Recent Advancement in Smoothed Finite Element Methods for Locking-free Analysis with Tetrahedral Elements

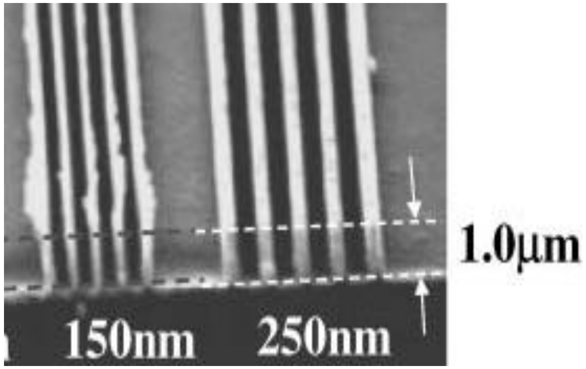
Yuki ONISHI
Tokyo Institute of Technology (Japan)

Motivation & Background

Motivation

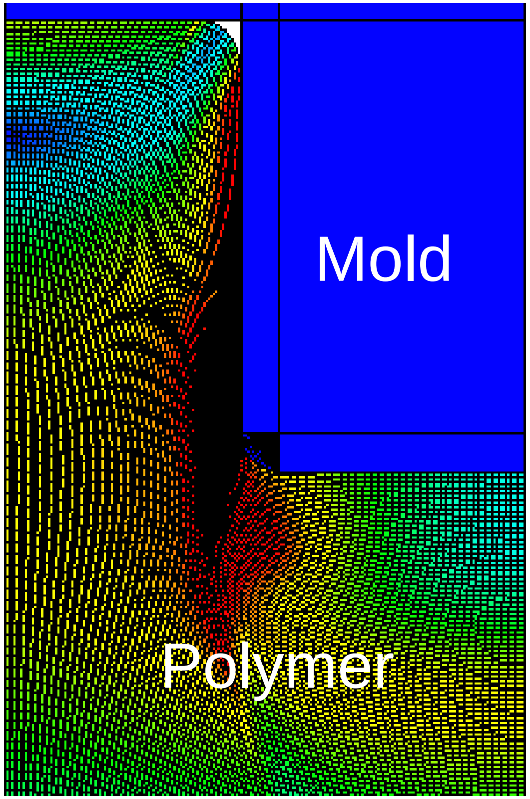
We want to analyze **severely large deformation** problems in solids ***accurately and stably!***

(Target: automobile tire, thermal nanoimprint, etc.)



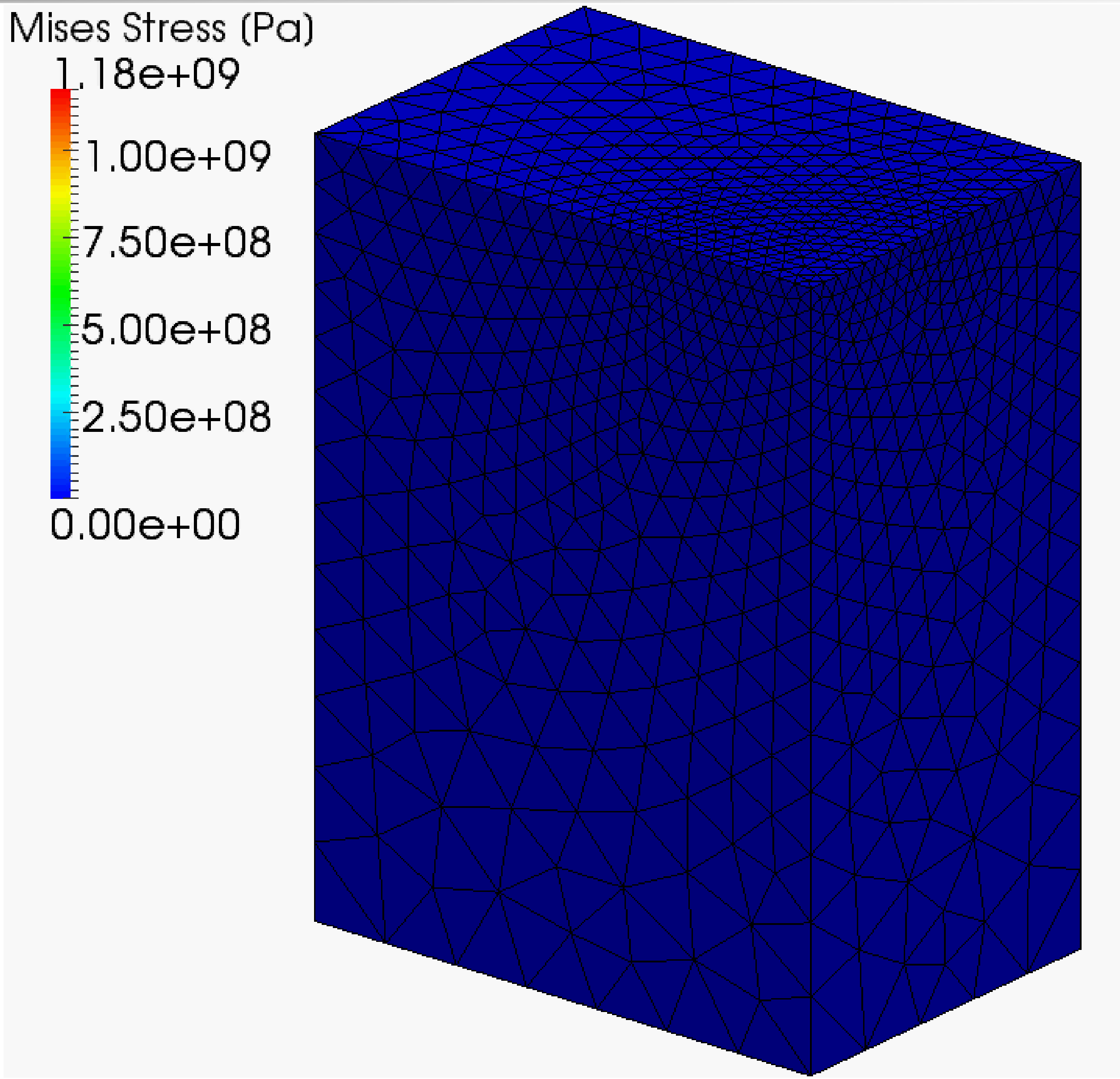
Background

Finite elements are **distorted** in a short time, thereby resulting in convergence failure.



Mesh rezoning method (*h*-adaptive mesh-to-mesh solution mapping) is indispensable.

Our First Result in Advance



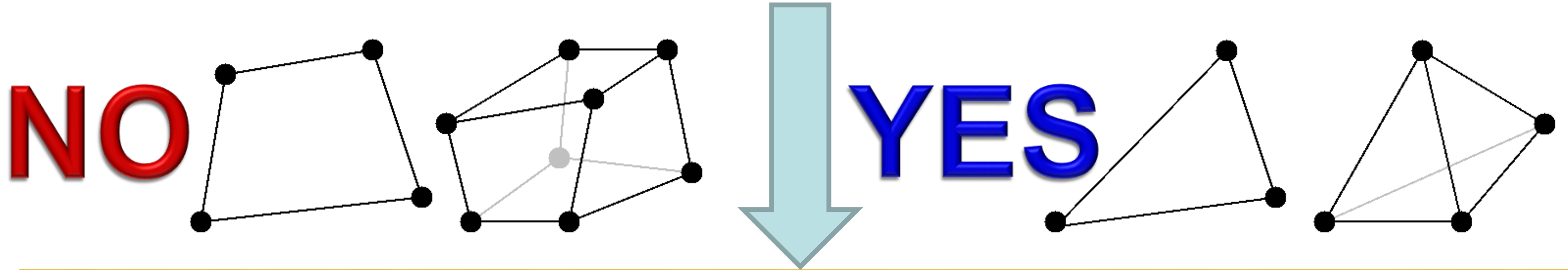
What we want to do:

- Static
- Implicit
- Large deformation
- Mesh rezoning

Issues

<u>The biggest issue</u>
<u>in large deformation mesh rezoning</u>

It is impossible to remesh arbitrary deformed 2D or 3D domains with quadrilateral or hexahedral elements.



We have to use triangular or tetrahedral elements...

However, the *standard* (constant strain) triangular or tetrahedral elements induce shear and volumetric locking easily, which leads to inaccurate results.

Conventional Methods

- Higher order elements:
  - ✗ Not volumetric locking free; Not effective in large deformation due to intermediate nodes.
- EAS elements:
  - ✗ Unstable.
- B-bar, F-bar and selective integration elements:
  - ✗ Not applicable to triangular/tetrahedral mesh.
- F-bar patch elements:
  - ✗ Difficult to construct good patches
- u/p hybrid (mixed) elements
  - ✗ No sufficient formulation for triangular/tetrahedral mesh is presented so far. (There are almost acceptable hybrid elements such as C3D4H or C3D10H of ABAQUS.)
- Smoothed finite elements:
  - ? Unknown potential (since 2009~). ***Let's try!***

Objective

Develop a locking-free, accurate and stable smoothed finite element method (S-FEM) with 4-node tetrahedral elements (T4) for large deformation problems

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- Quick survey of **3 classical S-FEMs**
- Introduction to *a recent* S-FEM: **hES-FEM**
- Introduction to *our new* S-FEM: **F-barES-FEM**
- Summary

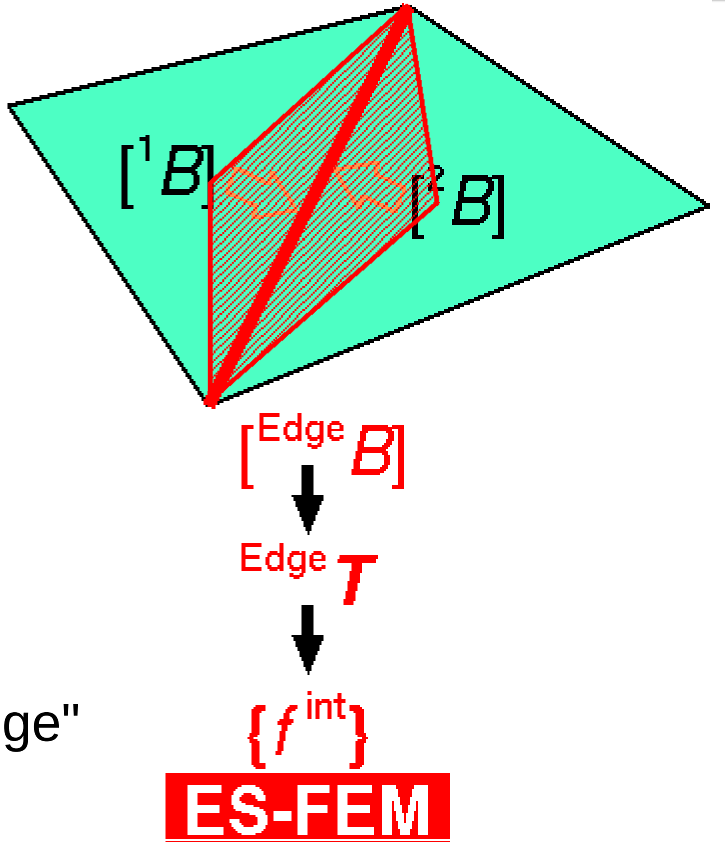
Quick survey of

3 Classical S-FEMs

(i) Edge-based S-FEM (ES-FEM)

- Calculate $[B]$ at element as usual.
- Distribute $[B]$ to the connecting edges and make $[\,{}^{\mathrm{Edge}}B]$.
- ***F*, *T*** etc and $\{f^{\mathrm{int}}\}$ are calculated on smoothed edge domains.

Generally accurate but induces volumetric locking.



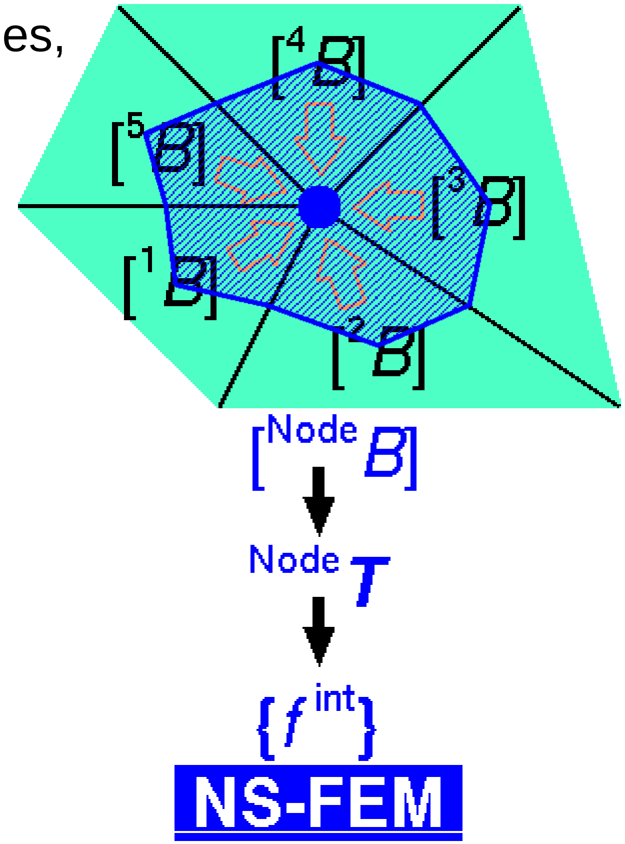
Substituting "face" for "edge" gives FS-FEM for 3D

(ii) Node-based S-FEM (NS-FEM)

- Calculate $[B]$ at element as usual.
- Distribute $[B]$ to the connecting nodes and make $[{}^{\mathrm{Node}}B]$
- ***F*, *T*** etc and $\{f^{\mathrm{int}}\}$ are calculated on smoothed node domains.

Generally not accurate but volumetric locking free.

(due to zero-energy modes, which are arisen in reduced integration finite elements as hour-glass modes)



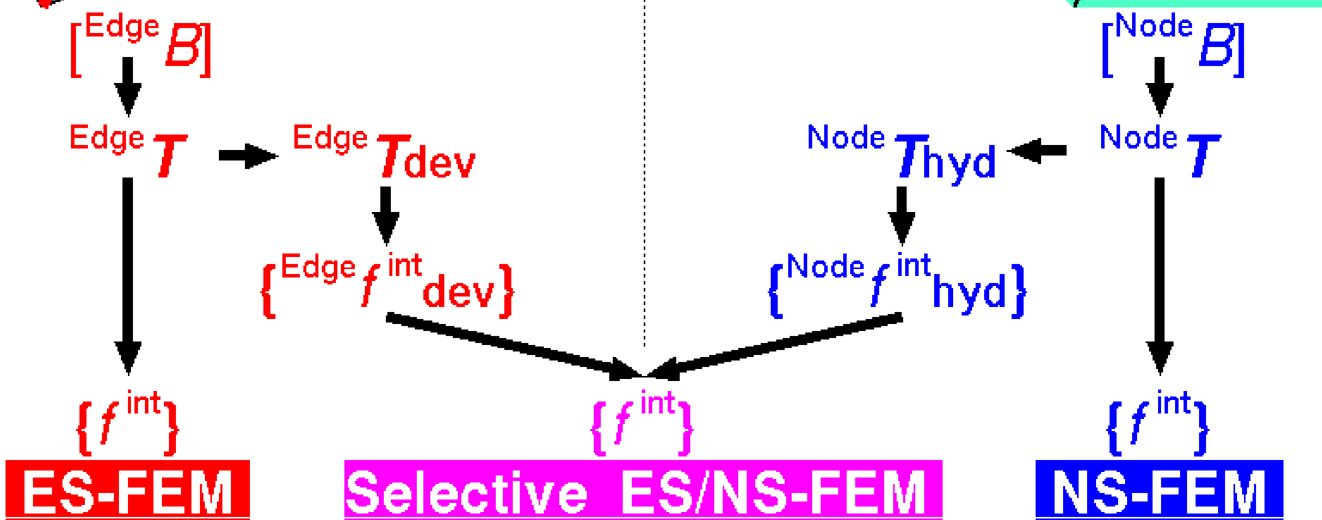
close to FVM with vertex-based control volume

(iii) Selective ES/NS-FEM

- Separate stress into "deviatoric part" and "hydrostatic part" and combine them .
- ***F*, *T*** etc and $\{f^{\,int}\}$ are calculated on both smoothed domains.

No locking & No zero-energy modes!!

Substituting "face" for "edge" gives FS/NS-FEM for 3D



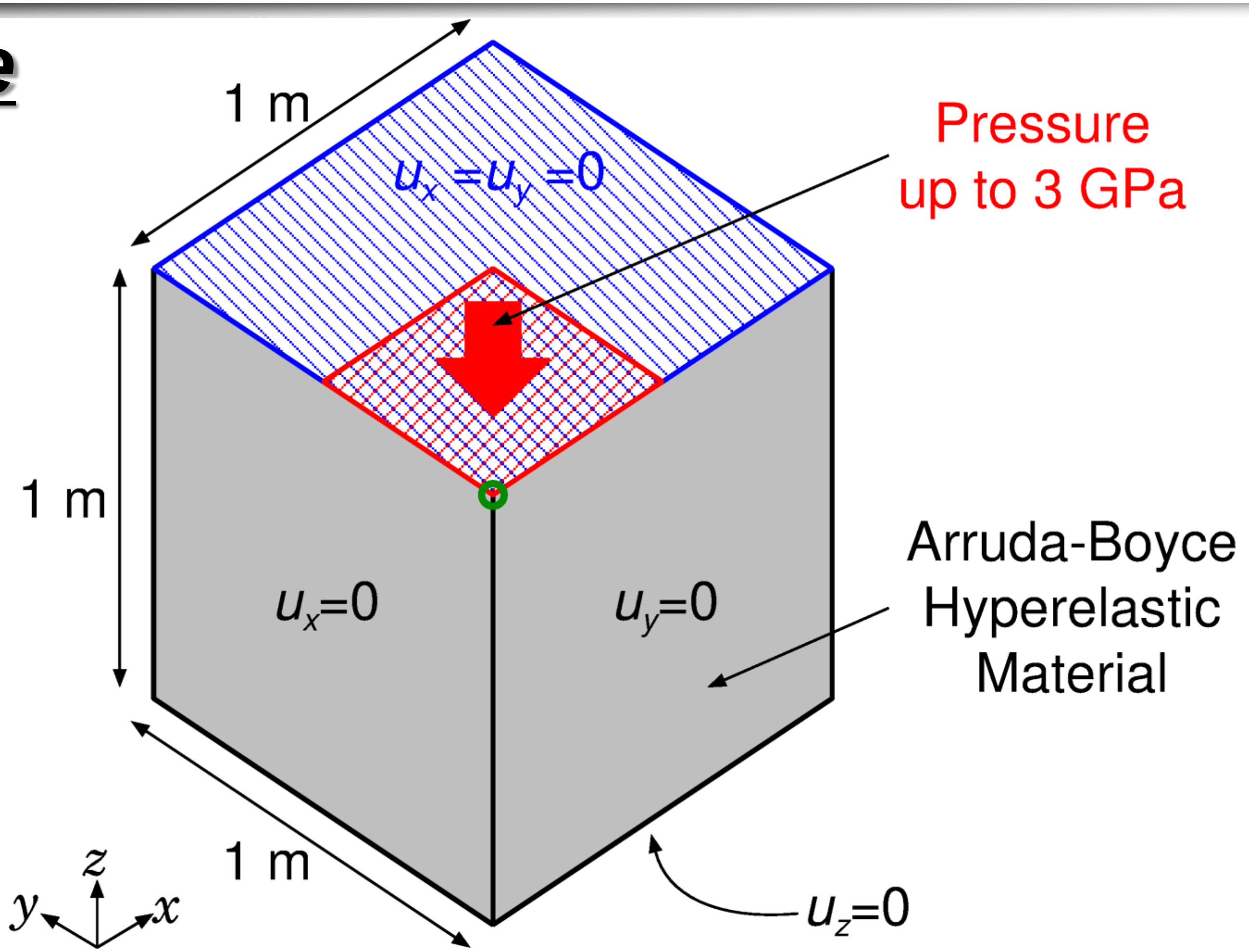
Characters of 3 Classical S-FEMs

- All the classical S-FEMs have an unique benefit: no increase in DOF.
  - The nodal displacement vectors are only the unknown. (No pressure or volumetric strain unknowns.)
  - Static condensation is unnecessary.

- All S-FEMs have a drawback: increase in the bandwidth of stiffness matrix [K].
  - [Bandwidth of ES-FEM-T4] $\simeq 2 \times$ [Bandwidth of standard FEM-T4])
  - [Bandwidth of NS-FEM-T4] = [Bandwidth of selective S-FEM-T4] $\simeq 4 \times$ [Bandwidth of standard FEM-T4])

Example: Compression of Hyperelastic Block

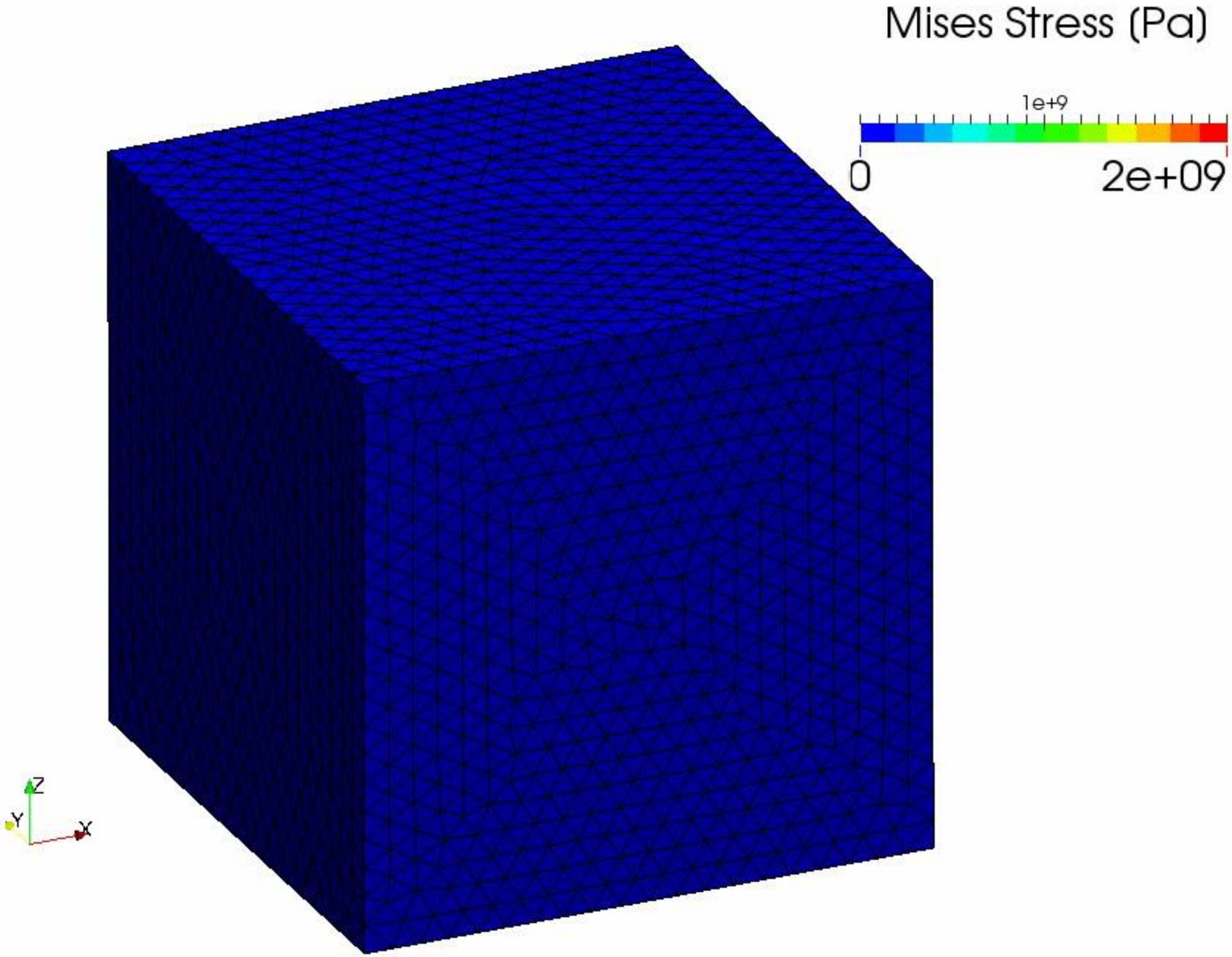
Outline



- Arruda-Boyce hyperelastic material ($\nu_{\mathrm{ini}} = 0.499$)
- Applying pressure on ¼ of the top face

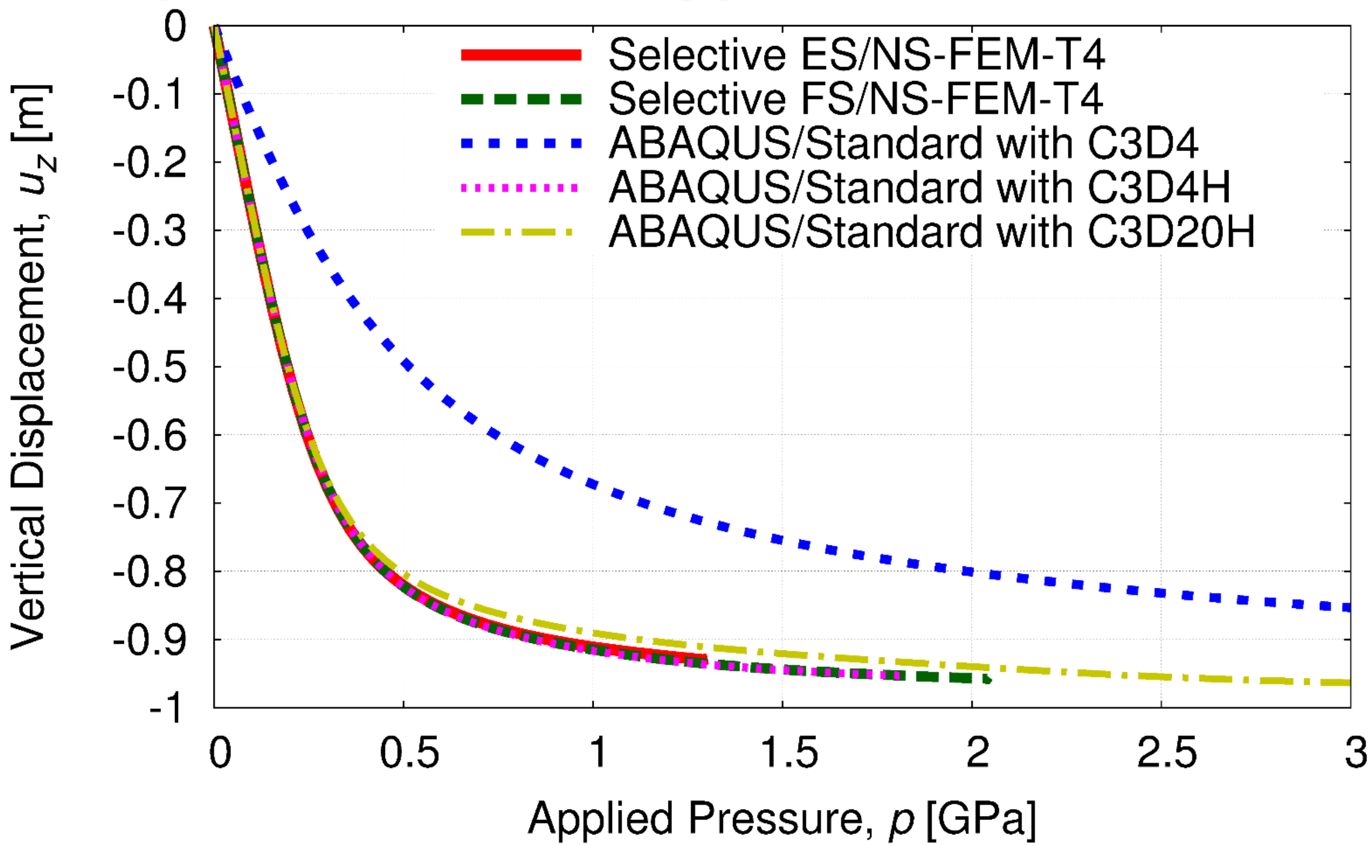
Example: Compression of Hyperelastic Block

Result of Selective ES/NS-FEM-T4



Example: Compression of Hyperelastic Block

Vertical Displacements vs. Applied Pressure



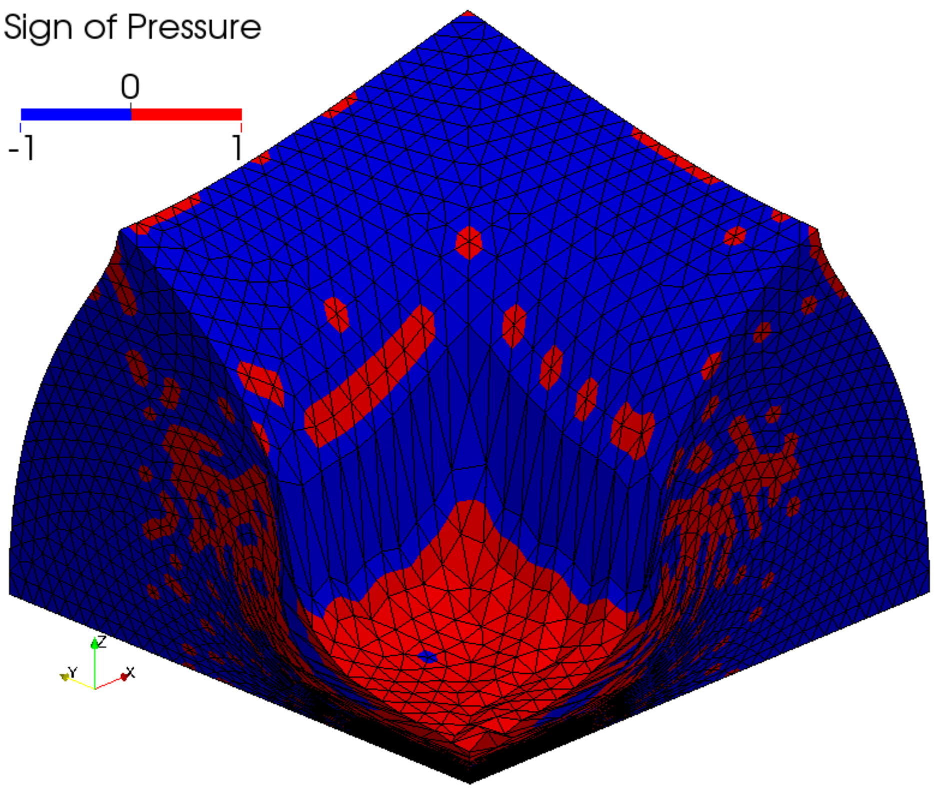
- Constant strain element (C3D4) locks quickly.
- Other elements including selective S-FEMs do not lock.

Selective S-FEMs are **locking-free** in large strain analysis!!

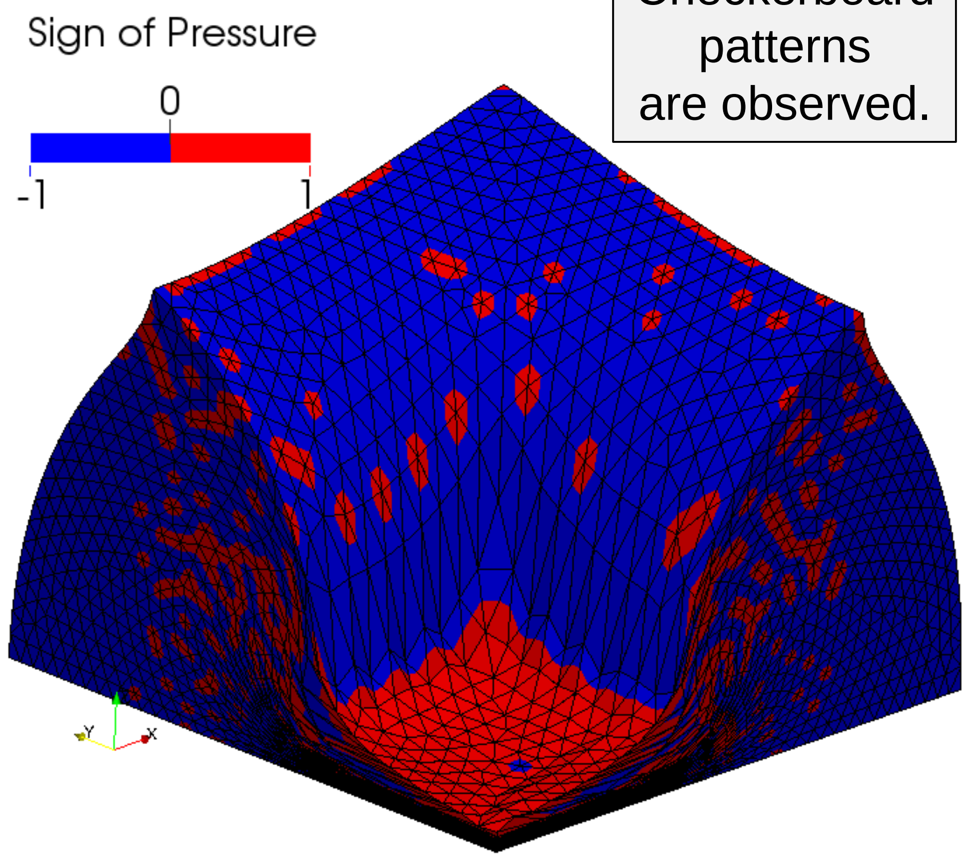
Example: Compression of Hyperelastic Block

Sign of Pressure

Checkerboard patterns are observed.



Selective **ES**/NS-FEM-T4



Selective **FS**/NS-FEM-T4

Pressure oscillation is present…

Characteristics of Classical S-FEMs

	Shear Locking	Volumetric Locking	Zero-Energy Mode	Pressure Oscillation
Standard FEM-T4	✗	✗	✓	✗
NS-FEM-T4	✓	✓	✗	✗
ES-FEM-T4 & FS-FEM-T4	✓	✗	✓	✗
Selective S-FEM-T4	✓	✓	✓	✗

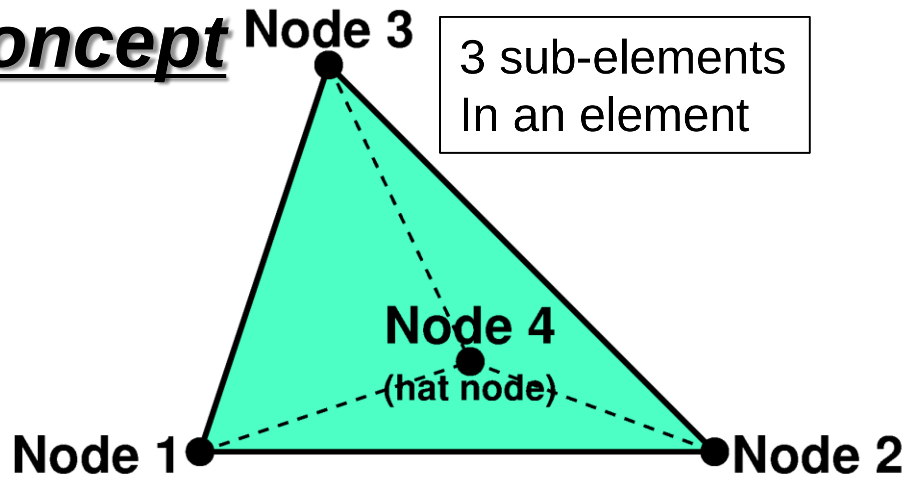
There still remain issues to be resolved.

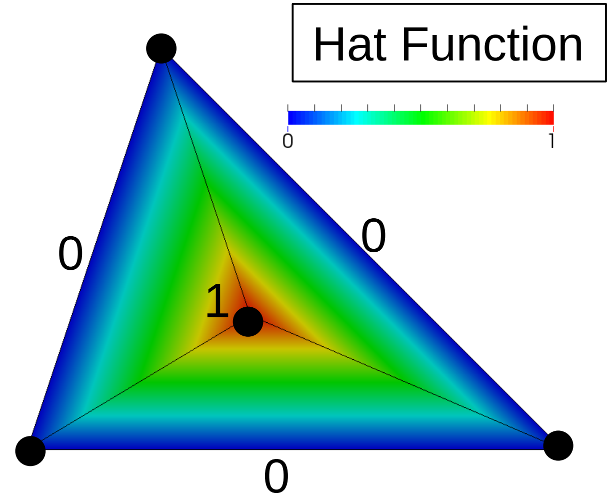
Introduction to *a Recent* S-FEM:

hES-FEM

Hat-enhanced ES-FEM (hES-FEM)

Concept



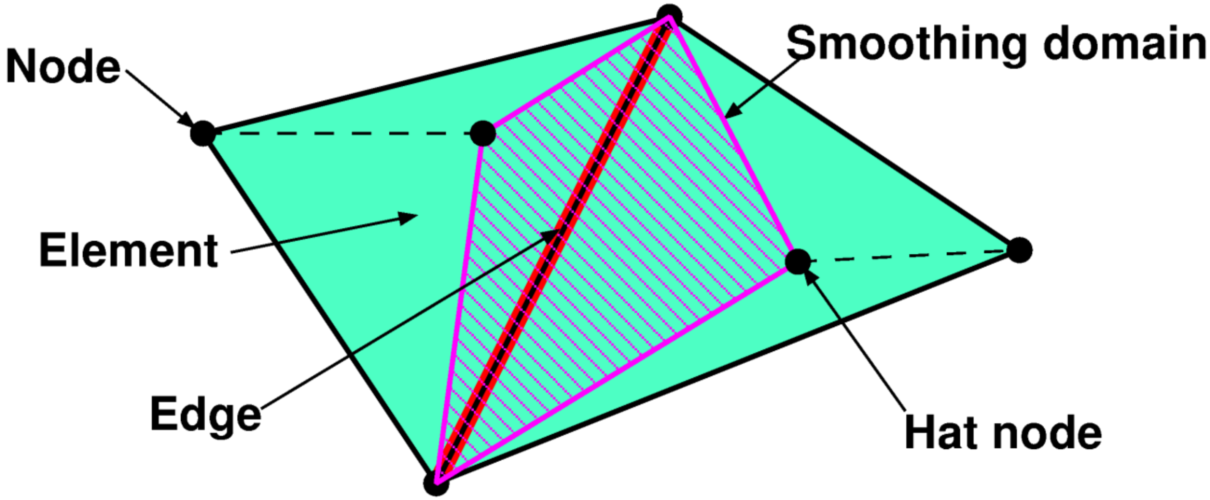


- An additional internal node (***hat node*** or bubble node) is put at each standard triangular element.
- Each hat node has its own displacement DOFs.
- The shape function of hat nodes is the hat function.
- Namely in hES-FEM, 3 sub-elements ($\Delta 124$, $\Delta 234$, $\Delta 314$) have the same shape functions as the standard triangular elements.

Hat-enhanced ES-FEM (hES-FEM)

Brief formulation

- Calculate $[B]$ at sub-element as usual.
- Distribute $[B]$ to the connecting edges and make $[{}^{\mathrm{Edge}}B]$.
- ***F, T*** etc and $\{f^{\mathrm{int}}\}$ are calculated on smoothed edge domains.



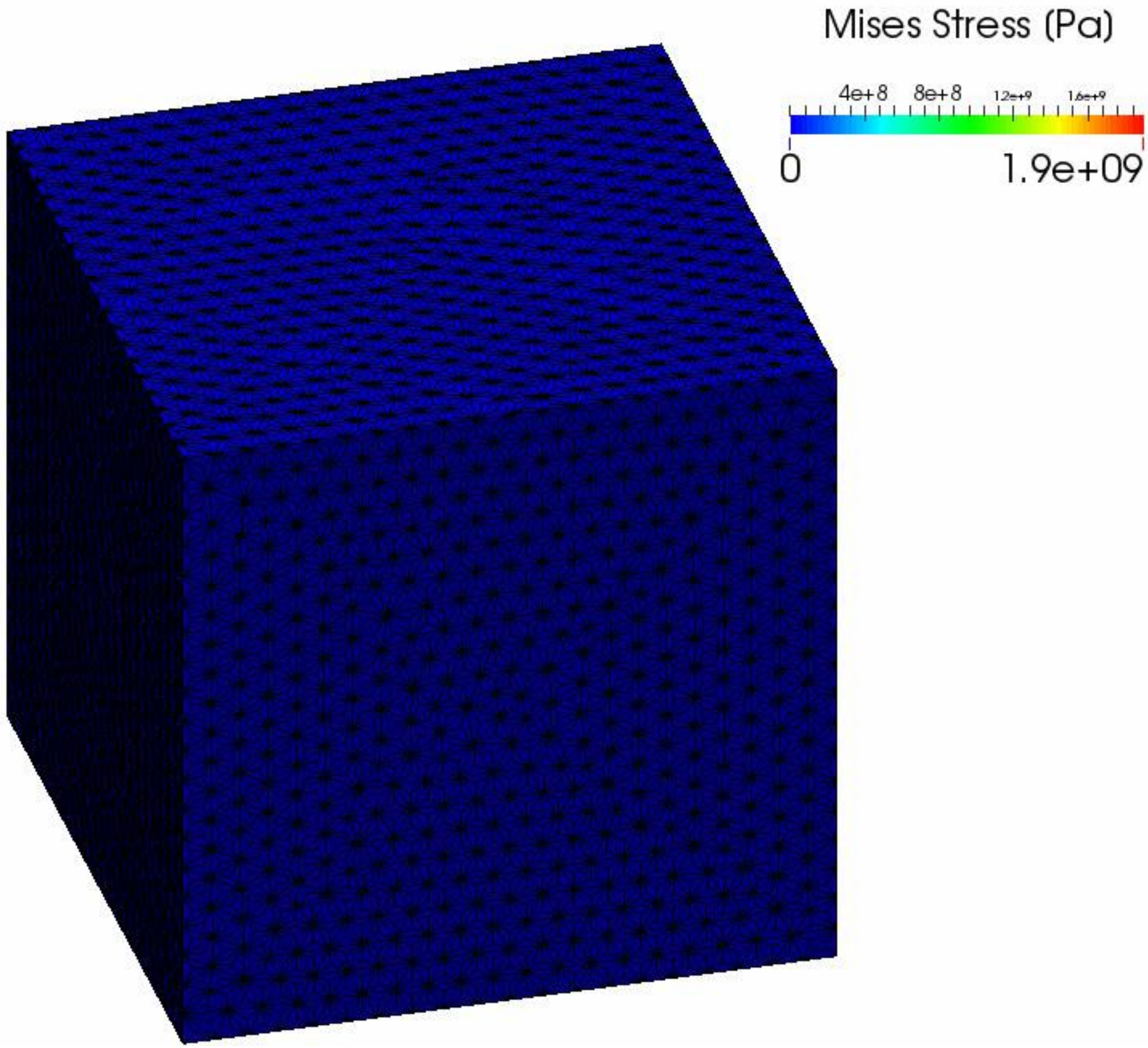
Segments between nodes and hat nodes are not regarded as edges.

$$[{}^{\mathrm{Edge}}B] \longrightarrow {}^{\mathrm{Edge}}\boldsymbol{F} \longrightarrow {}^{\mathrm{Edge}}\boldsymbol{T} \longrightarrow \{{}^{\mathrm{Edge}}f^{\mathrm{int}}\}$$

hES-FEM

Example: Compression of Hyperelastic Block

Result of hES-FEM

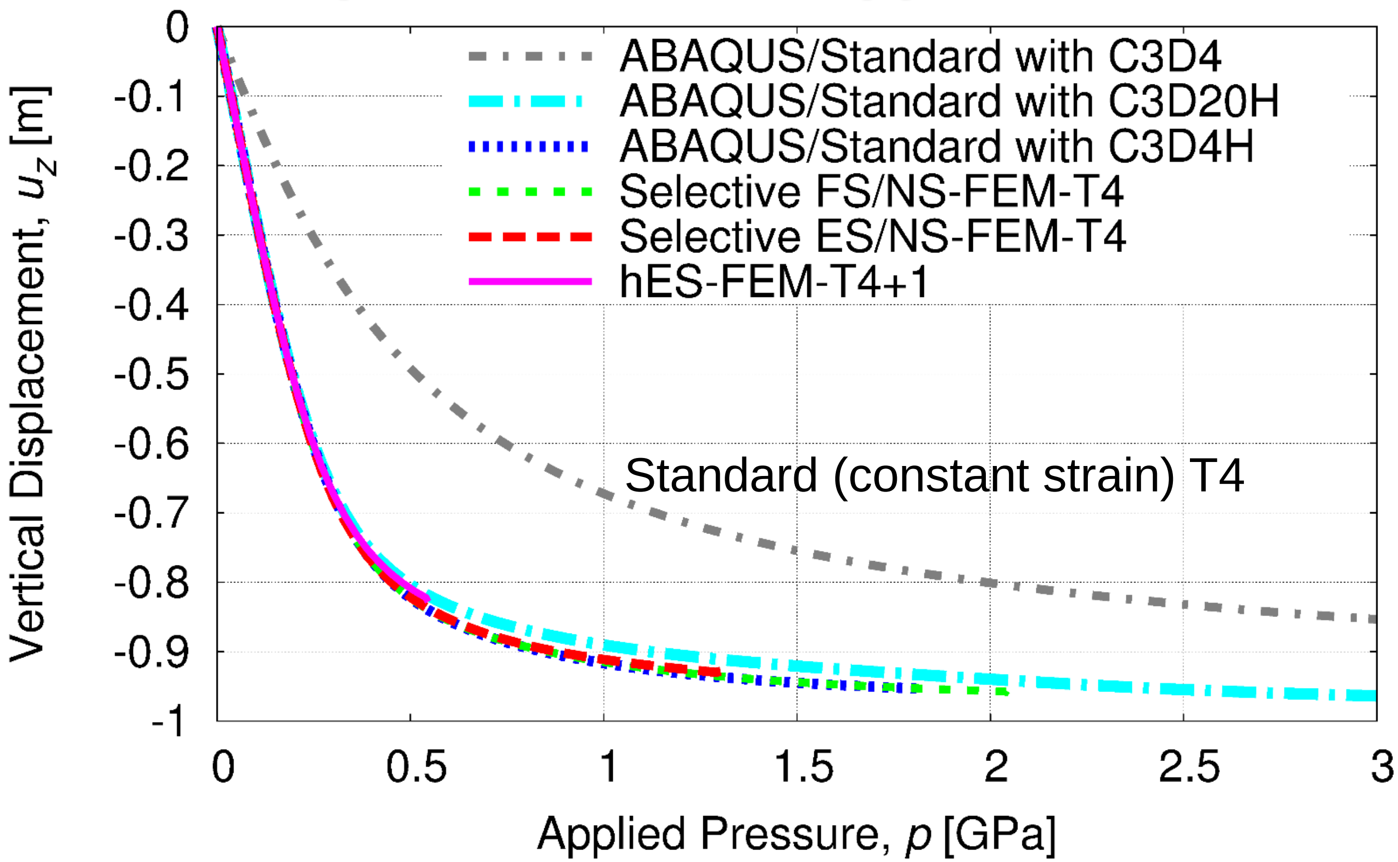


Mises stress distribution is smooth.

But, it got convergence failure at a relatively earlier stage due to pop out of hat nodes…

Example: Compression of Hyperelastic Block

Vertical Displacements vs. Applied Pressure

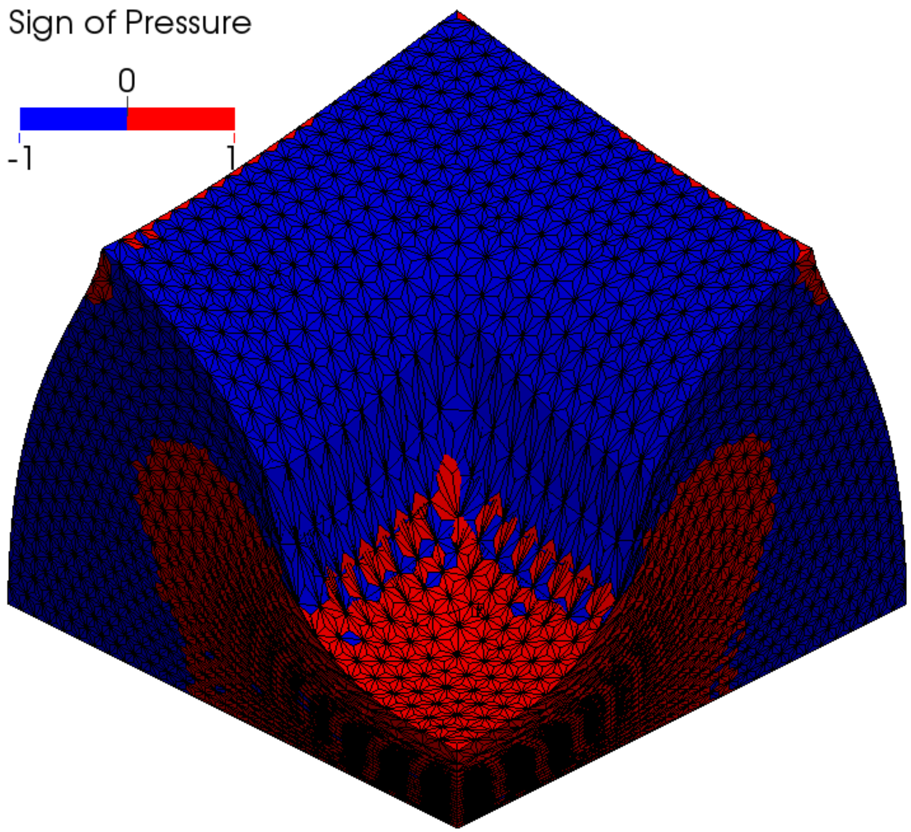


hES-FEM is loking-free but cannot bear with severe strain.

- Locking-free and accurate.
- hES-FEM stopped at 82% nominal compression, where as other locking-free methods stopped around 95% of that.

Example: Compression of Hyperelastic Block

Pressure distribution



Pressure oscillation is almost suppressed.

Characteristics of hES-FEM

■ Benefits

✓Locking-free.

✓Only displacement DOF.

✓Suppress pressure oscillation to some extent.

■ Drawbacks

✗ Early convergence failure due to pop out of internal nodes.

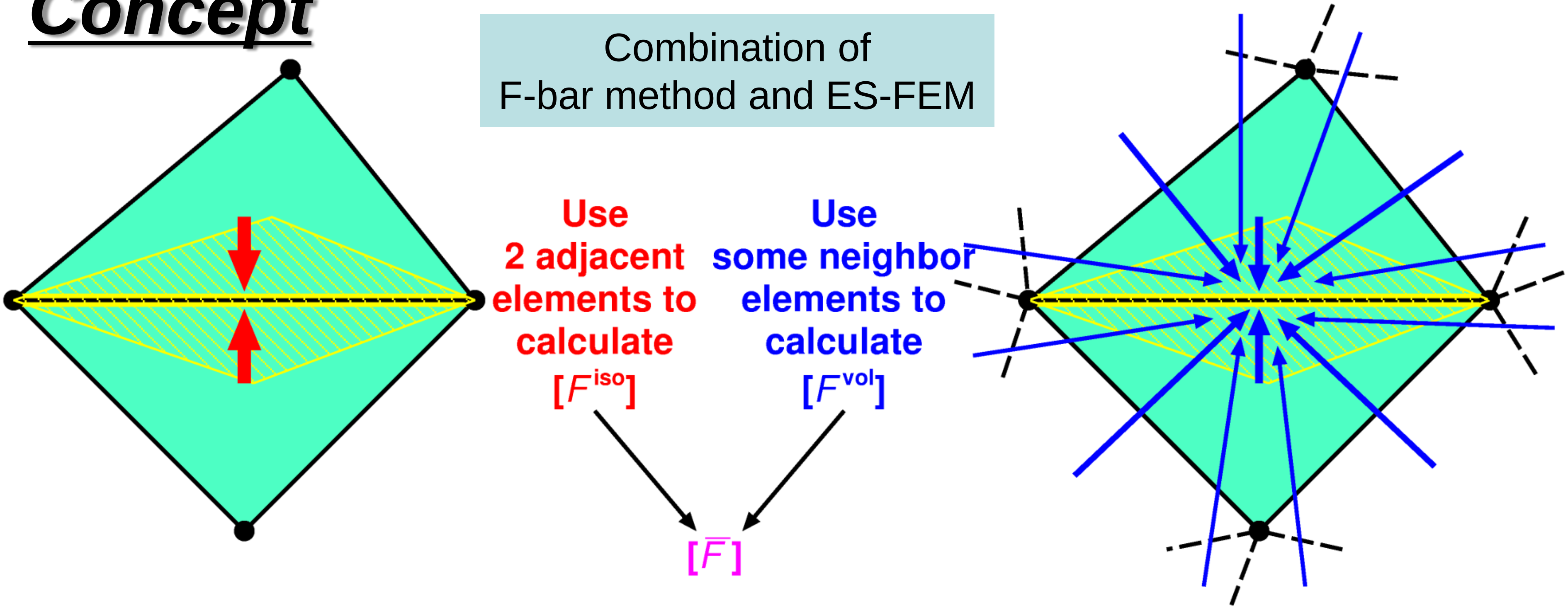
✗ Significant increase in total DOF.
In general unstructured T4 mesh,
[the num. of elements] $\simeq 5 \times$ [the num. of nodes].
i.e., total DOF of hES-FEM is 6 times larger…

Introduction to *our new* S-FEM:

F-barES-FEM

F-bar aided ES-FEM (F-barES-FEM)

Concept



- Use classical ES-FEM to calculate $[F^{\mathrm{iso}}]$.
- Use "cyclic smoothing" to calculate $[F^{\mathrm{vol}}]$.
- Apply F-bar method to obtain $[\bar{F}]$.

F-bar aided ES-FEM (F-barES-FEM)

Brief Formulation

1. Calculate J $(= \det([F])$ at elements as usual.

2. Smooth J of elements at nodes.
3. Smooth J of nodes at elements.
4. Repeat 2. and 3. as necessary.

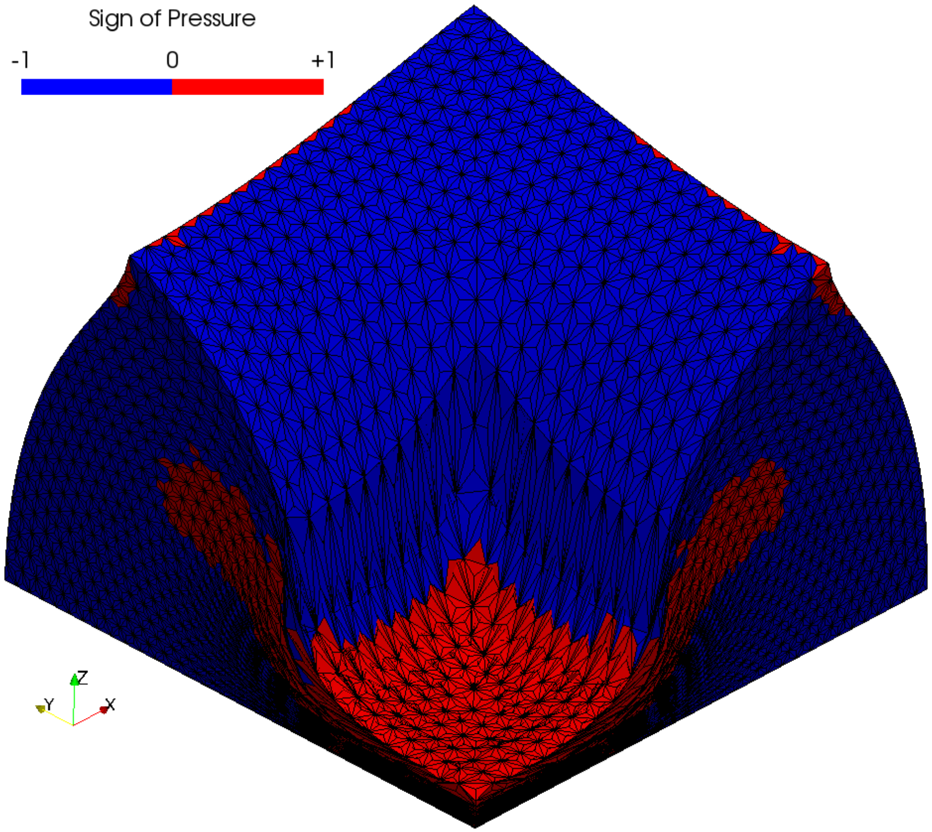
Cyclic Smoothing of J

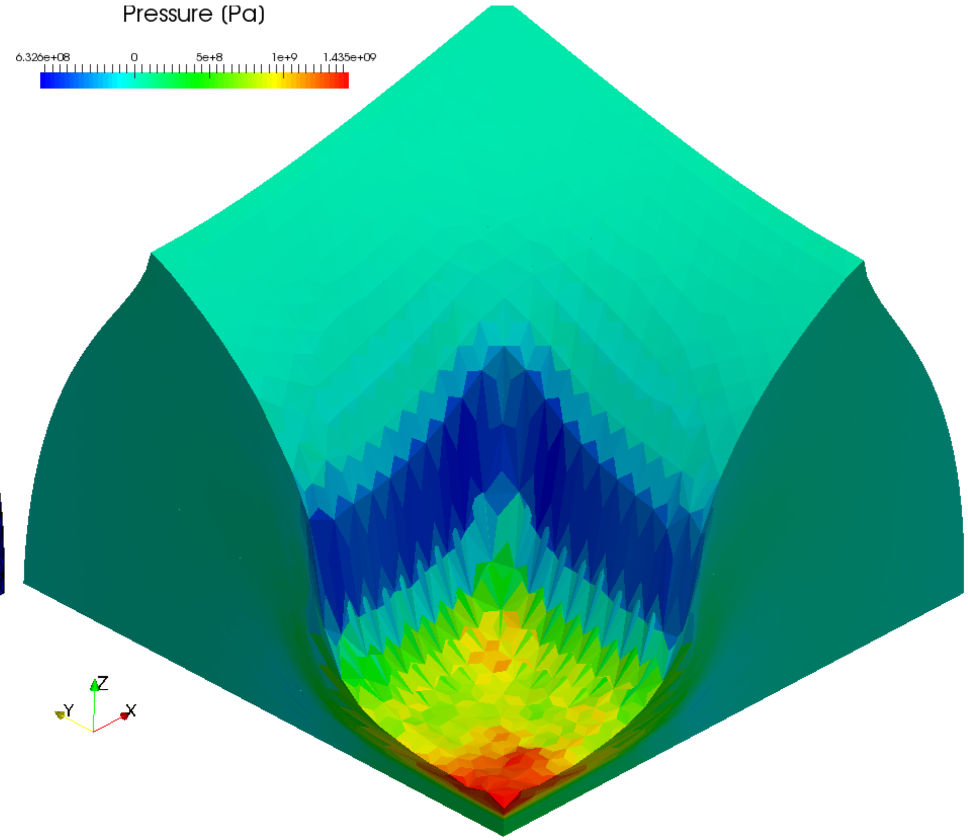
(Currently we employ 2 cycles.)

5. Smooth J of elements at edges.
6. Combine the cyclically smoothed $\bar{J}$ and $[F^{\mathrm{iso}}]$ of ES-FEM as $[\bar{F}] = \bar{J}^{1/3}\,[F^{\mathrm{iso}}]$.

Example: Compression of Hyperelastic Block

Pressure distribution



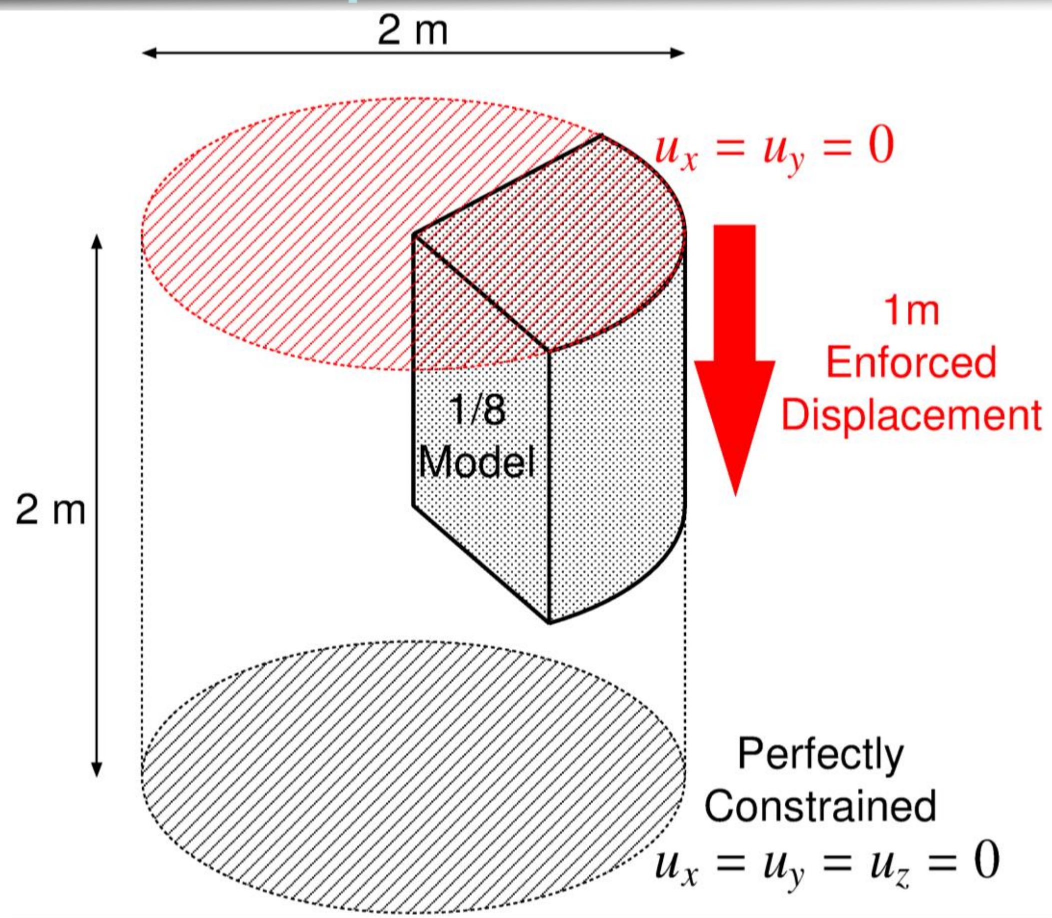


No checkerboard patterns are observed.

F-barES-FEM(2) suppresses pressure oscillation!!

Example: Compression of 1/8 Cylinder

Outline

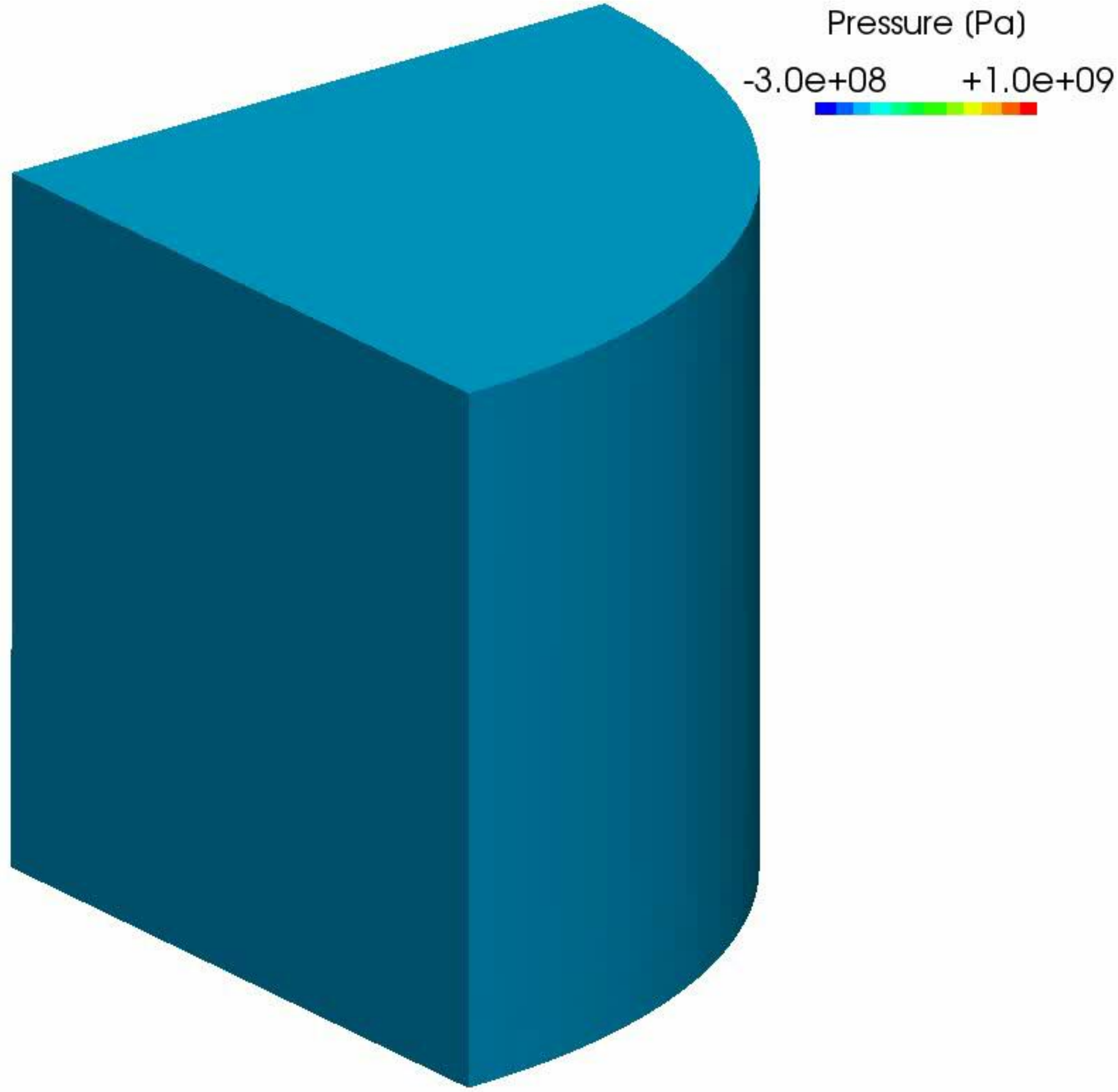


- Neo-Hookean hyperelastic material ($\nu_{\mathrm{ini}} = 0.499$).
- Enforced displacement is applied to the top surface.
- Stress singularity is present around the rim.

Example: Compression of 1/8 Cylinder

Result of F-bar ES-FEM(2)

50% nominal compression



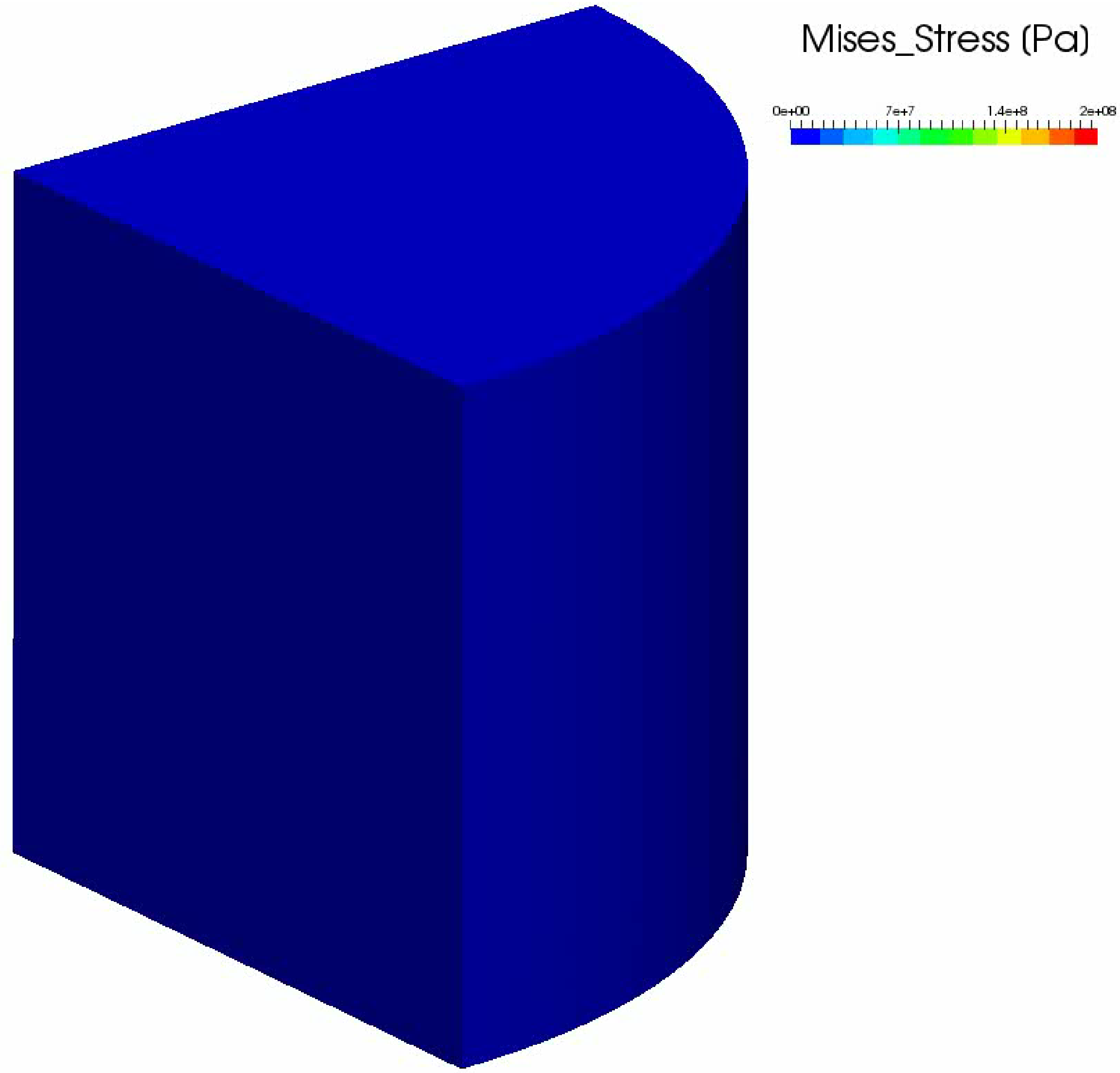
Almost smooth pressure distribution is obtained except just around the rim.

Example: Compression of 1/8 Cylinder

Result of F-bar ES-FEM(2)

50% nominal compression

Smooth Mises stress distribution is obtained except just around the rim.



Characteristics of F-barES-FEM

■ Benefits

- ✓Locking-free.
- ✓No increase in DOF.
- ✓Suppress pressure oscillation as increasing the number of cyclic smoothing.

■ Drawbacks

- ✗ Increase in bandwidth of stiffness matrix [*K*].
  In general unstructured T4 mesh,
  F-barES-FEM(1): 10 times wider than FEM-T4,
  F-barES-FEM(2): 20 times wider than FEM-T4.

But, exactly consistent [*K*] is not necessary and thus speed up can be expected.

Summary

Characteristics of S-FEMs

	Shear & Volumetric Locking	Zero-Energy Mode	Dev/Vol Coupled Material	Pressure Oscillation	Severe Strain
Standard FEM-T4	✗	✓	✓	✗	✓
Selective S-FEM-T4	✓	✓	✗	✗	✓
hES-FEM-T4	✓	✓	✓	✓	✗
F-bar ES-FEM-T4	✓	✓	✓	✓	✓

This is ✓ when the num. of cyclic smoothing is large enough; but, speed-up is necessary.

Take-Home Messages

1. Recent S-FEMs are about to realize locking-free, accurate and stable large deformation analysis of nearly incompressible solids with T4 elements.

2. Recent S-FEMs have no increase in DOF (or no increase in non-displacement DOF); thus, they have potential to be applied to various types of analysis such as eigen mode, harmonic response, explicit dynamic, etc..

Thank you for your kind attention!