Meshfree Large Deformation Analysis with Modified Formulation of Floating Stress-point Integration (FSPI)

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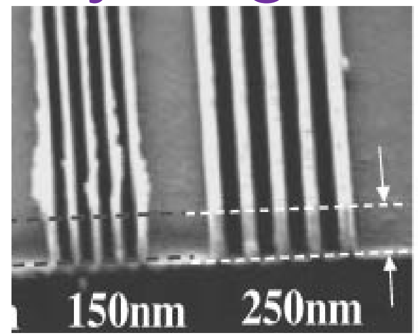
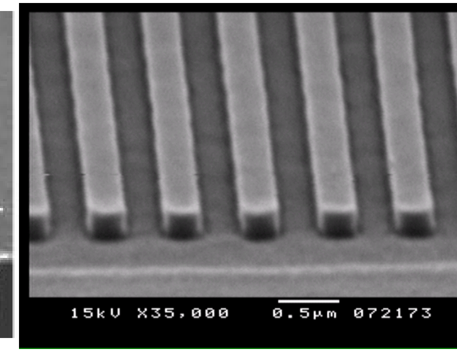
Backuground and Motivation

- We want to solve extremely large deformation problems easily!!

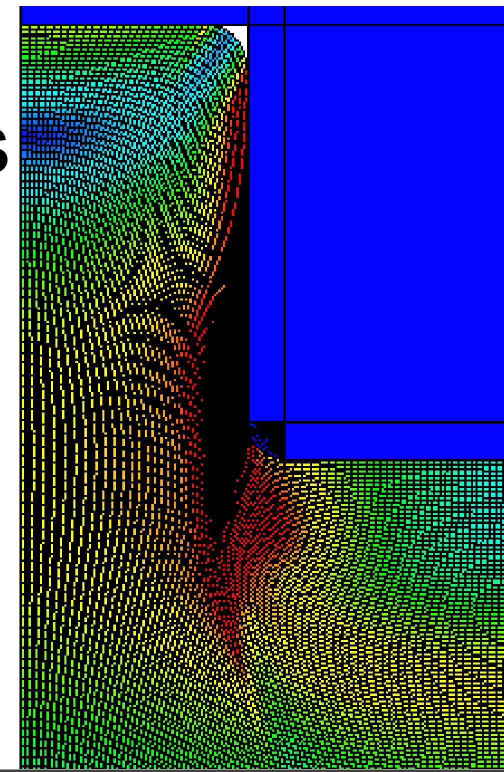
  Final target:

  thermal imprinting





- Using FEM, finite elements are easily distorted and bring analysis failure.



- Adaptive FEM is not convenient so far.

Challenge meshfree!!

3 Types of Domain Integration

- Background Cell Integration (EFGM)
  - Numerical diffusion arises through physical state interpolation.
- Nodal Integration (SCNI)
  - Doesn't get along with updated Lagrangian.
  - Zero-energy mode arises without artificial stabilization.
- Stress-Point Integration

We adopted this, and developed a new one named FSPI.

  - Move of SPs is required with deformation goes on.

There are only few researches especially for large deformation. (There is no standard formulation.)

Objective

Develop a new type of
stress-point integration meshfree method,
floating stress-point integration (FSPI),
for large deformation problems

In this presentation,
effectiveness of FSPI in cases of
elastic and ***elastoplastic*** materials
are presented.

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Outline of FSPI

■Concept of FSPI

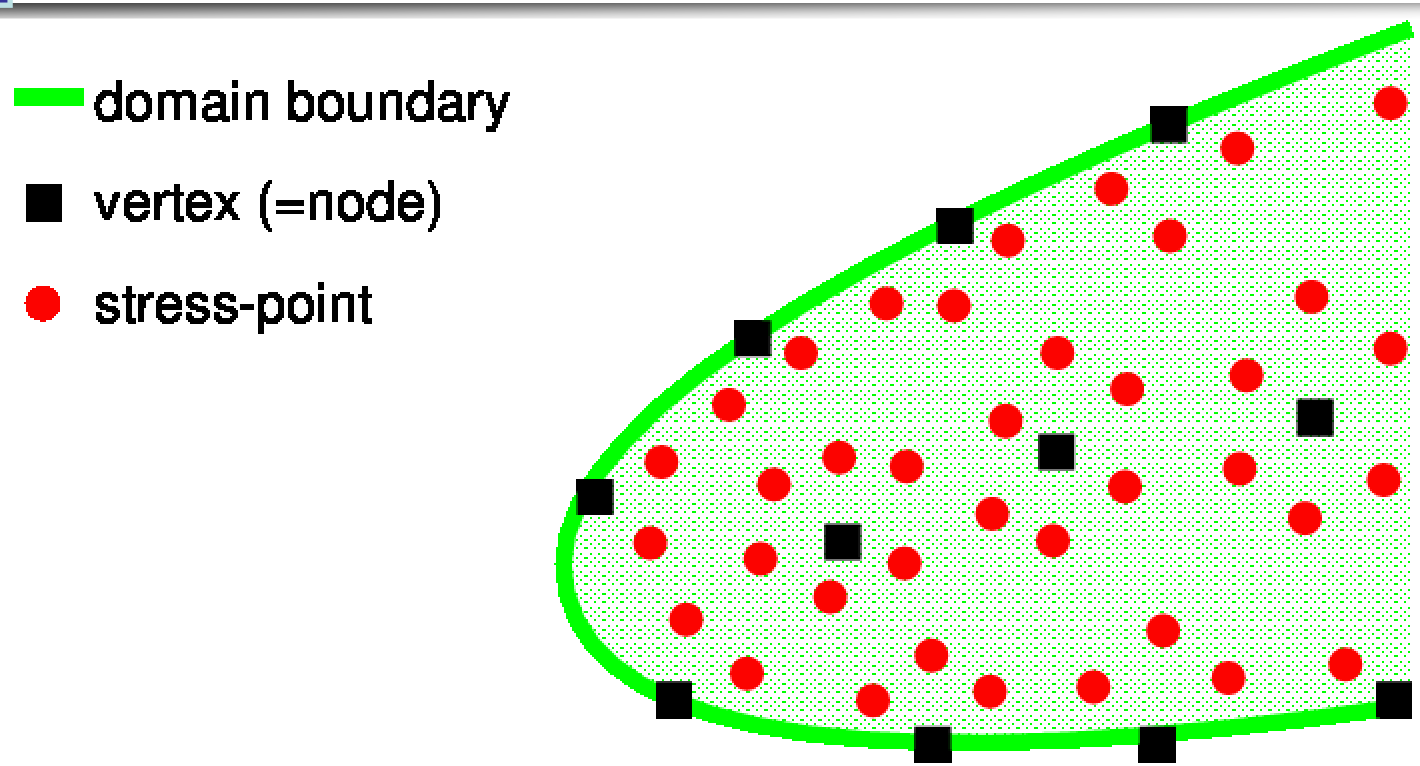
- Fully meshfree method for large deformation
- Use stress-points for domain integration
- Use updated Lagrangian procedure
- Use implicit time advancing sheme

Combination of these two is difficult to be realized in meshfree.

■Introduced Unique Techniques

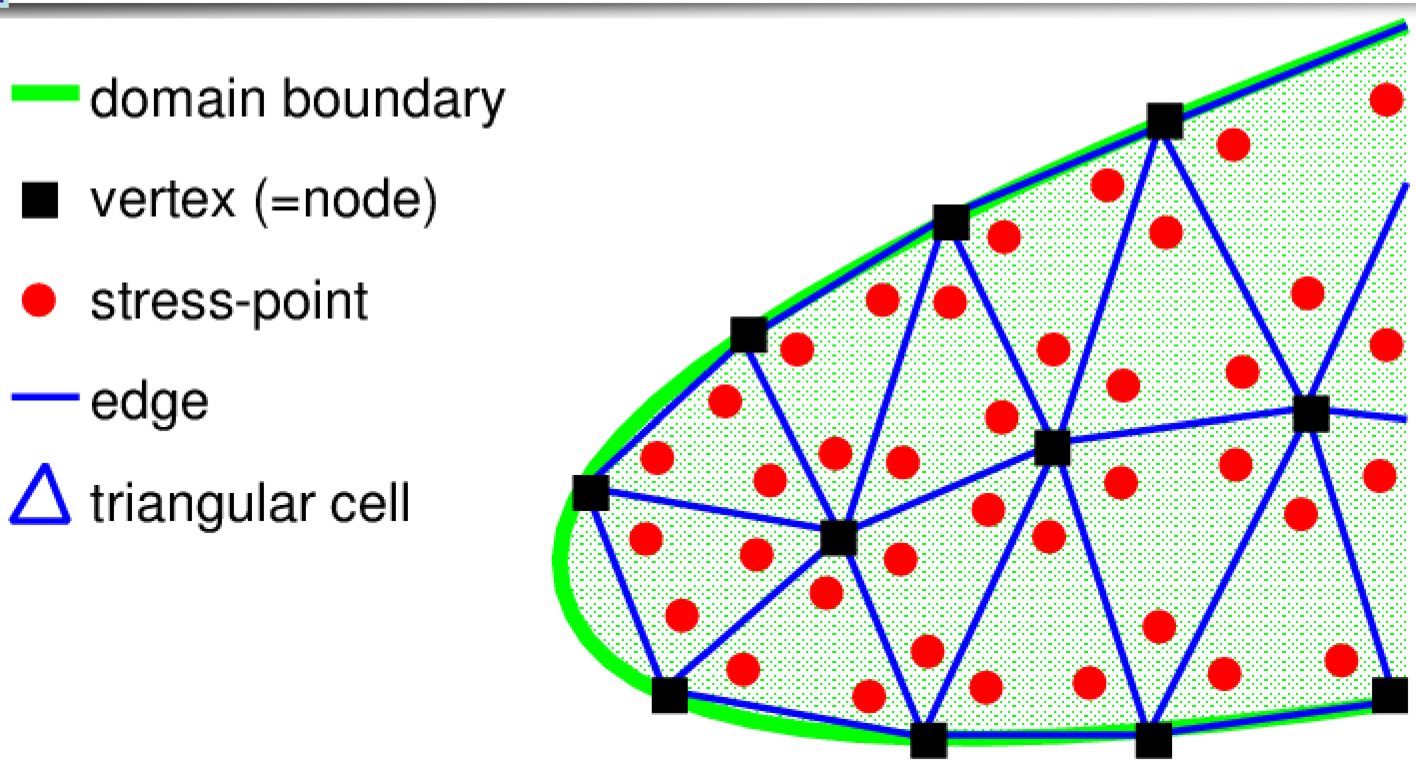
- Shape function construction with Robust MLS
- Incremental equilibrium equation
  for quasi-implicit time advancing scheme

Spacial Discretization & Initialization



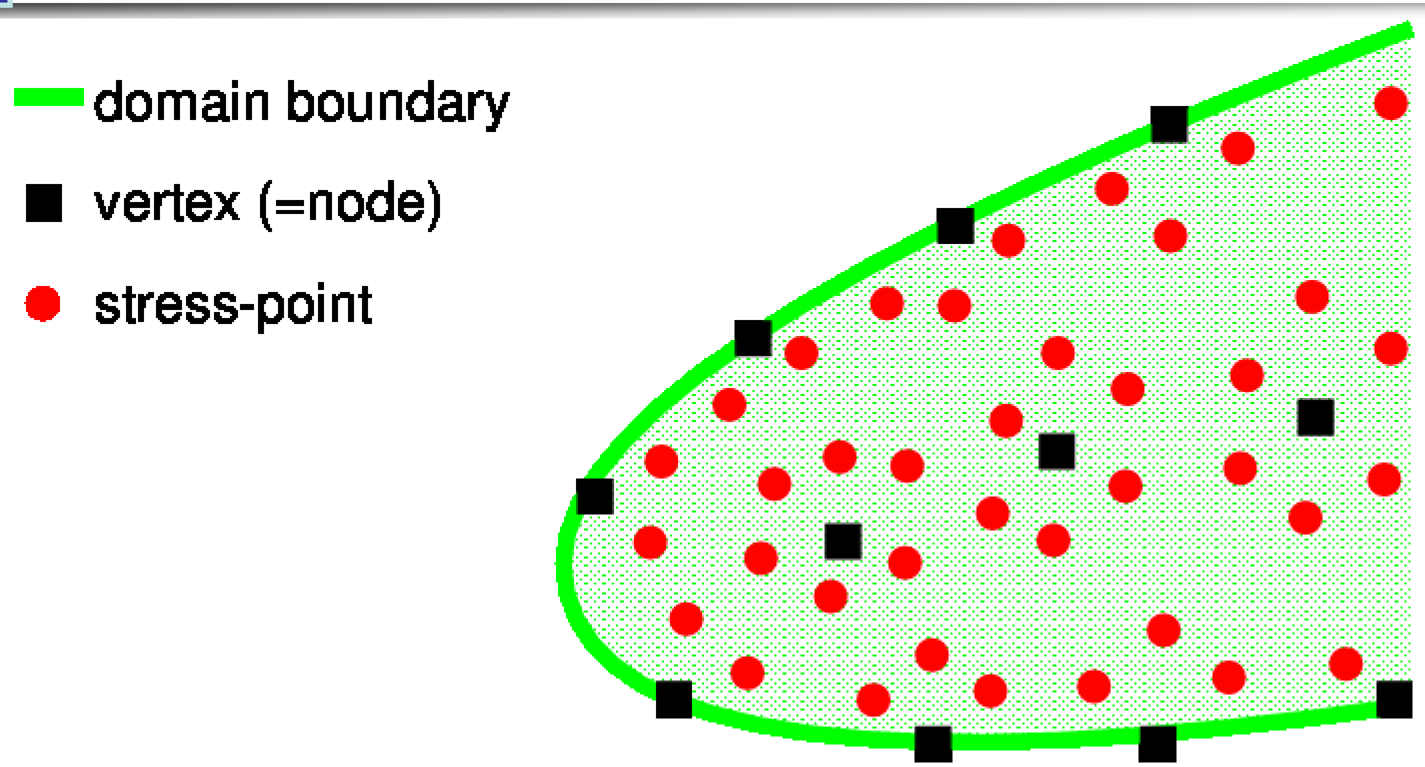
- Generate **nodes** and stress-points in the initial analysis domain
- Any generation methods will do.

Spacial Discretization & Initialization



- Currently, **nodes** and stress-points are generated with unstructured triangular meshes.
- Assign <u>initial corresponding volume of stress-points</u>, ${}^{I}V^{\mathrm{ini.}}$, as ${}^{I}V^{\mathrm{ini.}} = \frac{1}{3}{}^{I}V^{\mathrm{cell}}$.

Spacial Discretization & Initialization



- After the generation and the assignment, meshes are never referred any more.
- This way of node and stress-point generation is not an optimal one.

Shape Function

Robust Moving Least Squares (Robust MLS)

- Support radius for each stress-point I, ${}^{I}R$, is set dynamically and varies over time.

<Algorithm to set ${}^{I}R$>

set initial ${}^{I}R$ (small)

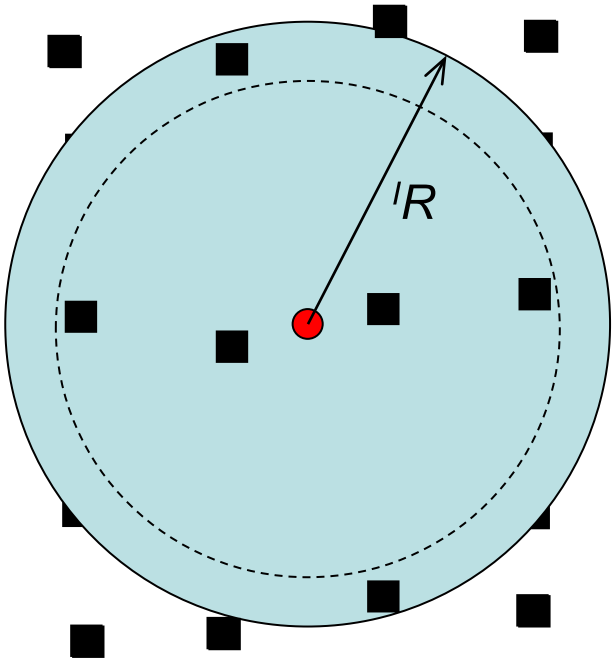
begin loop $\quad \boldsymbol{p} = \{1, x, y\}^T$

$\quad$ calculate $\boldsymbol{A} (= \sum_{J \in {}^{I}\mathbb{S}} {}^{I}w_J \, {}_J\boldsymbol{p}^T \, {}_J\boldsymbol{p})$

$\quad$ if $\mathrm{cond}(\boldsymbol{A}) < 1 \times 10^5$, break

$\quad {}^{I}R \longleftarrow 1.01 \times {}^{I}R$

end loop



Shape Function

- Weight function w (at a stress-point I to **a node** J)

$$ {}^{I}w_J = \begin{cases} 1/{}^{I}d_J - 1 & (0 < {}^{I}d_J < 1) \\ 0 & (1 \le {}^{I}d_J) \end{cases}, \quad {}^{I}d_J = \frac{\|{}_J\boldsymbol{x} - {}^{I}\boldsymbol{x}\|}{{}^{I}R} $$

- Shape function N and its derivatives N'

(at a stress-point I)

$$ \{{}^{I}N\} = \{{}^{I}p\}\ [{}^{I}A]^{-1}\ [{}^{I}B], $$

$$ \{{}^{I}N'_i\} = \frac{\partial\{{}^{I}N\}}{\partial {}^{I}x_i} = \left(\frac{\partial\{{}^{I}p\}}{\partial {}^{I}x_i}\right)[{}^{I}A]^{-1}[{}^{I}B] + \{{}^{I}p\}\left(\frac{\partial[{}^{I}A]^{-1}}{\partial {}^{I}x_i}\right)[{}^{I}B] + \{{}^{I}p\}[{}^{I}A]^{-1}\left(\frac{\partial[{}^{I}B]}{\partial {}^{I}x_i}\right). $$

same as the original MLS except ${}^{I}R$

Time Advancing Scheme

Fully-implicit Time Advancing

- Start of a time increment
  - Start of the Newton-Raphson loop
    - ◆Update support, w, N, and N' of SPs
    - ◆Calculate strain, stress, etc. of SPs
    - ◆Calculate $\{f^{int.}\}$, $\{f^{ext.}\}$, and $[K]$
    - ◆If $|| \{f^{ext.}\} - \{f^{int.}\}||$ is small, break
    - ◆Solve $[K]\ \{\delta u\} = \{f^{ext.}\} - \{f^{int.}\}$
    - ◆Update $\{\Delta u\} = \{\Delta u\} + \{\delta u\}$
    - ◆Update disp. of Nodes and SPs
  - Update all physical variables

Support, shape function, etc. are changed every time in Newton-Raphson loop.

↓

A node can get in-and-out of the support repetitively

↓

Unstable

Time Advancing Scheme

Quasi-implicit Time Advancing

- Start of a time increment
  - Update support, w, N, and N' of SPs
  - Start of the Newton-Raphson loop
    - ◆Update support, w, N, and N' of SPs
    - ◆Calculate strain, stress, etc. of SPs
    - ◆Calculate $\{f^{\text{int.}}\}$, $\{f^{\text{ext.}}\}$, and $[K]$
    - ◆If || $\{f^{\text{ext.}}\}$ - $\{f^{\text{int.}}\}$|| is small, break
    - ◆Solve $[K]\{\delta u\} = \{f^{\text{ext.}}\} - \{f^{\text{int.}}\}$
    - ◆Update $\{\Delta u\} = \{\Delta u\} + \{\delta u\}$
    - ◆Update disp. of Nodes and SPs
  - Update all physical variables

B matrix changed suddenly.

↓

$\{f^{\text{int.}}\}$ $\left(= \sum [B]^T \{\sigma\} V\right)$ also changed suddenly.

↓

$$\lim_{\Delta t \to +0} \{\Delta u\} \neq \{0\}$$

↓

Unstable

Time Advancing Scheme

Quasi-implicit Time Advancing + Incremental Equalibrium Equation

- Start of a time increment
  - Update support, w, N, and N' of SPs
  - Start of the Newton-Raphson loop
    - Update support, w, N, and N' of SPs
    - Calculate strain, stress, etc. of SPs
    - Calculate $\{\Delta f^{\text{int.}}\}$, $\{\Delta f^{\text{ext.}}\}$, and $[K]$
    - If $\| \{\Delta f^{\text{ext.}}\} - \{\Delta f^{\text{int.}}\} \|$ is small, break
    - Solve $[K]\{\delta u\} = \{\Delta f^{\text{ext.}}\} - \{\Delta f^{\text{int.}}\}$
    - Update $\{\Delta u\} = \{\Delta u\} + \{\delta u\}$
    - Update disp. of Nodes and SPs
  - Update all physical variables

$$\lim_{\Delta t \to +0} \{\Delta u\} = \{0\}$$

Stable!!

Incremental Equilibrium Equation

$$\int_v \dot{\boldsymbol{\Pi}}_t^T(t) : \delta \boldsymbol{F}_t(t)\ \mathrm{d}v = \int_s \dot{\underline{\tilde{\boldsymbol{t}}}}_t(t) \cdot \delta \boldsymbol{u}\ \mathrm{d}s$$

Virtual Work Equation in Rate Form without body force term

" $\dot{}$ ":	Material time derivative
" $_t$ ":	Denoting in the current configuration
" δ ":	Denoting the variation
$\boldsymbol{\Pi}(t)$:	1st Piola-Kirchhoff stress tensor
$\boldsymbol{F}(t)$:	Deformation gradient tensor
$\underline{\tilde{\boldsymbol{t}}}(t)$:	Surface traction vector
$\boldsymbol{u}$:	Displacement vector

Incremental Equilibrium Equation

$$\int_v \dot{\boldsymbol{\Pi}}_t^T(t) : \delta \boldsymbol{F}_t(t)\ \mathrm{d}v = \int_s \dot{\tilde{\underline{\boldsymbol{t}}}}_t(t) \cdot \delta \boldsymbol{u}\ \mathrm{d}s$$

Linearize time derivative

$$\dot{\boldsymbol{\Pi}}_t^T(t) \longrightarrow \Delta\, {}^I\boldsymbol{\Pi}_t^T / \Delta t$$

$$\dot{\tilde{\underline{\boldsymbol{t}}}}_t(t) \longrightarrow \Delta\, {}_J\underline{\boldsymbol{t}}_t / \Delta t$$

Galerkin discretization

$$\delta \boldsymbol{F}_t(t) = \frac{\partial \delta \boldsymbol{u}}{\partial \boldsymbol{x}} \longrightarrow \sum_{J \in {}^I\mathbb{J}} {}^I\boldsymbol{N}'_J \delta\, {}_J\boldsymbol{u} = [{}^IB_{\mathrm{N}}]\{\delta u\}$$

$$\{\Delta f^{\mathrm{ext.}+}\} - \{\Delta f^{\mathrm{int.}+}\} = \{0\}$$

$\{\Delta f^{\mathrm{ext.}+}\}$: Incremental external force vector array

$$= \int_\Gamma [\widetilde{N}]^T \{\Delta \underline{t}_t\}\ \mathrm{d}\Gamma \simeq \{f^{\mathrm{ext.}+}\} - \{f^{\mathrm{ext.}}\}$$

$\{\Delta f^{\mathrm{int.}+}\}$: Incremental internal force vector array

$$= \sum_{I \in \mathbb{I}_\Omega} \int_{{}^I\Omega} [\widetilde{B}_{\mathrm{N}}]^T \{\Delta \Pi_t^{T+}\}\ \mathrm{d}\Omega \simeq \sum_{I \in \mathbb{I}_\Omega} [{}^I\widetilde{B}_{\mathrm{N}}]^T \{\Delta\, {}^I\Pi_t^{T+}\}\, {}^IV^+$$

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Elastic Constitutive Equation

- Common isotropic elastic model
- Exactly the same as the default elastic model of ABAQUS/Standard.

$$\boldsymbol{T} = \boldsymbol{C} : \boldsymbol{E} \qquad (\text{i.e.,} \;\; \mathring{\boldsymbol{T}} = \boldsymbol{C} : \boldsymbol{D}).$$

$\boldsymbol{T}$: Cauchy stress,

$\boldsymbol{E}$: Hencky strain,

$\mathring{\boldsymbol{T}}$: Jaumann rate of Cauchy stress,

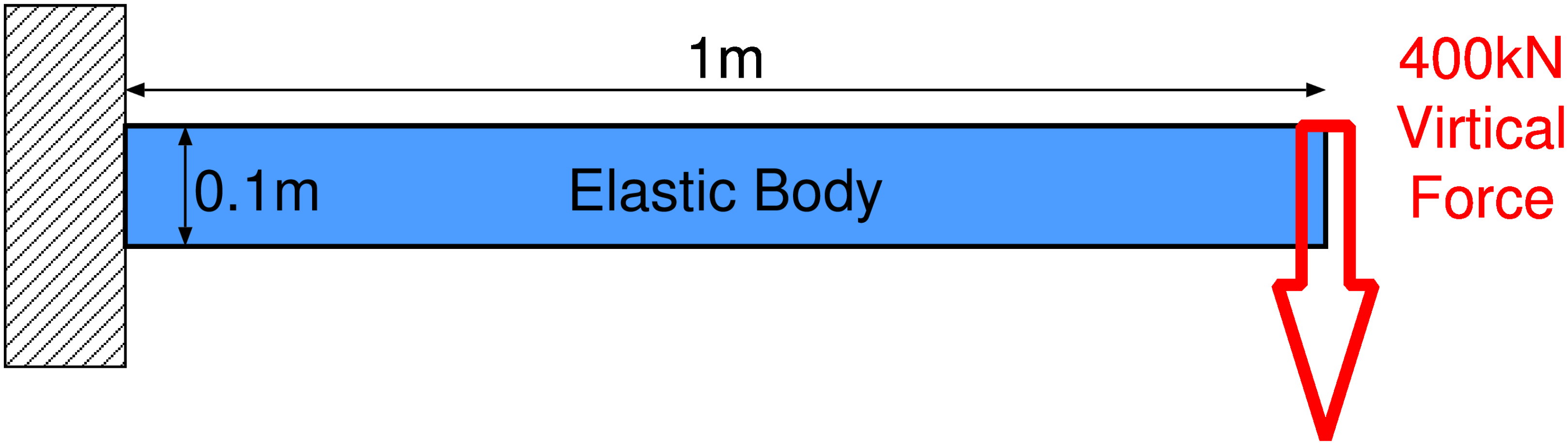
$\boldsymbol{D}$: Rate of deformation tensor,

$$C_{ijkl} = \lambda \delta_{ij} \delta_{kl} + 2\mu \delta_{ik} \delta_{jl},$$

λ, μ : Lame's parameters,

δ : Kronecker's delta

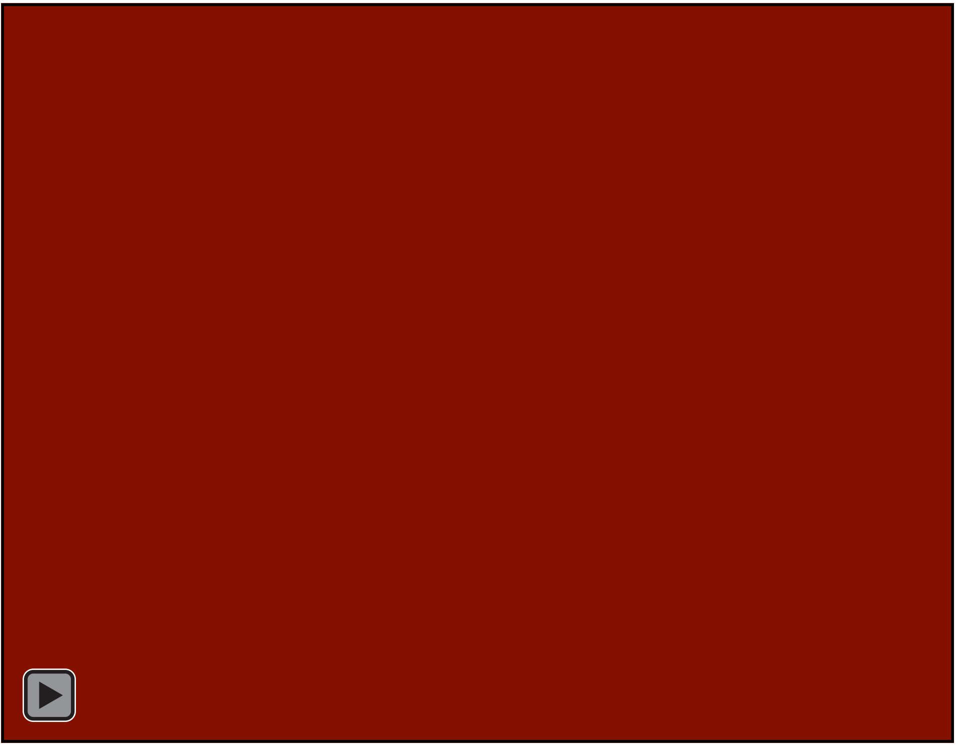
Elastic Cantilever Bending



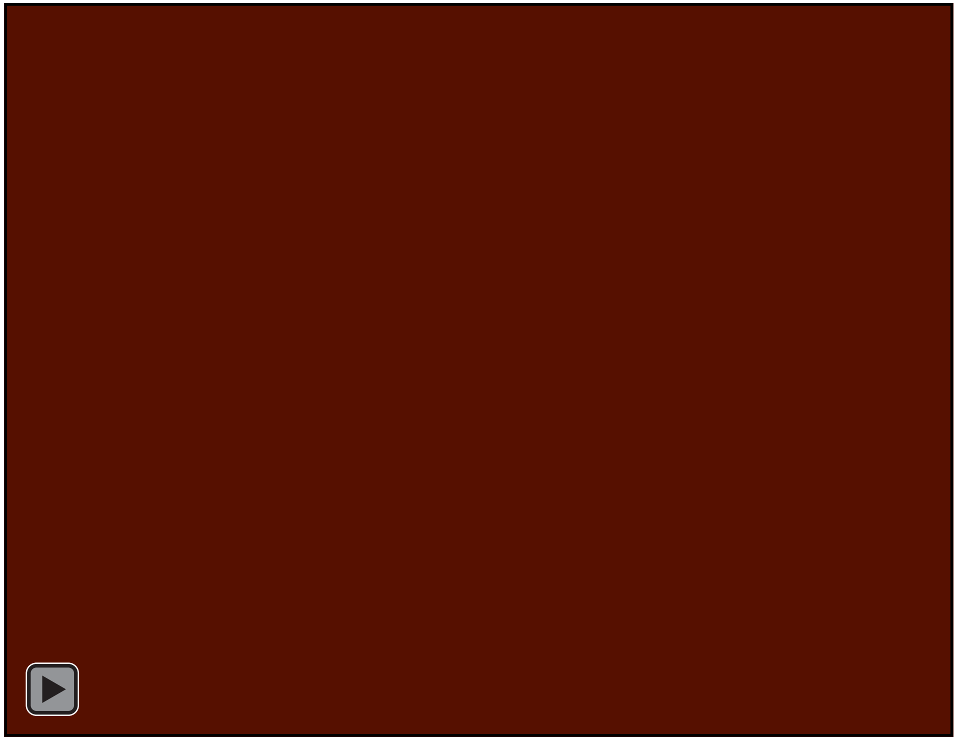
- Quasi-static, Plane strain
- Young's modulus: 1GPa, Poisson's ratio: 0.3
- The num. of nodes: 335, The num of SPs: 1450
- 400kN concentrated force to downward dir.
- Compared to ABAQUS/Standard with 8-node 2nd-order quadrilateral elements (CPE8)

Elastic Cantilever Bending

Distributions of Mises Stress

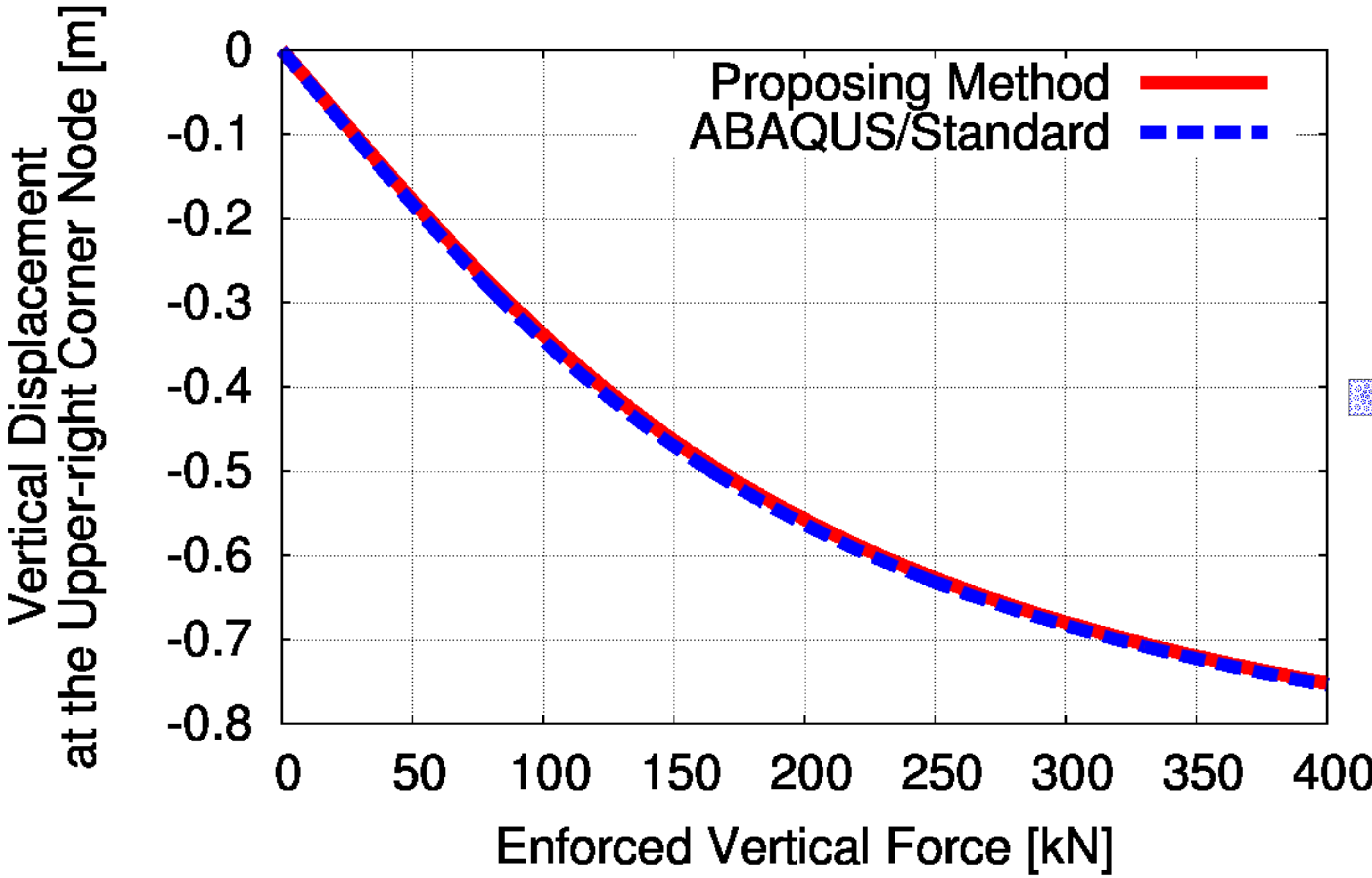


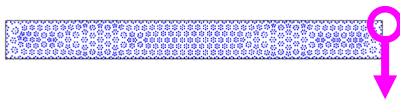
FSPI Meshfree



ABAQUS/Standard

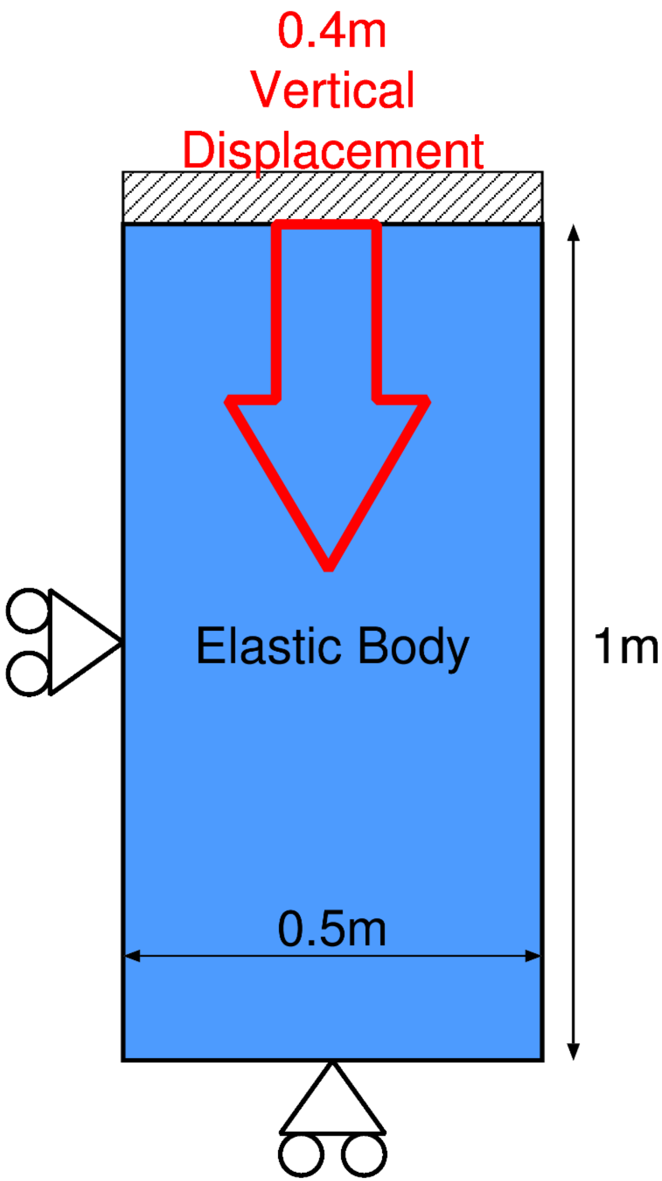
Elastic Cantilever Bending





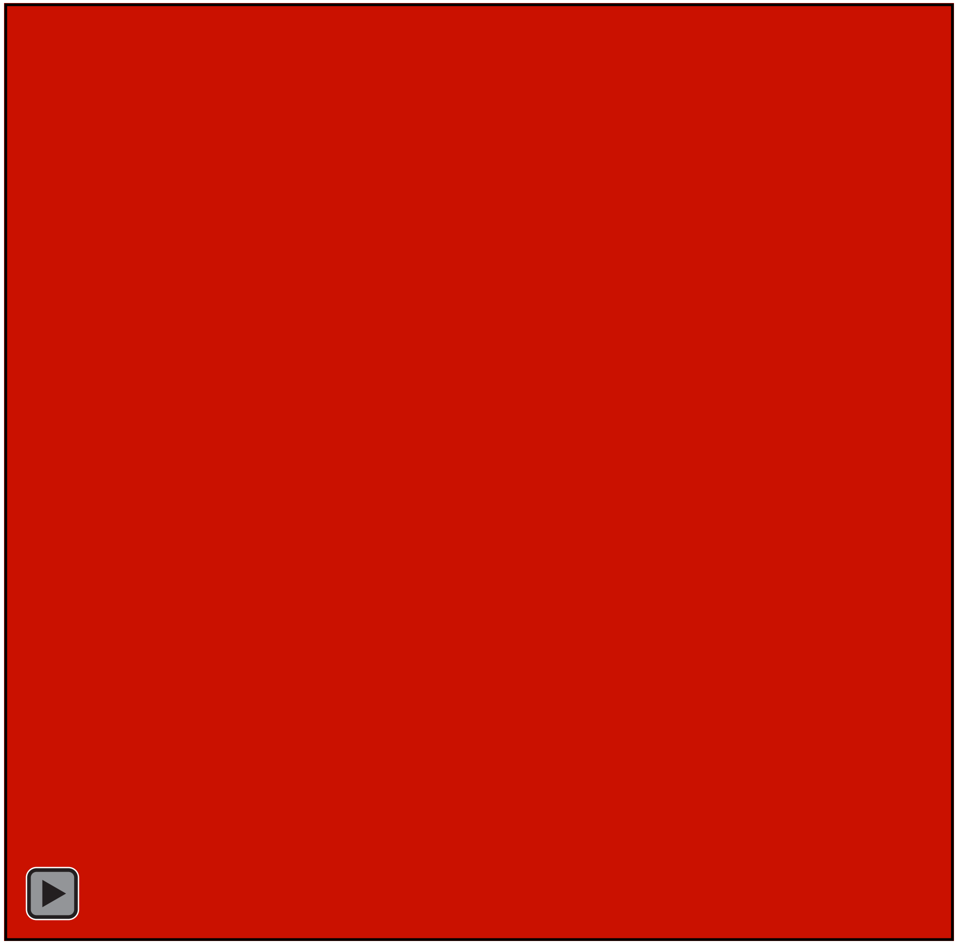
- Displacement error is less than 0.3%.
- Proposing method is shear locking free.

Elastic Uniaxial Compression

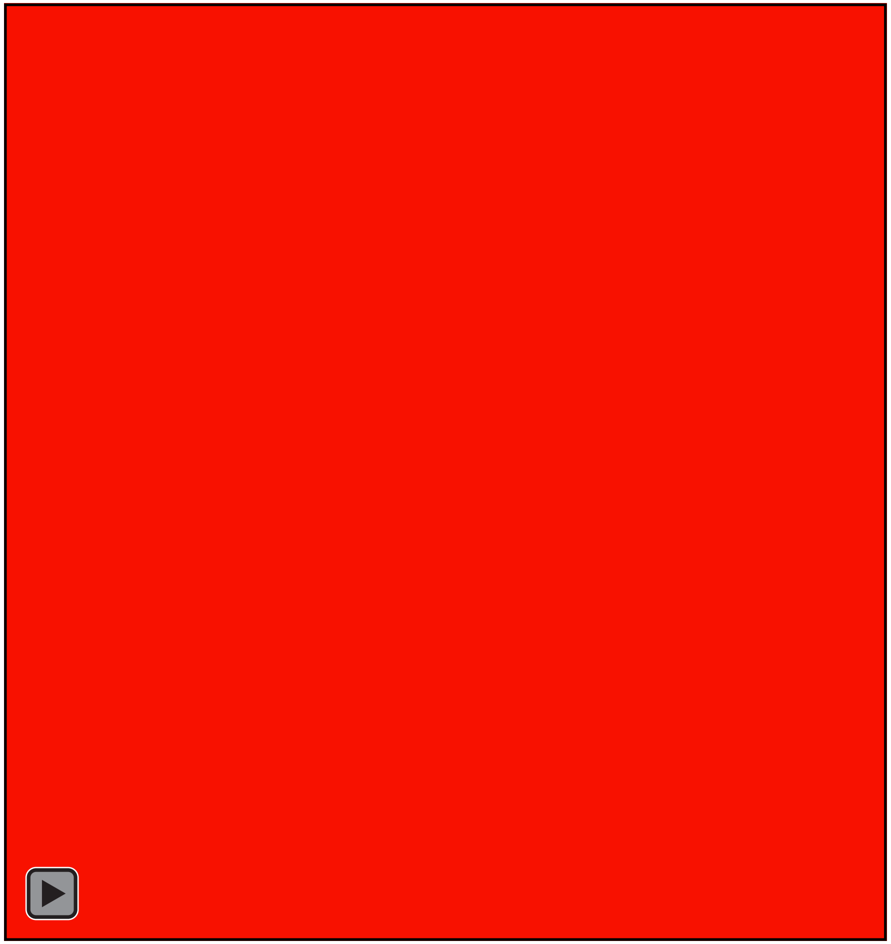


- Quasi-static, Plane strain
- Young's modulus: 1GPa, Poisson's ratio: 0.45
- The num. of nodes: 528, The num. of SPs: 2455
- 0.4m displacement to downward dir.
- Compared to ABAQUS/ Standard with 3-node 1st-order triangular elements (CPE3)

Elastic Uniaxial Compression



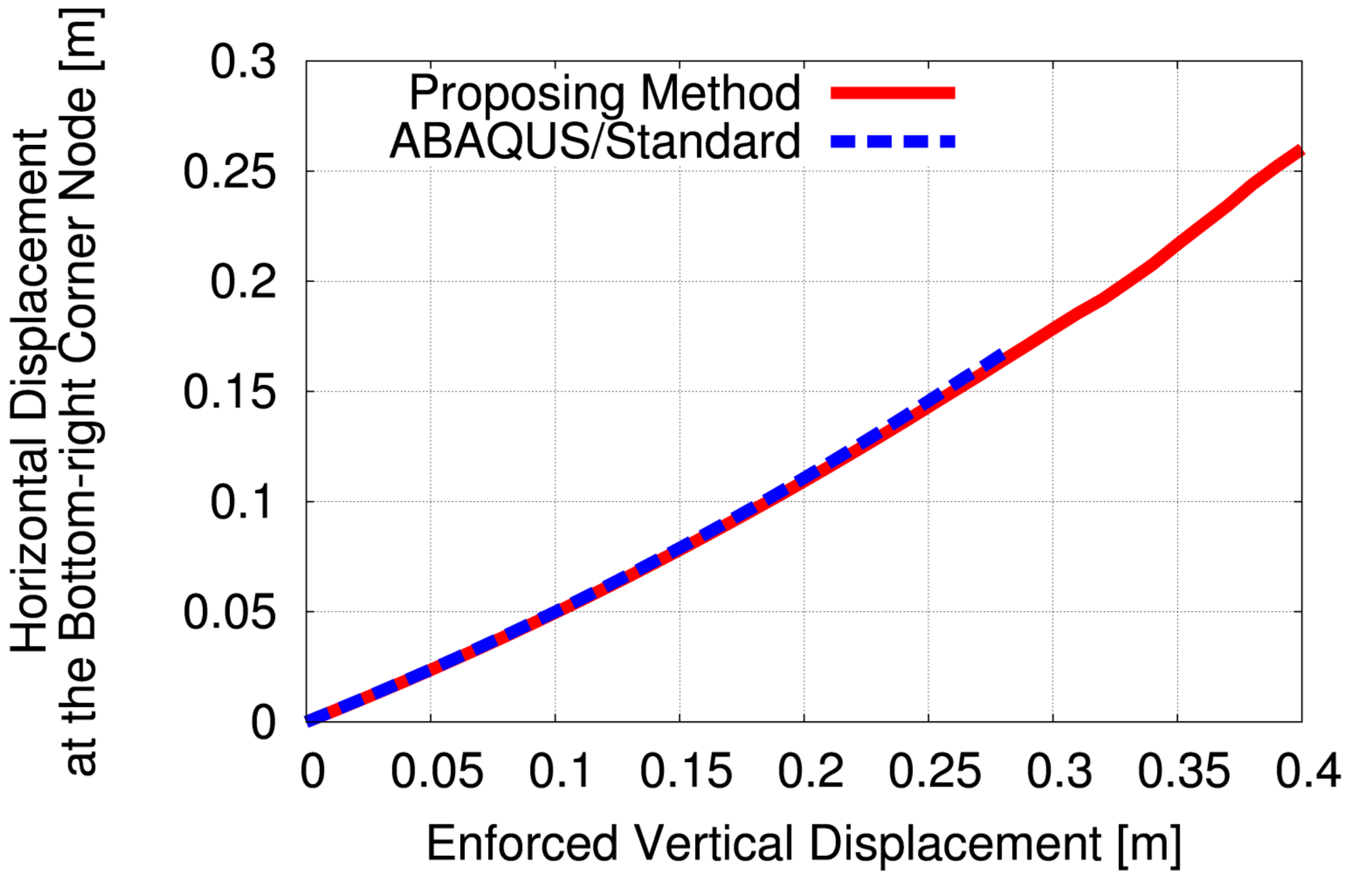
FSPI Meshfree

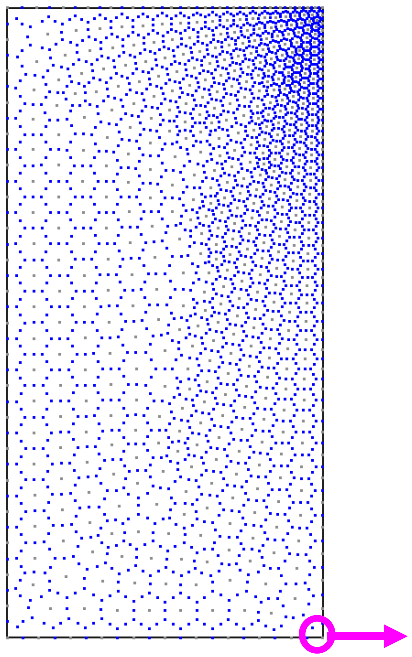


ABAQUS/Standard

■ ABAQUS stopped at 0.28m displacement.

Elastic Uniaxial Compression

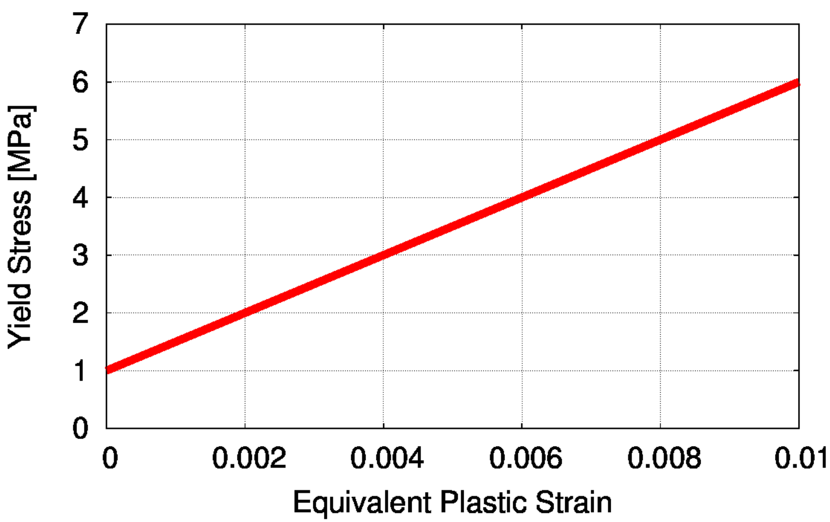




- 3% displacement difference at 0.28m disp.
- Treatments at the concave corner is necessary.

Elasto-plastic Constitutive Equation

- Classical elasto-plastic model with
  - von Mises yield criterion
  - associated flow rule
  - isotropic hardening rule



- Exactly the same as the default elasto-plastic model of ABAQUS/Standard.

$$\boldsymbol{T} = \boldsymbol{C} : \boldsymbol{E}_{\rm el} \quad \left(\text{i.e.,} \quad \overset{\circ}{\boldsymbol{T}} = \boldsymbol{C} : \boldsymbol{D}_{\rm el}\right).$$

Young's Modulus: 1GPa
Poisson's Ratio: 0.3

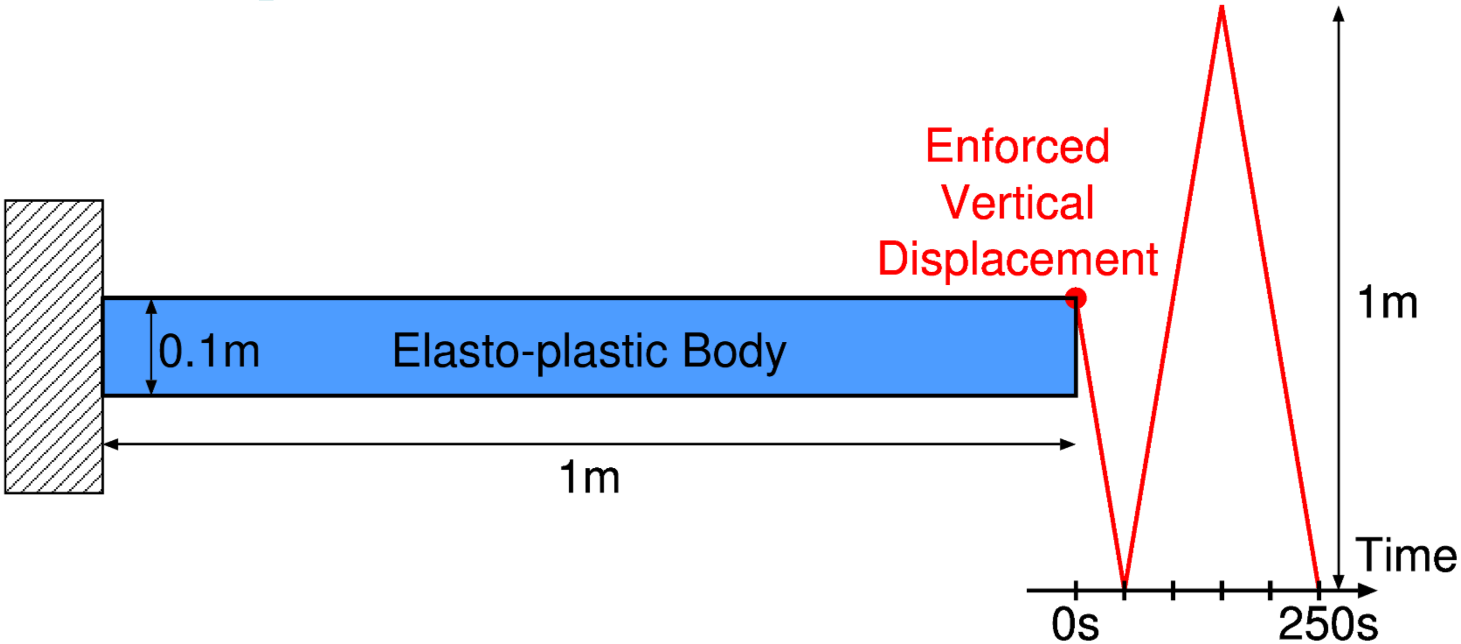
$\boldsymbol{T}$: Cauchy stress,

$\boldsymbol{E}_{\rm el}$: Elastic part of Hencky strain,

$\overset{\circ}{\boldsymbol{T}}$: Jaumann rate of Cauchy stress,

$\boldsymbol{D}_{\rm el}$: Elastic part of rate of deformation tensor.

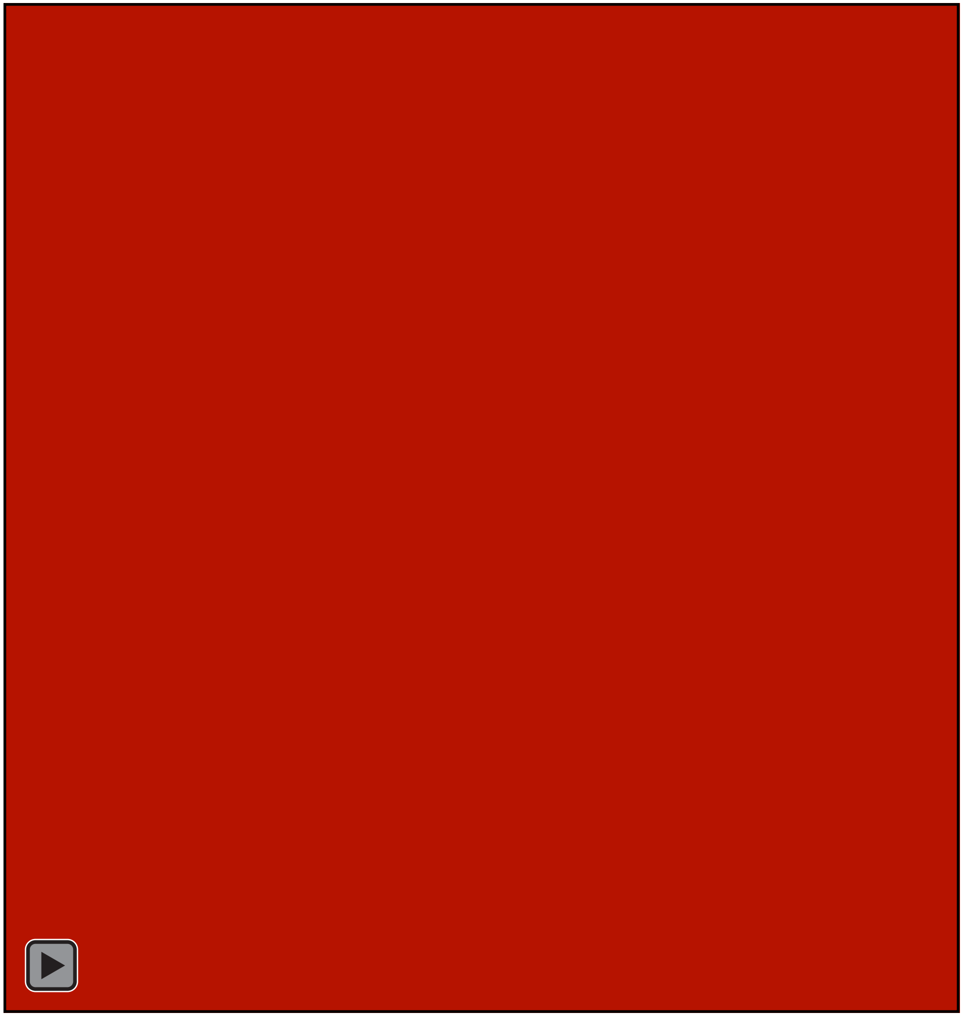
Elasto-plastic Cantilever Bending



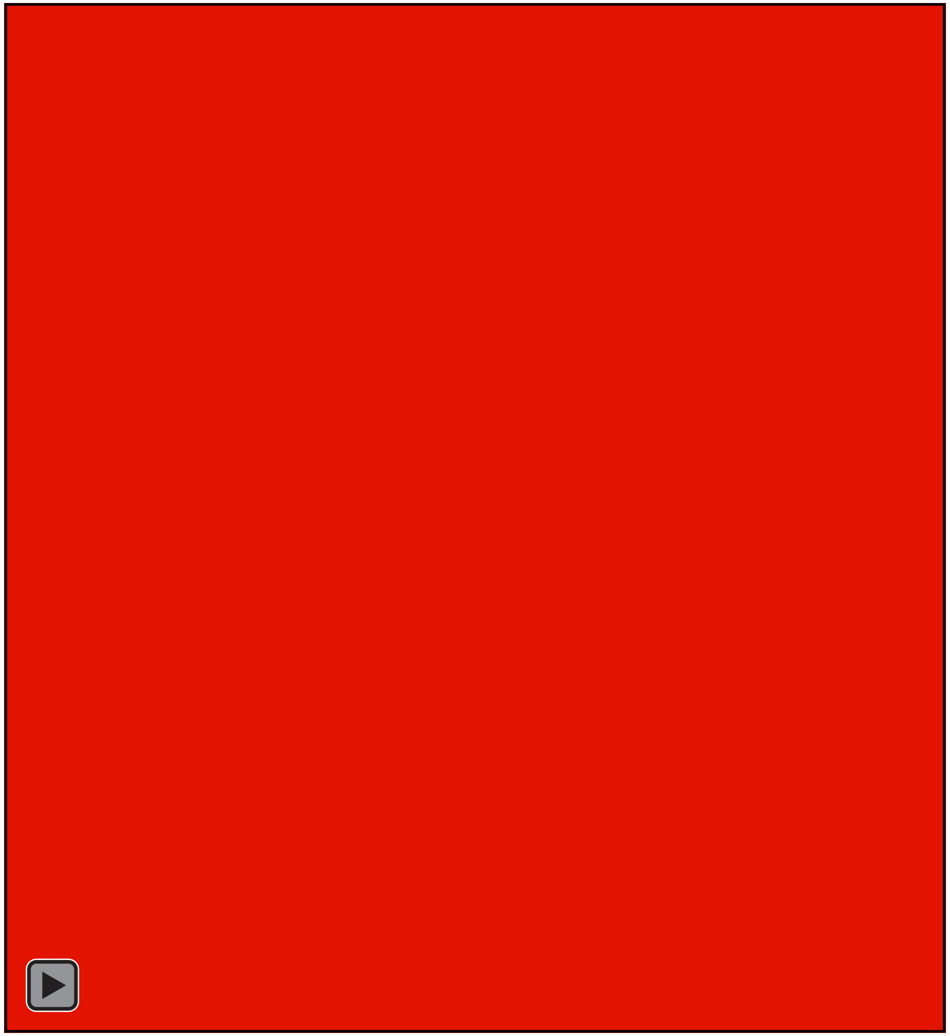
- Quasi-static, Plane strain
- Num. of Nodes: 335, Num of SPs: 1450
- Oscillating vertical disp. enforced
- Compared to ABAQUS/Standard with 8-node 2nd-order quadrilateral elements (CPE8)

Elasto-plastic Cantilever Bending

Distributions of Equivalent Plastic Strain



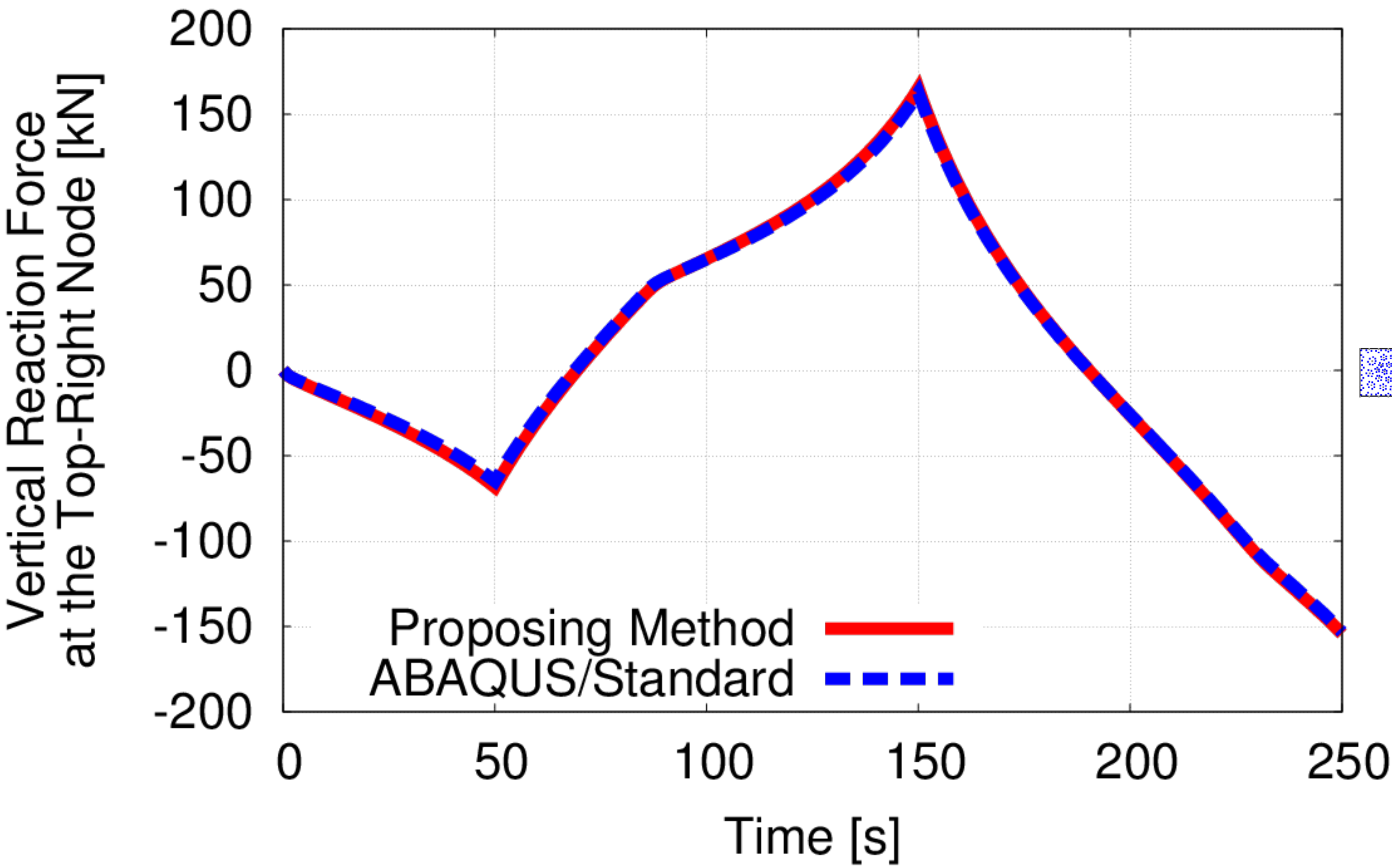
FSPI Meshfree

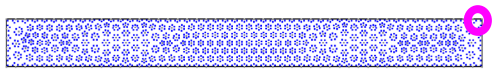


ABAQUS/Standard

Elasto-plastic Cantilever Bending

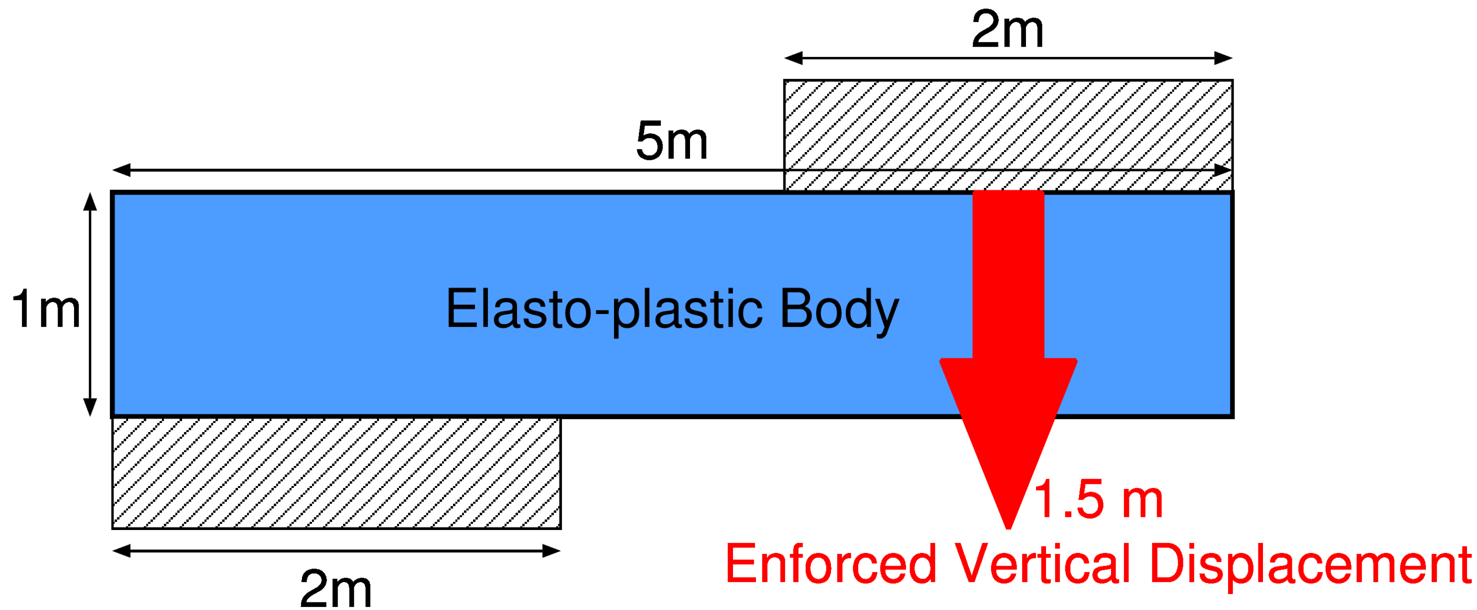
Time-history of Vertical Reaction Force at the Bounding Node





- Error of reaction force is less than 0.3%
- No shear locking in elasto-plastic case too

Elasto-plastic Bar Shearing



- Quasi-static, Plane strain
- Shearing with 1.5m vertical displacement
- The num. of nodes: 1052, The num. of SPs: 3156
- Compared to ABAQUS/Standard with 3-node 1st-order triangular elements (CPE3)

Elasto-plastic Bar Shearing

Distributions of
Equivalent Plastic Strain

FSPI
Meshfree

ABAQUS
/Standard

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1. Introduction and Objective

2. Formulation of
   Floating Stress-Point Integration (FSPI)
   Meshfree Method

3. Examples of Analysis

4. **Conclusion**

Summary and Future Work

■ Summary

- A new meshfree method for large deformation analysis, FSPI meshfree method, was proposed.
- Its formulation based on robust MLS and incremental equilibrium equation was presented.
- A patch test and a few examples of elasto-plastic large deformation analysis were performed to verify that the method had enough accuracy.

■ Future Work

- solve contact problems
- feature of adding node and stress-point
- adaptive FEM based on the incremental equilibrium equation

Appendix

List of Specifications

	Standard FEM	FSPI Meshfree
Node	Yes	Yes
Element	Yes	**No** (only for initialization)
Evaluation Point	Integration Point	**Stress-Point**
Shape Func.	in Element	**with MLS**
Integration Correction	Unnecessary	**Scaling Type Correction**
Time-advancing	Fully implicit	**Quasi-implicit** (shape func. etc. are explicit)
Reference Configuration	Updated/Total Lagrange	Updated Lagrange
Equilibrium Eq.	Standard Form	**Incremental Form**

Update Equations for SP variables

- Location $^I x$

$$
{}^I\boldsymbol{x}^{\mathrm{trial}} \longleftarrow {}^I\boldsymbol{x} + \sum_{J \in {}^I\mathbb{S}} {}^I\phi_J \left({}_J\boldsymbol{x}^{\mathrm{trial}} - {}_J\boldsymbol{x} \right)
$$

$\boldsymbol{x}$: Current Posiotion, S: Set of Nodes in Support,
ϕ: Shape Function

- Corresponding volume $^I V$

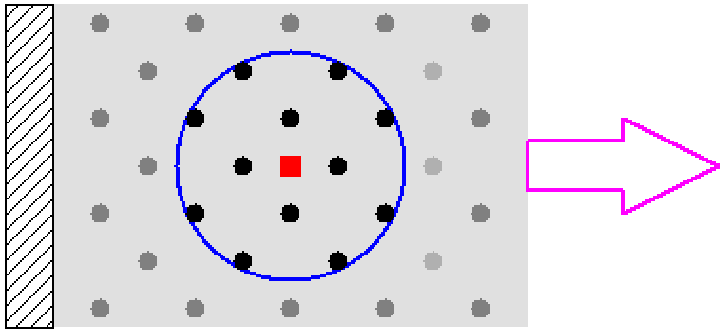
$$
{}^I V^{\mathrm{trial}} \longleftarrow {}^I V^{\mathrm{initial}} \det({}^I\boldsymbol{F}^{\mathrm{trial}})
$$

V^{initial}: Initial Corresponding Volume
$\boldsymbol{F}$: Deformation Gradient

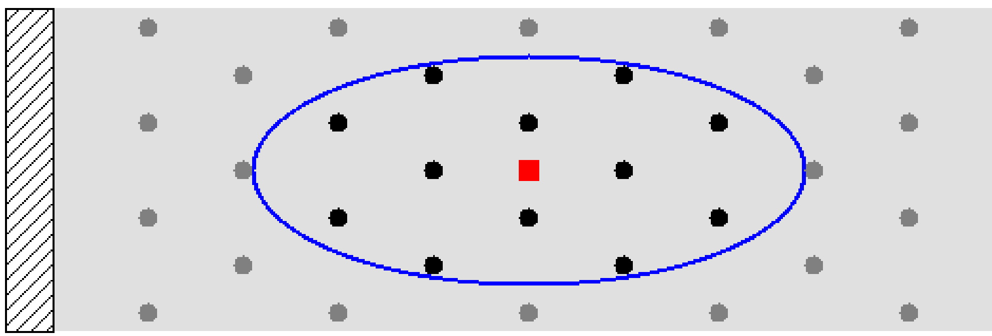
MLS with Total/Updated Lagrange

- In case of wide horizontal stretch:

Before

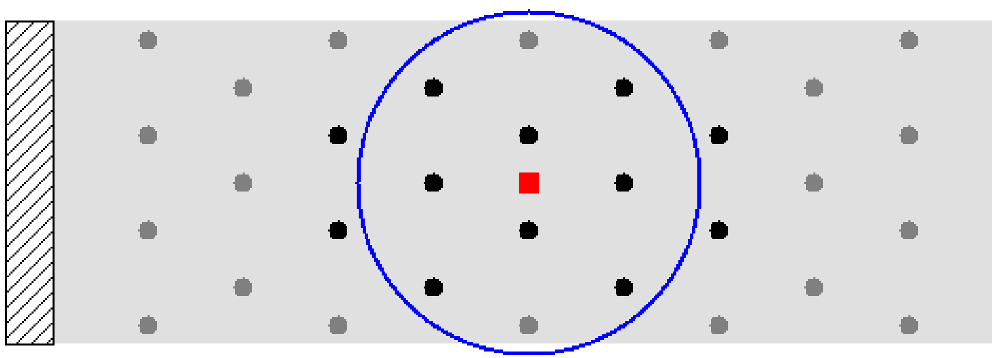


After (Total-Lagrange)



Support is widened
↓
Unsuitable for very large deformation and rezoning

After (Updated-Lagrange)



Support is dynamic circles
↓
Having nodes to start/stop relation (Same as adaptive FEM)