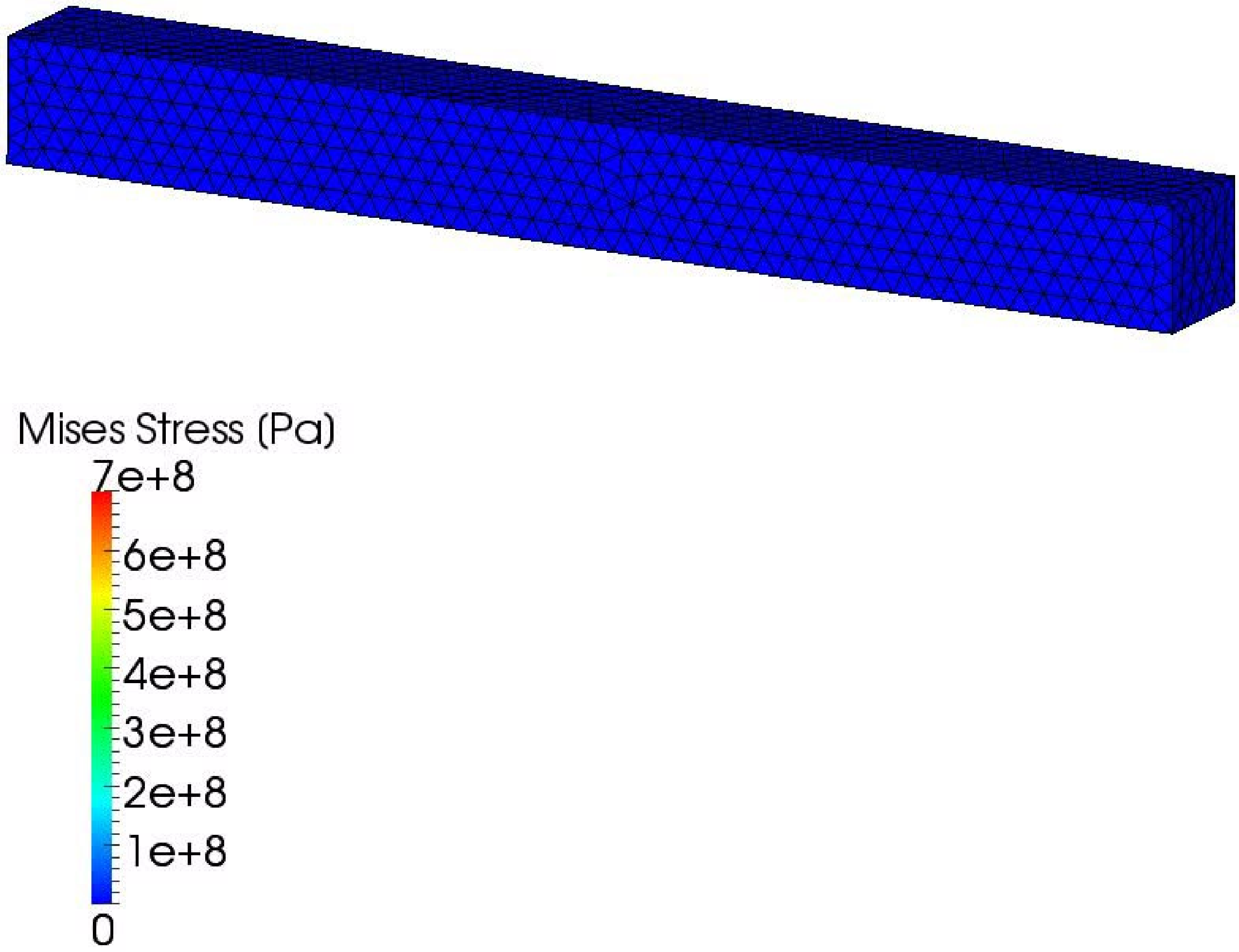
$$[T] = 2C_{10}\frac{\mathrm{Dev}(\bar{B})}{J} + \frac{2}{D_1}(J-1)[I]$$

with a constant C_{10}(=1 GPa) and various D_1s.

- Our selective FS/NS-FEM with 9560 tetrahedral elements (and 2288 nodes) is performed.
- ABAQUS/Standard with 1250 *C3D20H* (2nd-order hybrid hexahedral) elements (and 6696 nodes) is also performed.
- No mesh rezoning is taken place for this test.

Verification of Our Selective S-FEM

Results with $D_1 = 2\ \mathrm{PPa}^{-1}$ (ν_0=0.499999)

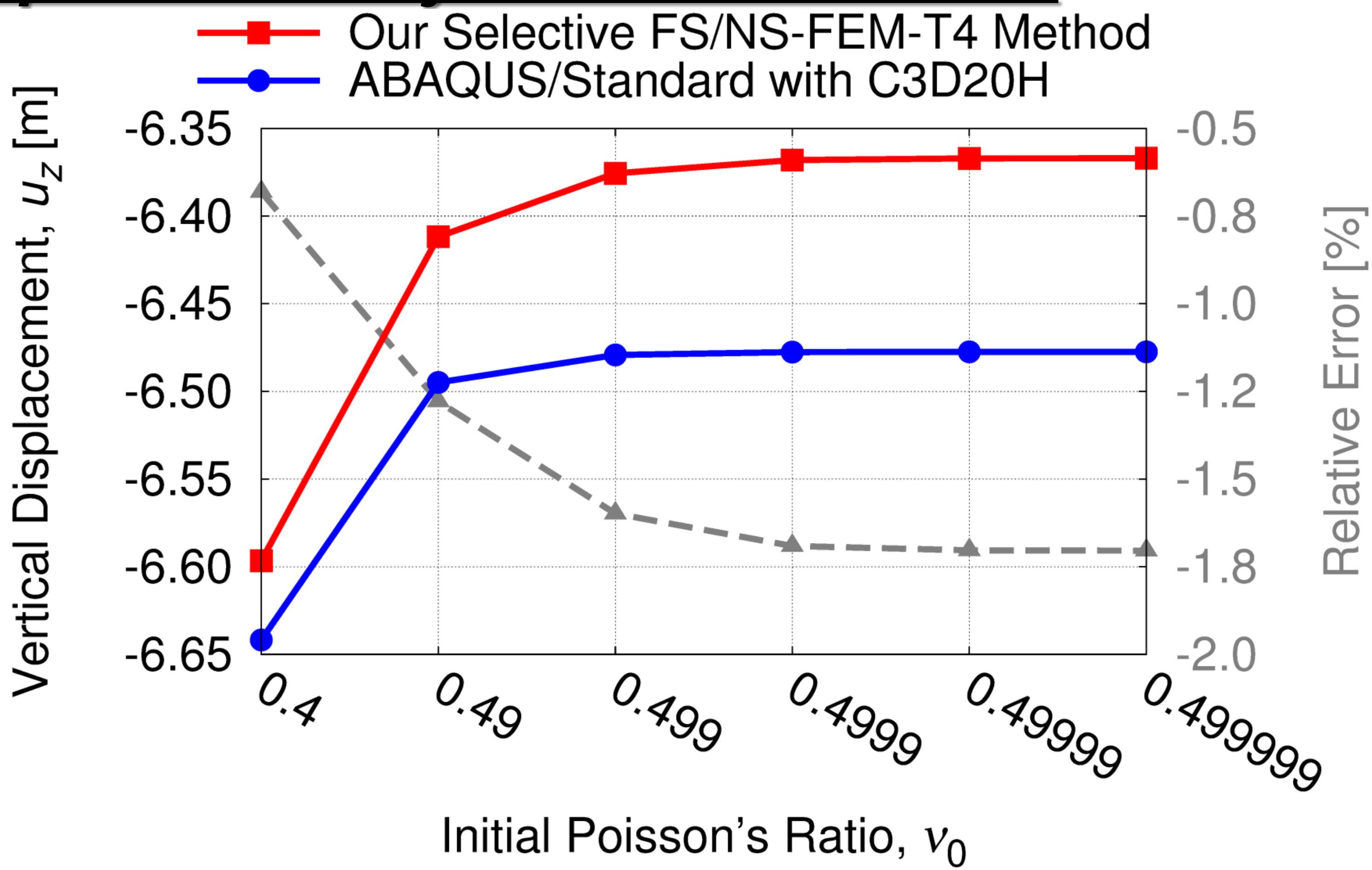


The amount of vertical deflection is about 6.5 m.

If we use constant strain tetrahedral, the amount of vertical deflection is about only 0.1 m.

Verification of Our Selective S-FEM

Comparison to Hybrid Element Case



Our selective S-FEM can treat any material model and is free from locking!!

Characteristics of Our S-FEM

Advantage

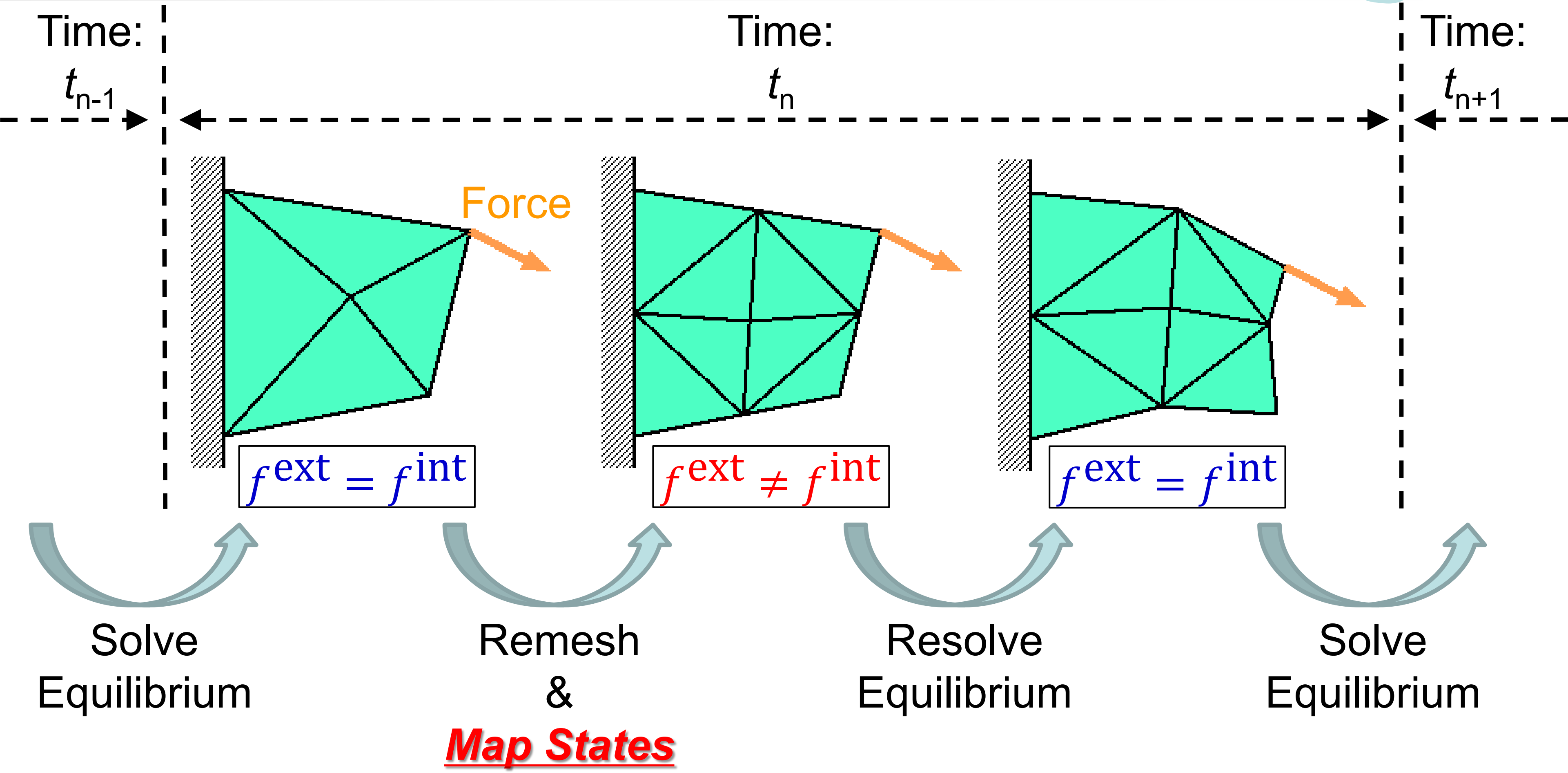
- Locking free
- no increase of DOF
  (The unknown is displacement vector $\{u\}$ only.)
- easy to implement

Disadvantage

- increase of matrix band width
- cannot treat perfectly incompressible material
- no smoothing effect with super coarse mesh

Part 2:
Procedure of our mesh rezoning method

Procedure of Mesh Rezoning



The way of mapping varies with the material constitutive model.
(e.g. Elasto-plastic models necessitate some kind of correction.)

Mapping of Stress/Strain States

For *Elastic or Hyperelastic* Materials

i.e., $[T] = [T([F])]$

- Map initial position $\{x^{\text{initial}}\}$ at nodes, and then remake deformation gradient $[F]$ at edges & nodes.

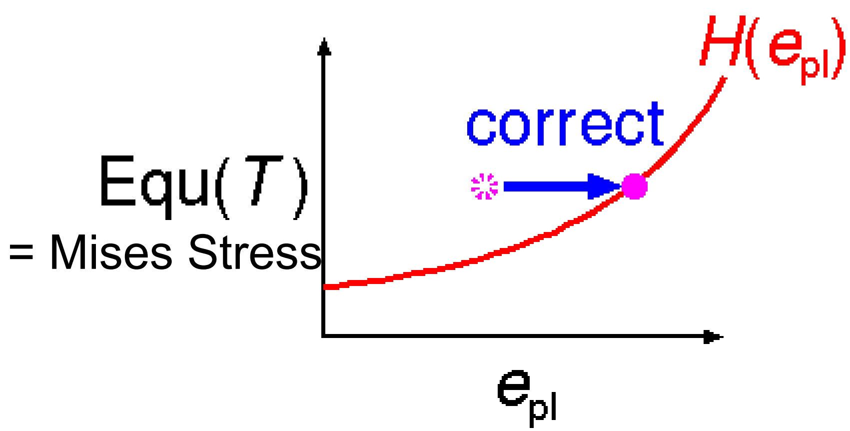
Each node preserve its initial position so that the domain can spring back to the initial shape after unloading.

Mapping of Stress/Strain States

For Elasto-Plastic Material in Total Strain Form

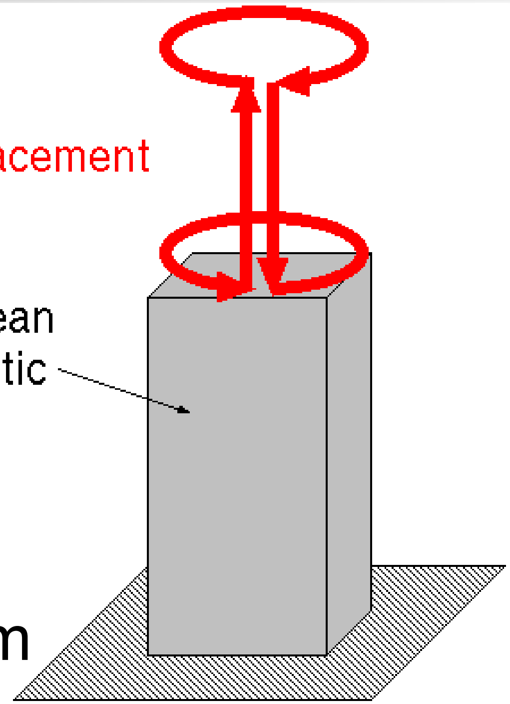
$$\text{e.g., } [T] = [T([F], [E_{\mathrm{pl}}], e_{\mathrm{pl}}; H(e_{\mathrm{pl}}))]$$

- Map initial position $\{x^{\mathrm{initial}}\}$ at nodes, and then remake deformation gradient $[F]$ at edges & nodes.
- Map history dependent variables, plastic strain $[E_{\mathrm{pl}}]$ and equivalent plastic strain e_{pl}.
- Correct e_{pl} to satisfy $\mathrm{Equ}([T]) = H(e_{\mathrm{pl}})$



Part 3:
Examples of Analysis

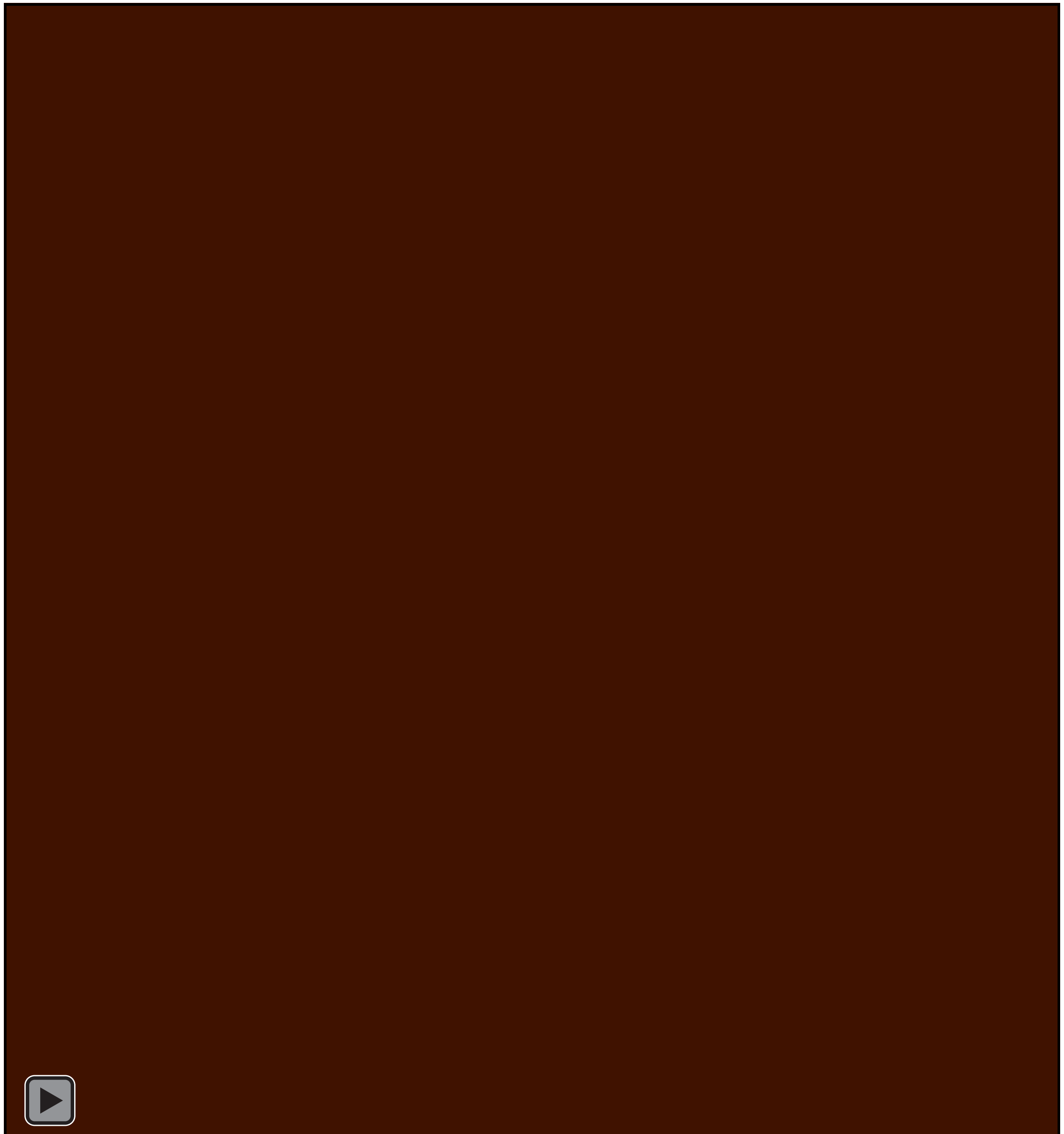
Twist and Stretch of Hyperelastic Body



- Static, 1 m x 2 m x 4 m
- Neo-Hookean hyperelastic body of $C_{10} = 1\ \mathrm{GPa}$ and $D_1 = 400\ \mathrm{GPa}^{-1}$ ($\nu_0 = 0.48$)
- Twist up to 360 deg. $\Longrightarrow$ Stretch up to 100% nominal strain $\Longrightarrow$ Twist back $\Longrightarrow$ Shrink back
- Our selective FS/NS-FEM with tetrahedral elements
- Global mesh rezoning every 90 deg. and 50% stretch/shrink

Twist and Stretch of Hyperelastic Body

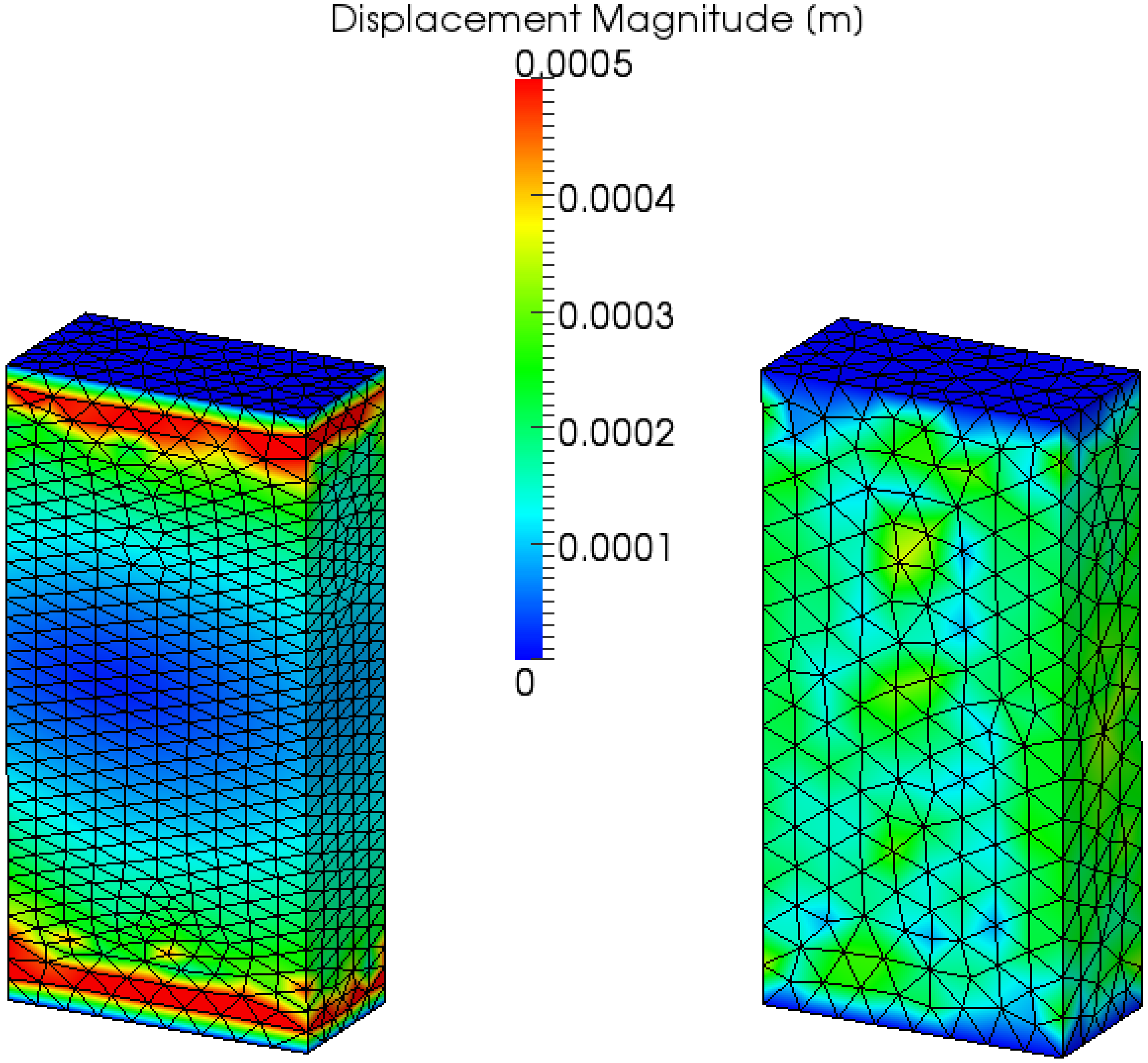
Our selective FS/NS-FEM with mesh rezoning



Our selective FS/NS-FEM without mesh rezoning

Twist and Stretch of Hyperelastic Body

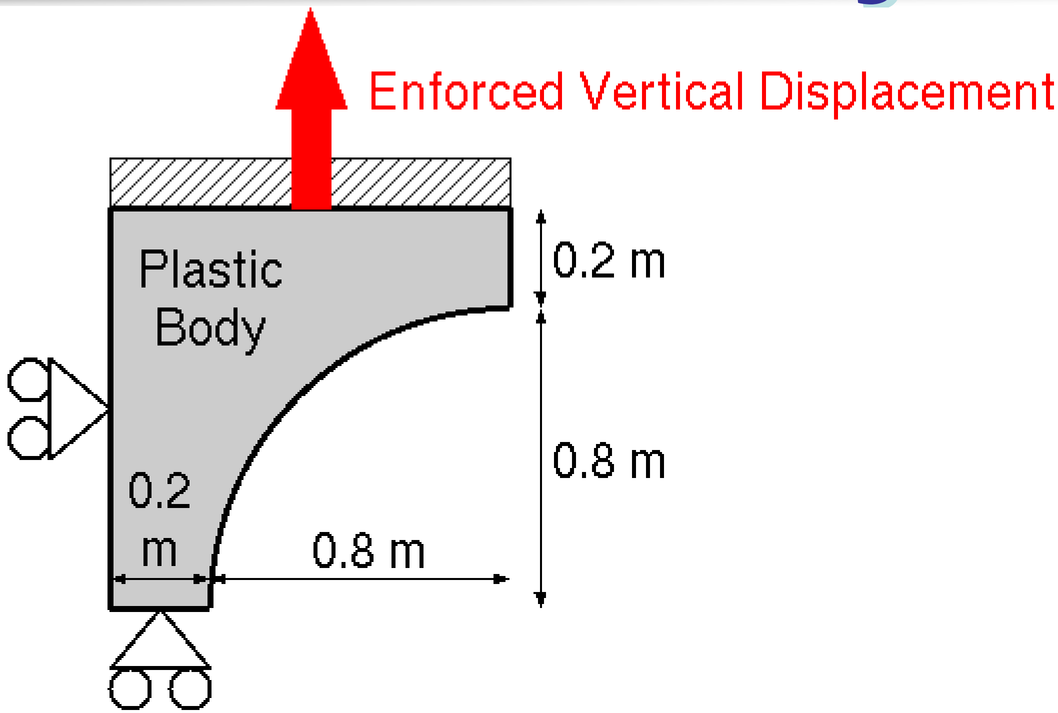
Residual Displacement



Necking of Elasto-Plastic Body

Outline

- Static, Plane-strain
- 1/4 of test piece
- horizontal constraint on left edge
- vertical constraint on bottom edge



- horizontal constraint and enforced displacement on top edge
- Mesh rezoning every 0.05 m displacement
- Our selective ES/NS-FEM with triangular elements

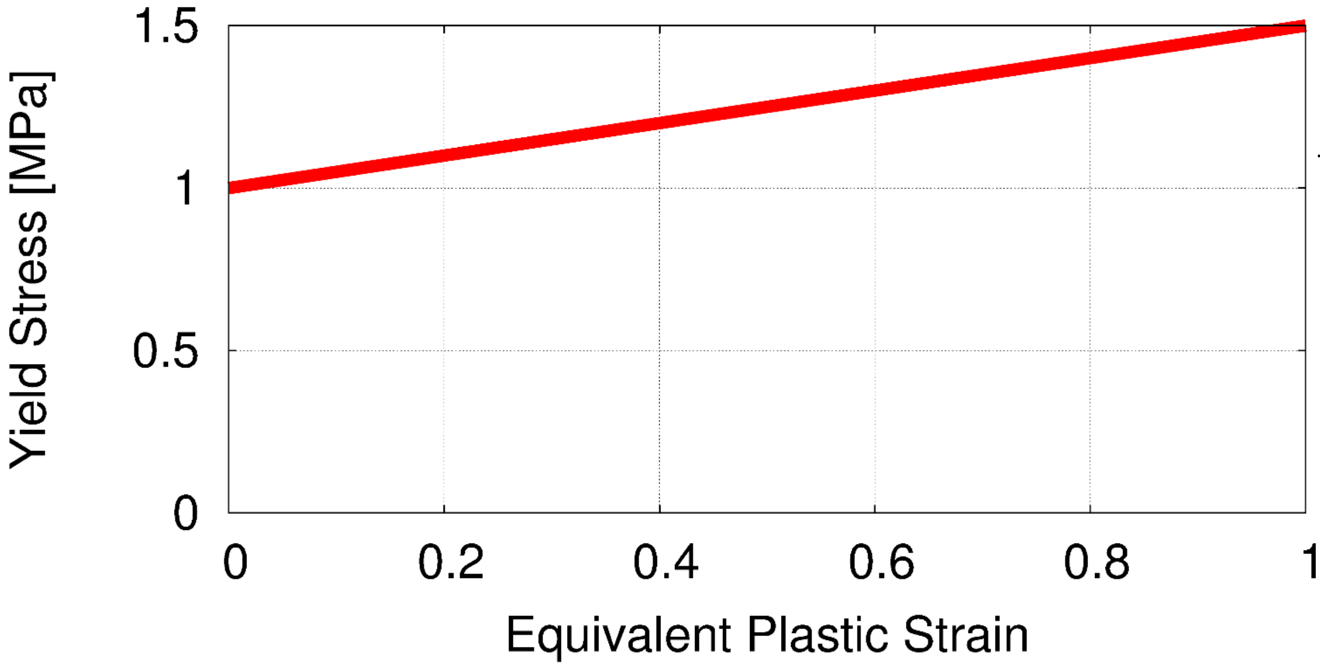
Necking of Elasto-Plastic Body

Material Constitutive Model

Hencky's elasto-plastic material, $\boldsymbol{T} = \boldsymbol{C} : \boldsymbol{h}_{\mathrm{el}}/J$,
with von Mises yield criterion and isotropic hardening.

Young's Modulus: 1 GPa, Poisson's Ratio: 0.3,

Yield Stress: 1 MPa, Hardening Coeff.: 0.5 MPa.



very close to **elastic-perfect plastic** model

Necking of Elasto-Plastic Body

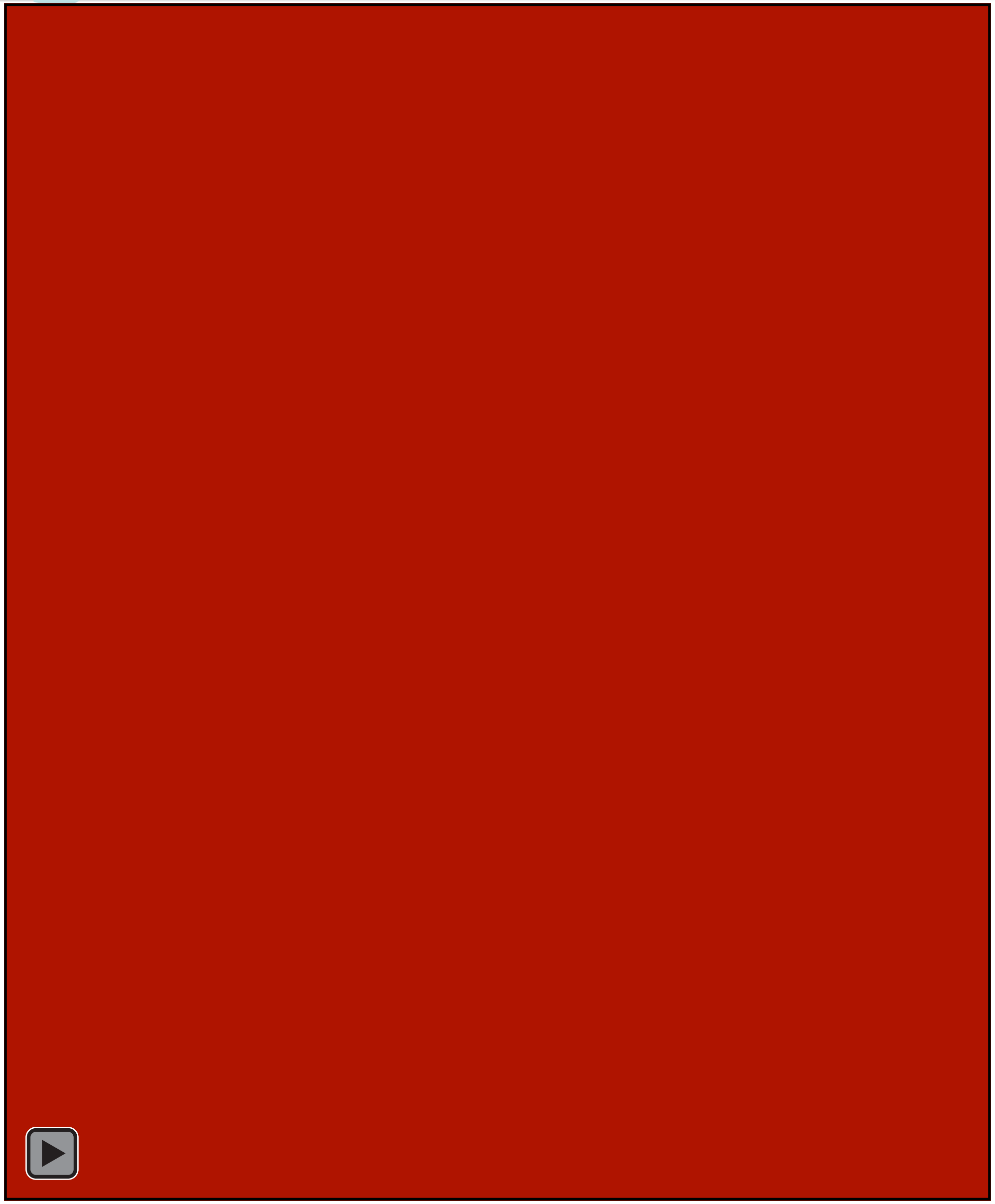
ABAQUS /Standard with constant strain triangular elements

Strange deformation is obtained due to:

- locking of triangular elements
- absence of mesh rezoning

Necking of Elasto-Plastic Body

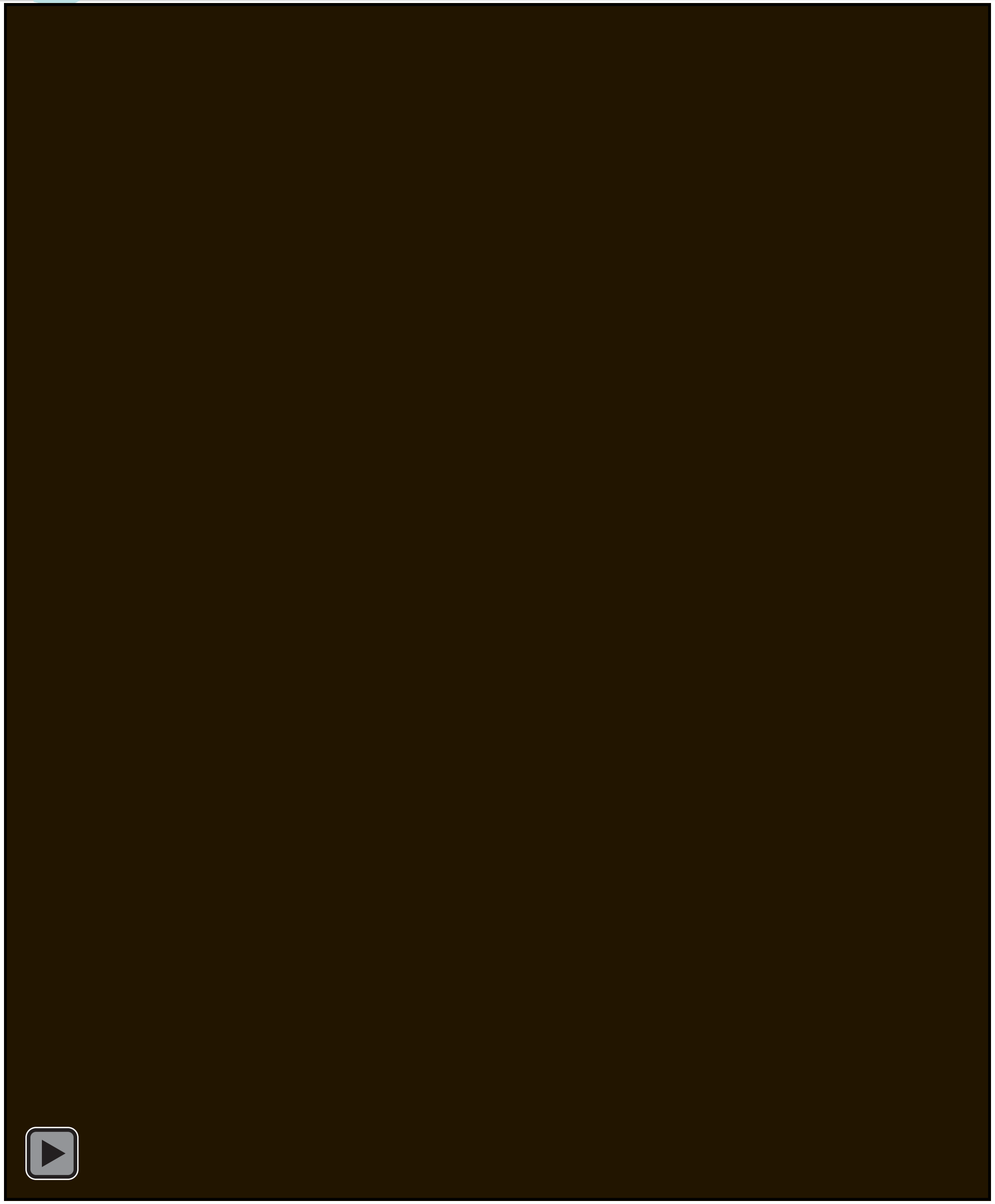
Our selective ES/NS-FEM with mesh rezoning



Typical deformation of necking test is obtained.

Necking of Elasto-Plastic Body

Zoom-in view around the center



Summary

Take-Home Messages

1. Our modified selective S-FEM with triangular or tetrahedral elements is locking free and very easy to implement.

2. Our S-FEM goes well together with mesh rezoning.

3. Our S-FEM is worth using even without mesh rezoning.

Summary and Future Work

Summary

- A new static-implicit mesh rezoning method for severely large deformation analysis is proposed.
- It adopts our modified selective S-FEM, which separates stress into deviatoric part and hydrostatic part.
- Its accuracy are verified with hyperelastic material and elasto-plastic material.

Future Work

- More V&V
- Local mesh rezoning
- Apply to contact forming, crack propagation, etc.

Thank you for your kind attention.
I appreciate your question in slow and easy English!!