Performance of Smoothed Finite Element Methods with Tetrahedral Elements in Large Deformation Elasto-Plastic Analysis

Yuki ONISHI
Tokyo Institute of Technology, Japan

Motivation

Motivation

We want to accurately and stably analyze **severe large deformation** of solids in **any shape** with finite elements.

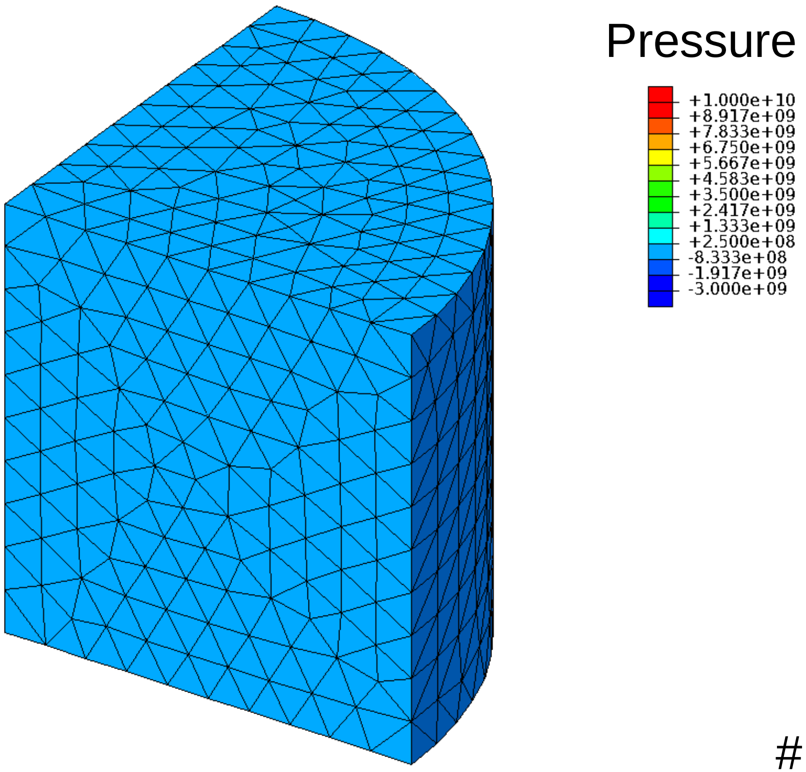
Issues

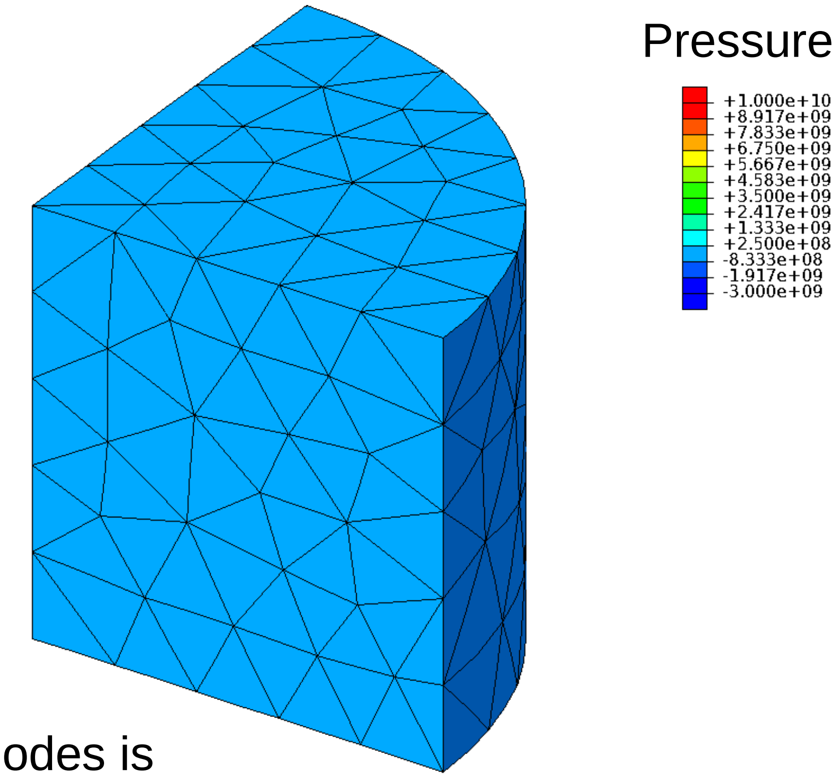
- Severe large deformation of arbitrary body
  ⟹ No Hex mesh. **Only Tet** mesh.
- 1st order standard tetrahedral (constant strain) element
  (e.g. C3D4 in ABAQUS) for materials with **incompressibility**
  ⟹ Shear/volumetric locking and pressure oscillation.
- 2nd order u/p hybrid tetrahedral element
  (e.g., C3D10H, C3D10MH in ABAQUS)
  ⟹ Low accuracy in severe large deformation.
  Convergence difficulty in contact.

Researches on FE formulations for 1st order tetra (T4) are still active especially for rubber-like or elasto-plastic materials.

An Example for Rubber-like Material

Material: neo-Hookean **hyperelastic**, $\nu_{\mathrm{ini}} = 0.49$





# of Nodes is almost the same.

<u>1st order hybrid T4 (C3D4H)</u>

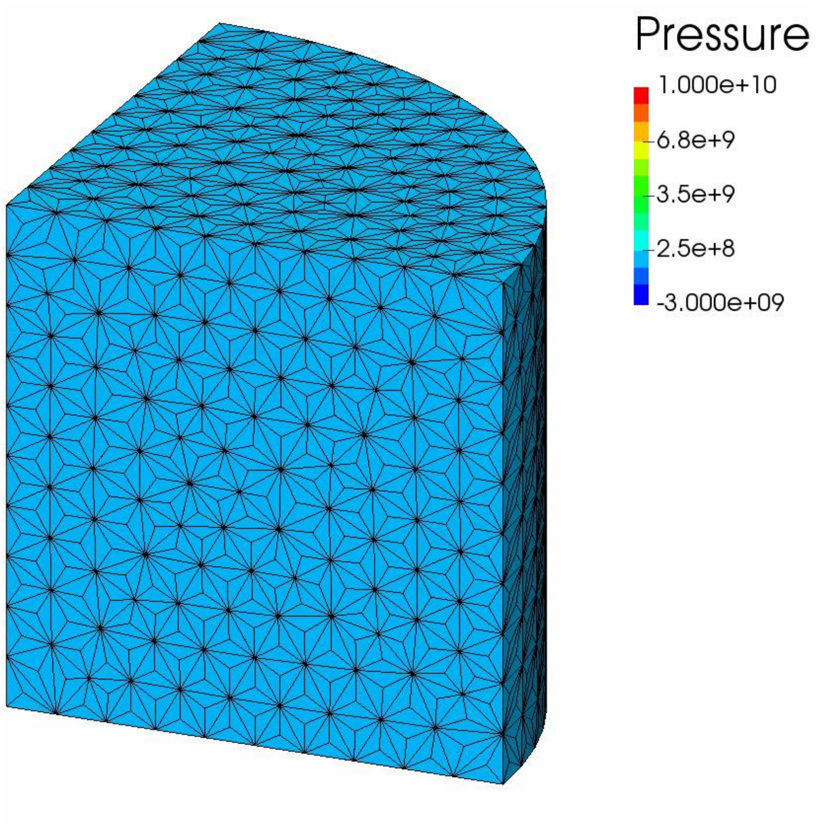
- ✓ No shear/volumetric locking
- ✗ Pressure oscillation
- ✗ Corner locking

<u>2nd order modified hybrid T10 (C3D10MH)</u>

- ✓ No shear/volumetric locking
- ✗ Decrease in interpolation accuracy
- ✗ Early convergence failure

An Example for Rubber-like Material

Material: neo-Hookean **hyperelastic**, $\nu_{\rm ini} = 0.49$



# of Nodes is exactly the same as the C3D4H case.

F-barES-FEM-T4 (a new type of smoothed finite element method (**S-FEM**))

- ✓ No shear/volumetric locking
- ✓ Less pressure oscillation
- ✓ Less corner locking

F-barES-FEM-T4 has excellent accuracy on **rubber-like** materials.
How about it on **elasto-plastic** materials?

Objective

Apply the new type of S-FEM,
F-barES-FEM-T4,
to large deformation problems
of elasto-plastic materials.

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- Methods: Quick introduction of F-barES-FEM-T4
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Methods

Quick introduction of F-barES-FEM-T4

(F-barES-FEM-T3 in 2D is explained for simplicity.)

What is S-FEM?

S-FEM: **S**moothed **F**inite **E**lement **M**ethod

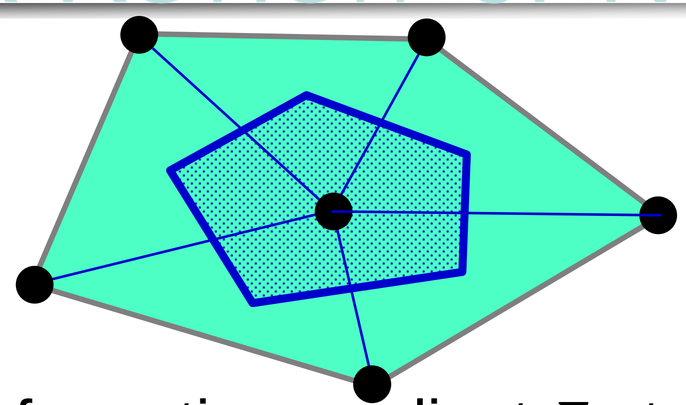
- A new sort of *strain smoothing* technique (since 2007).
- Strain is smoothed **across elements**.
- Various types of S-FEMs:
  - Basic types
    - Node-based S-FEM (NS-FEM)
    - Face-based S-FEM (FS-FEM)
    - Edge-based S-FEM (ES-FEM)
  - Selective types (e.g. ES/NS-FEM)
  - Bubble types (e.g. bES-FEM)
  - NEW **F-bar type (e.g. F-barES-FEM)**

✗ Spurious zero-energy mode or Volumetric locking, Pressure oscillation, Corner locking.

✓ Good in hyperelastic case.
? Unknown in elasto-plastic case.

Quick Review of NS-FEM

For triangular (T3) or tetrahedral (T4) elements.



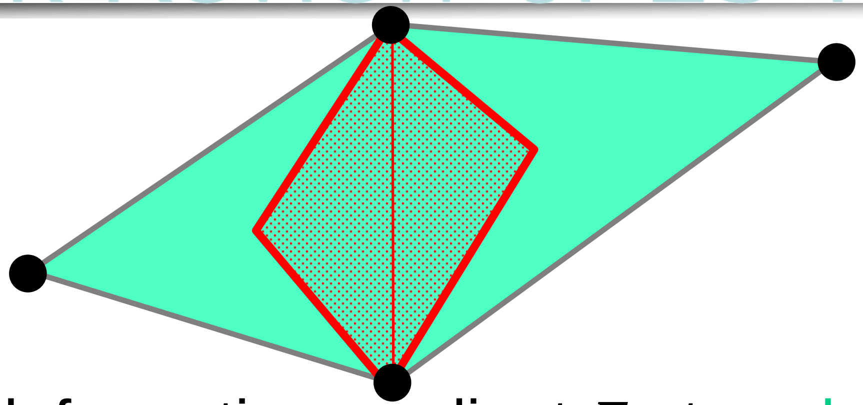
Algorithm:

1. Calculate the deformation gradient $\boldsymbol{F}$ at each element as usual.
2. Distribute the deformation gradient $\boldsymbol{F}$ to the connecting nodes with area weights to make ${}^{\mathrm{Node}}\widetilde{\boldsymbol{F}}$ at each node.
3. Use ${}^{\mathrm{Node}}\widetilde{\boldsymbol{F}}$ to calculate the stress, nodal force and so on.

NS-FEM **avoids shear/volumetric locking** in T3/T4 elements.
Yet, it **suffers from zero-energy modes, pressure oscillation and corner locking...**

Quick Review of ES-FEM

For triangular (T3) or tetrahedral (T4) elements.



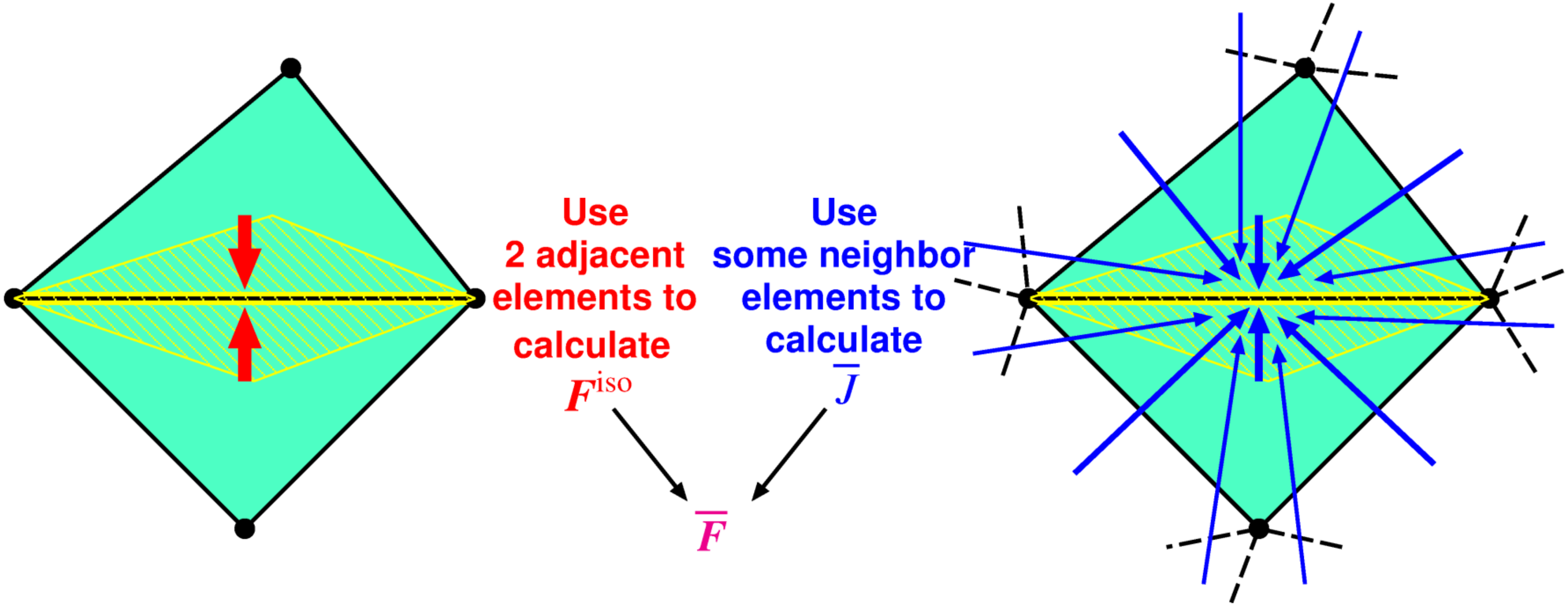
Algorithm:

1. Calculate the deformation gradient $\boldsymbol{F}$ at each element as usual.
2. Distribute the deformation gradient $\boldsymbol{F}$ to the connecting edges with area weights to make ${}^{\mathrm{Edge}}\tilde{\boldsymbol{F}}$ at each edge.
3. Use ${}^{\mathrm{Edge}}\tilde{\boldsymbol{F}}$ to calculate the stress, nodal force and so on.

ES-FEM **avoids shear locking** in T3/T4 elements.
Yet, it **suffers from volumetric locking, pressure oscillation and corner locking…**

Quick Introduction of F-barES-FEM

Concept: combination of F-bar method and ES-FEM



- $^{\text{Edge}}\tilde{F}^{\text{iso}}$ is given by ES-FEM.
- $^{\text{Edge}}\bar{J}$ is given by cyclic nodal smoothing.
- $^{\text{Edge}}\bar{F}$ is calculated in the manner of F-bar method:

$$^{\text{Edge}}\bar{F} = {}^{\text{Edge}}\bar{J}^{1/3}\ {}^{\text{Edge}}\tilde{F}^{\text{iso}}.$$

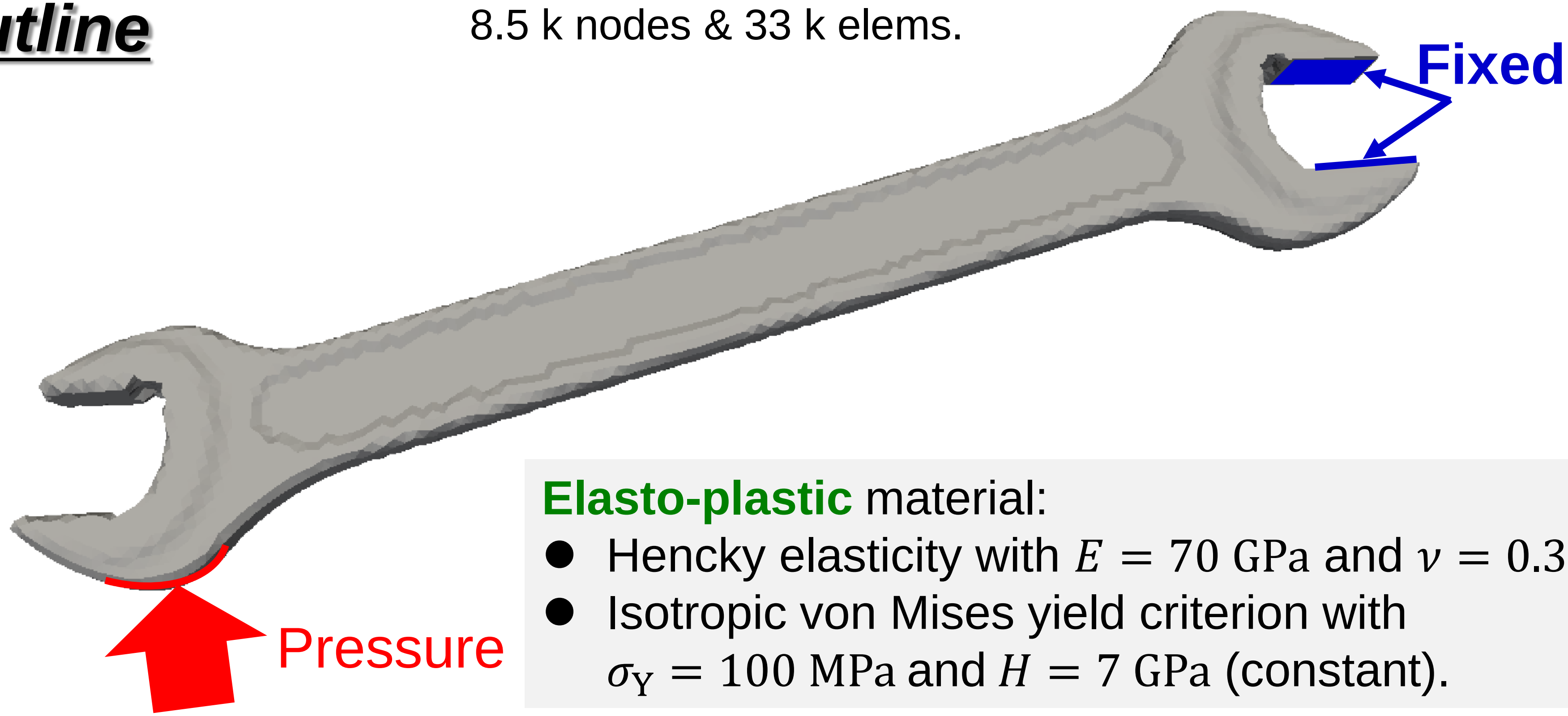
Results

A few verification analyses

Bending of Elasto-Plastic Spanner

Outline

8.5 k nodes & 33 k elems.



Elasto-plastic material:

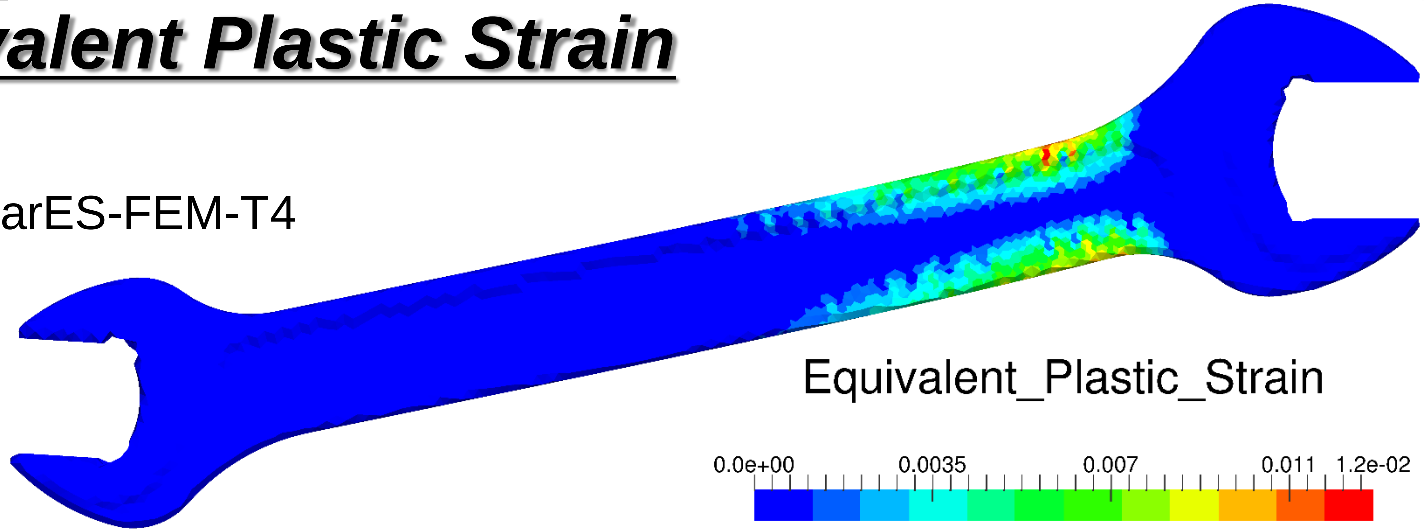
- Hencky elasticity with $E = 70$ GPa and $\nu = 0.3$.
- Isotropic von Mises yield criterion with $\sigma_Y = 100$ MPa and $H = 7$ GPa (constant).

- 2 faces are perfectly constrained.
- Pressure is applied to a side part of the spanner.
- Compared to ABAQUS C3D4H with the same unstructured T4 mesh.

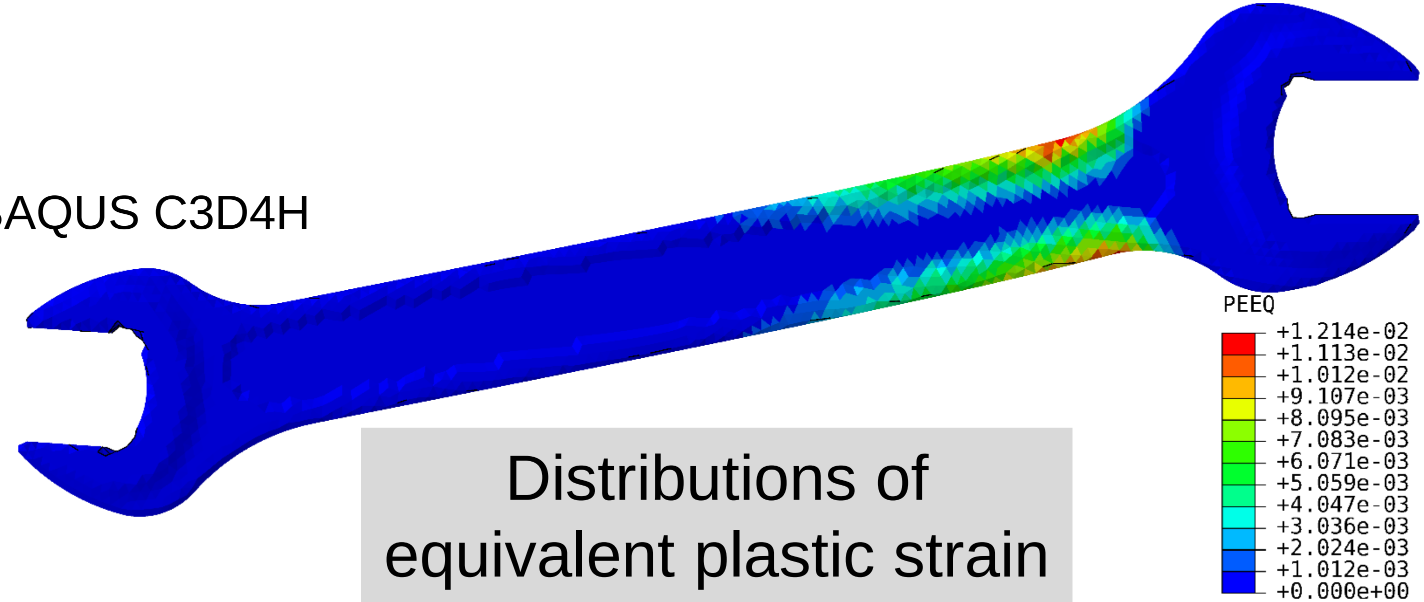
Bending of Elasto-Plastic Spanner

Equivalent Plastic Strain

F-barES-FEM-T4



ABAQUS C3D4H

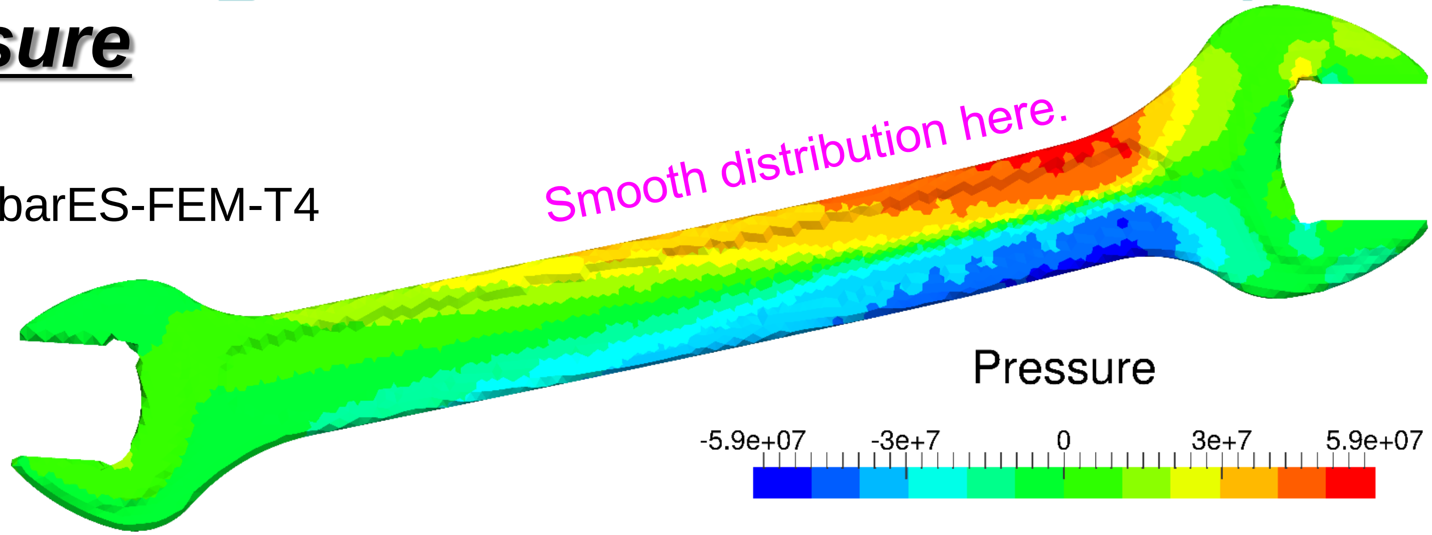


Distributions of equivalent plastic strain are about the same.

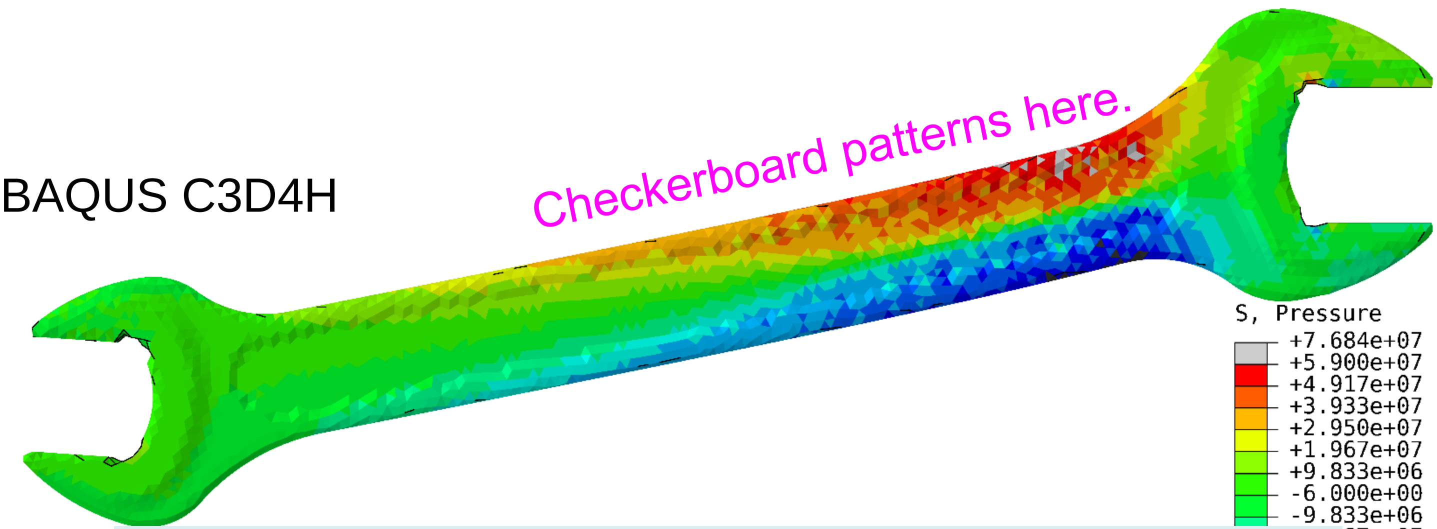
Bending of Elasto-Plastic Spanner

Pressure

F-barES-FEM-T4



ABAQUS C3D4H

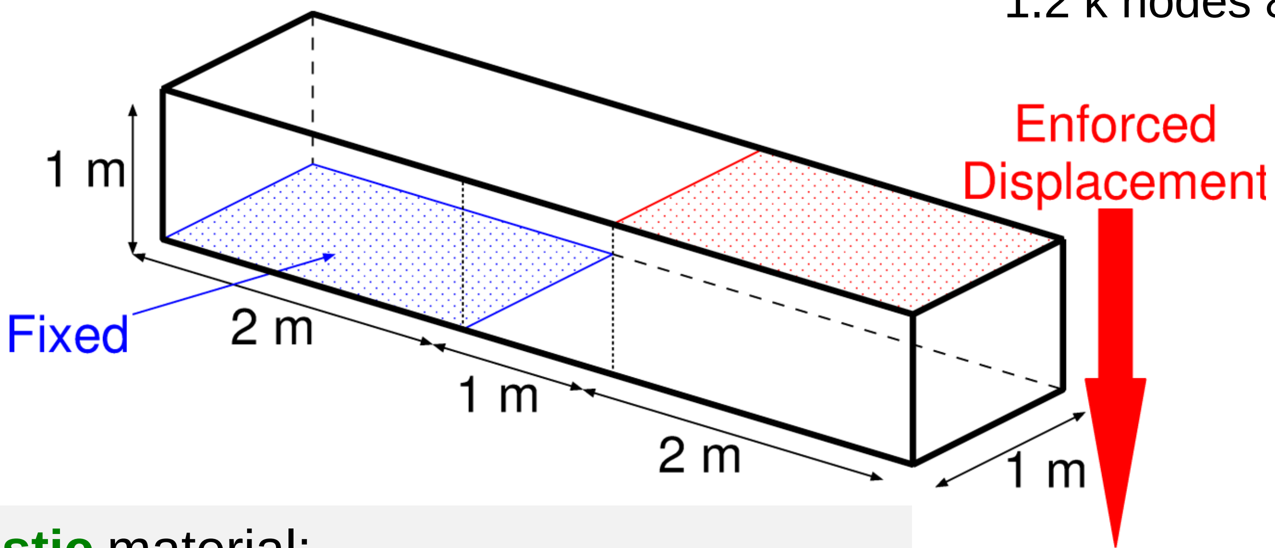


ABAQUS C3D4H suffers from pressure oscillation even in a small deformation elasto-plastic case.

Shearing & Tensioning of Elasto-Plastic Bar

Outline

1.2 k nodes & 4.8 k elems.



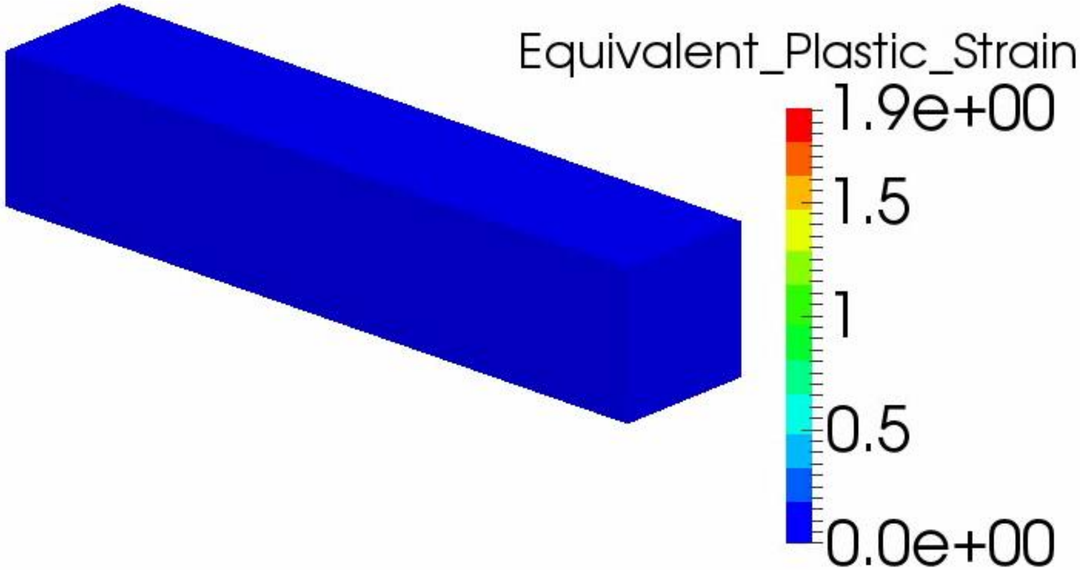
Elasto-plastic material:

- Hencky elasticity with $E = 1\ \mathrm{GPa}$ and $\nu = 0.3$.
- Isotropic von Mises yield criterion with $\sigma_{\mathrm{Y}} = 1\ \mathrm{MPa}$ and $H = 0.1\ \mathrm{GPa}$ (constant).

- Blue face is perfectly constrained.
- Red face is constrained in plane and pressed down.
- Compared to ABAQUS C3D4H with the same unstructured T4 mesh.

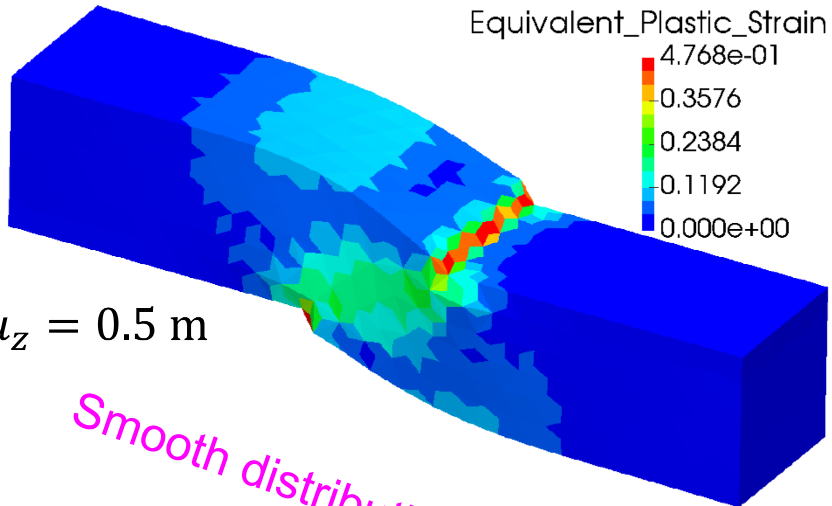
Shearing & Tensioning of Elasto-Plastic Bar

Result of F-bar ES-FEM (Equiv. Plastic Strain)



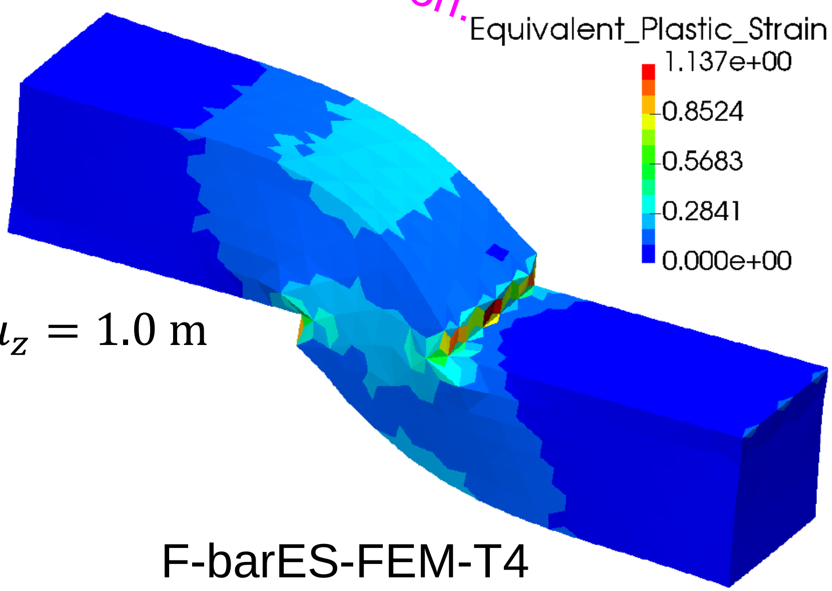
Shearing & Tensioning of Elasto-Plastic Bar

Equivalent Plastic Strain



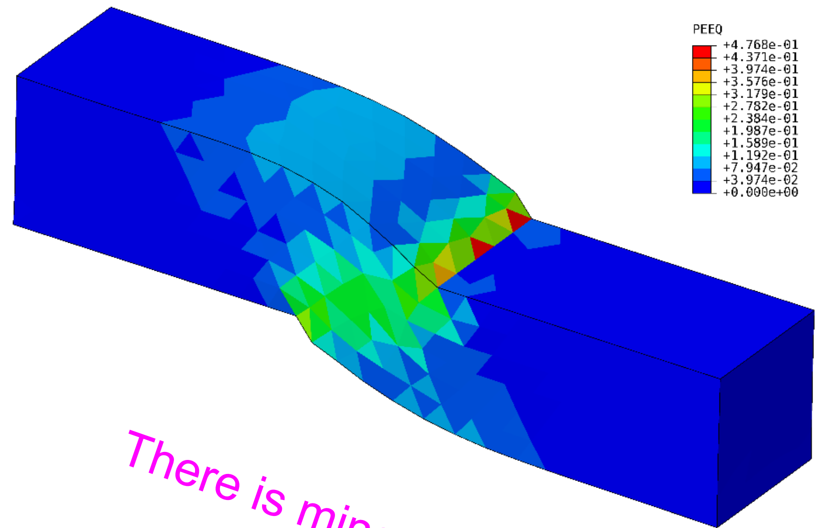
$u_z = 0.5\ \mathrm{m}$

Smooth distribution.

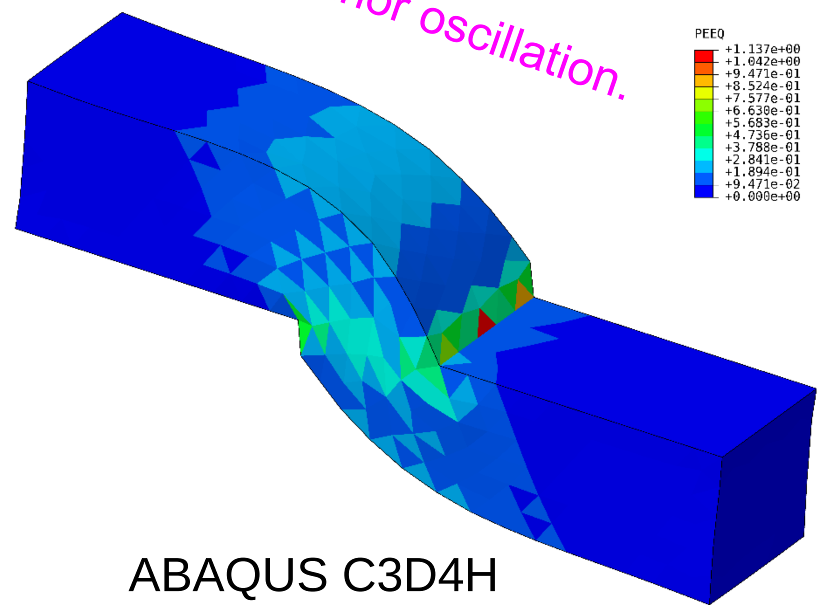


$u_z = 1.0\ \mathrm{m}$

F-barES-FEM-T4



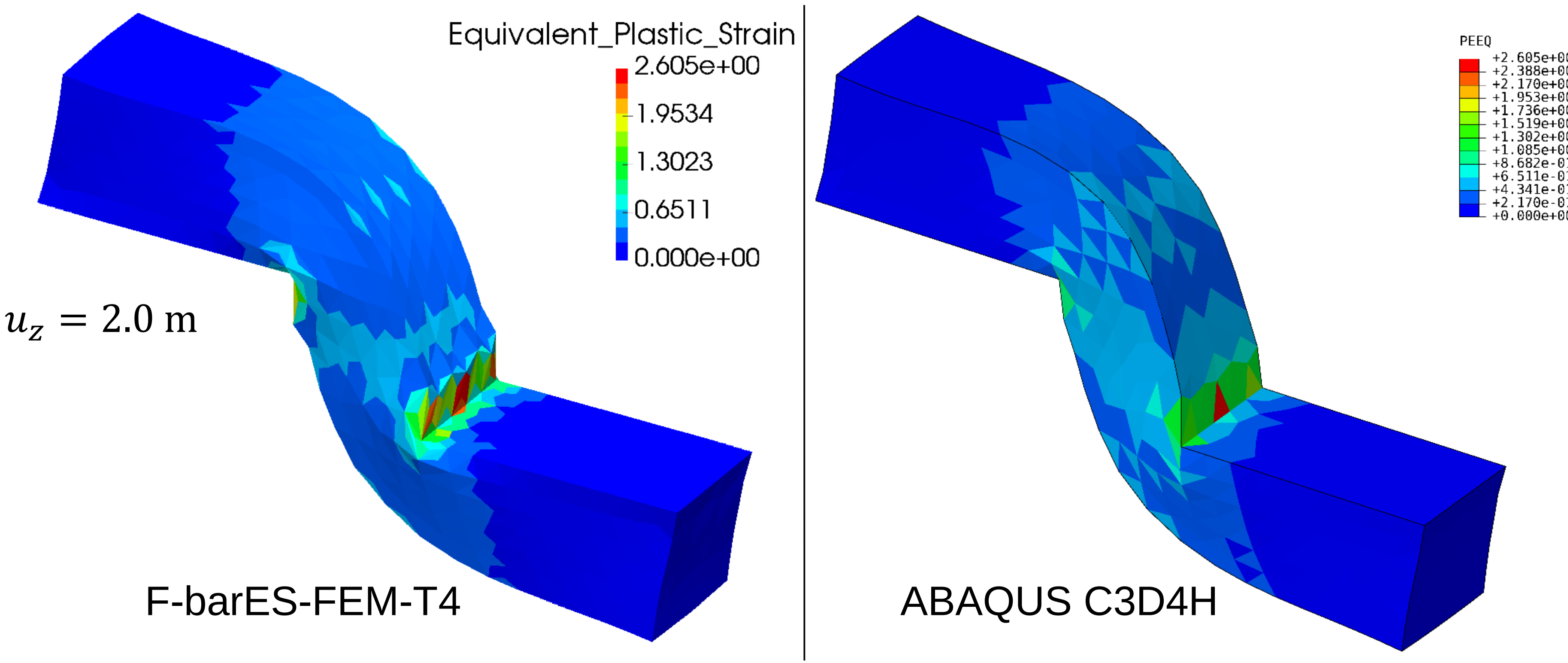
There is minor oscillation.



ABAQUS C3D4H

Shearing & Tensioning of Elasto-Plastic Bar

Equivalent Plastic Strain

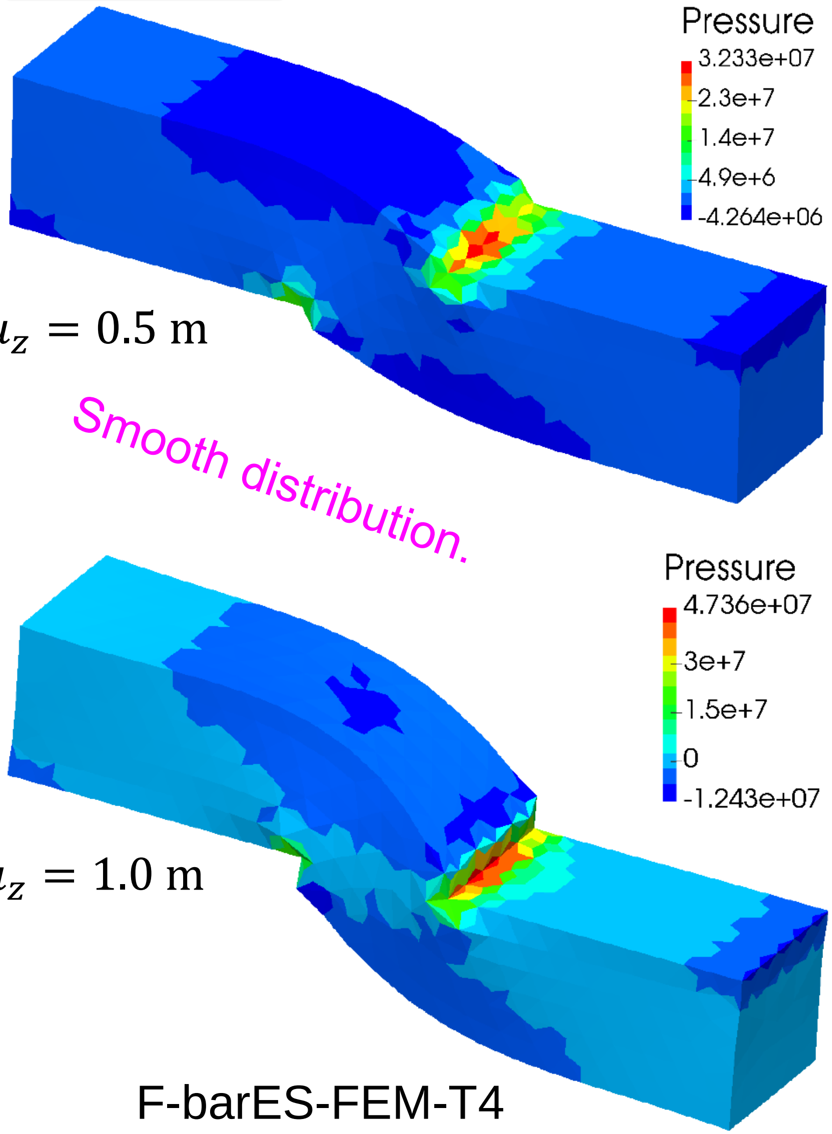


F-barES-FEM-T4 | ABAQUS C3D4H

Accuracy of equivalent plastic strain seems no much different.

Shearing & Tensioning of Elasto-Plastic Bar

Pressure

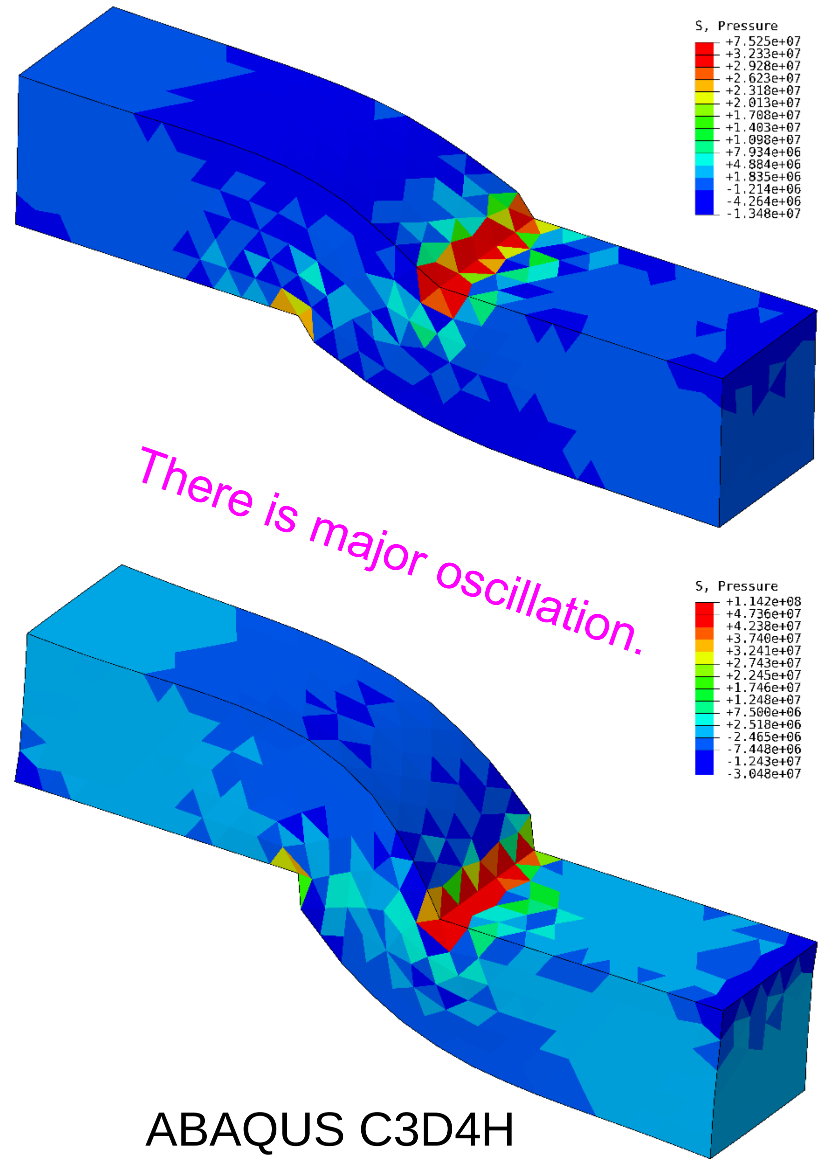


$u_z = 0.5$ m

Smooth distribution.

$u_z = 1.0$ m

F-barES-FEM-T4



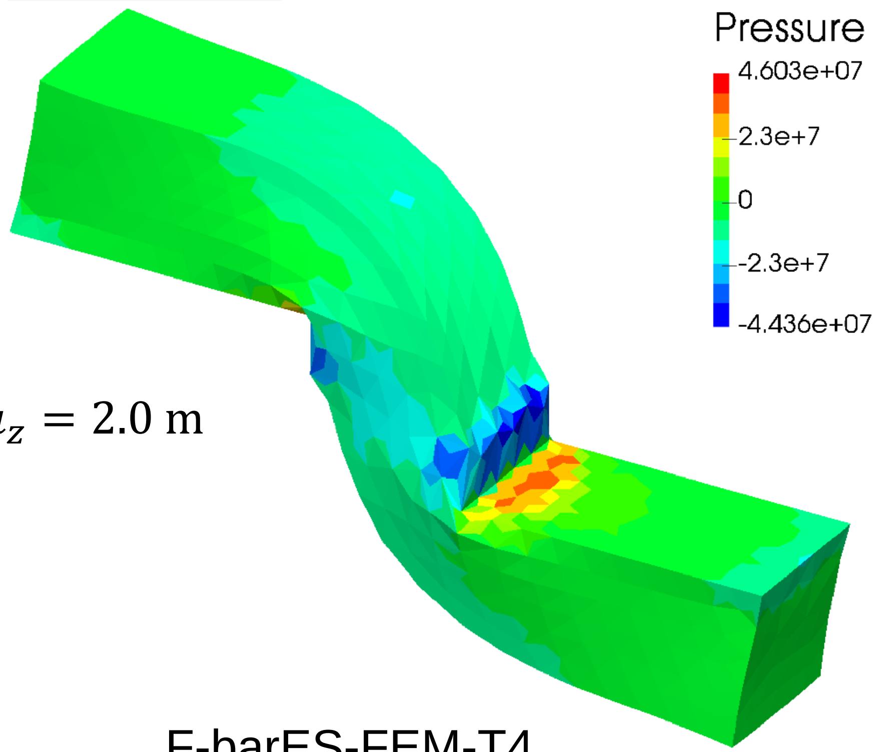
There is major oscillation.

ABAQUS C3D4H

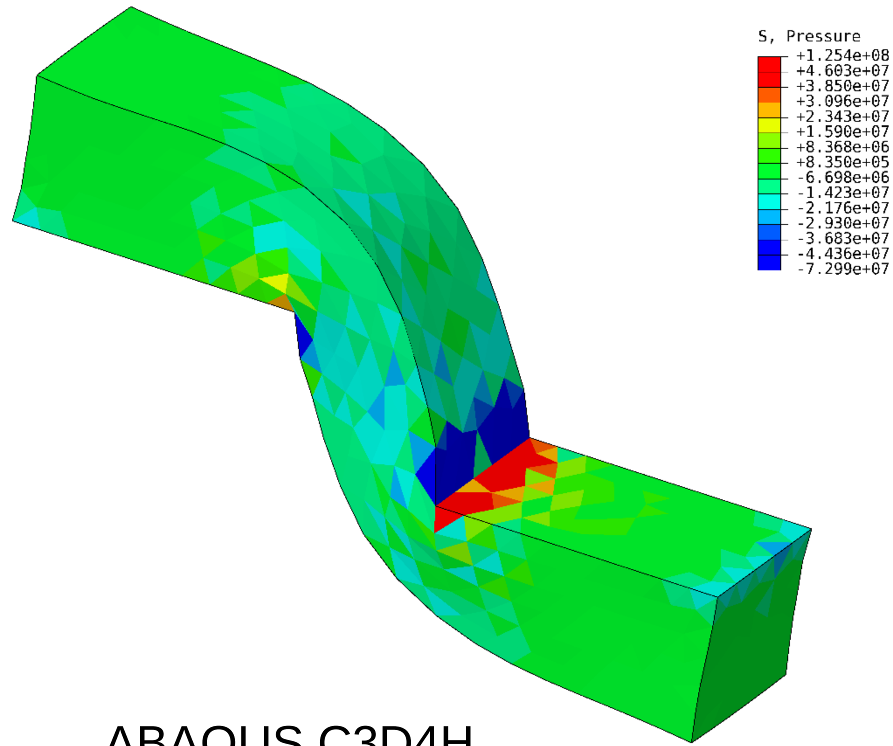
Shearing & Tensioning of Elasto-Plastic Bar

Pressure

$u_z = 2.0\ \mathrm{m}$



F-barES-FEM-T4



ABAQUS C3D4H

Accuracy of pressure is quite different
due to the pressure oscillation of ABAQUS C3D4H.

Summary

Benefits and Drawbacks of F-barES-FEM-T4

Benefits

- ✓ Locking-free with 1st -order tetra meshes.
  No difficulty in severe strain or contact analysis.
- ✓ No increase in DOF.
  No intermediate nodes. No need for static condensation.
- ✓ No restriction of material constitutive model.
  Pressure dependent models are acceptable.
- ✓ Less pressure oscillation and corner locking.

Drawbacks

- ✗ The more cyclic smoothing necessitates the more CPU time.

The Take-Home Message

If you are interested in elasto-plastic problems that have

- 3D bulk complex shapes,
- severe large deformation or contact, and especially
- pressure dependent constitutive models,

then, please consider using F-barES-FEM-T4.

Thank you for your kind attention!