Recent advances in smoothed finite element methods with tetrahedral elements

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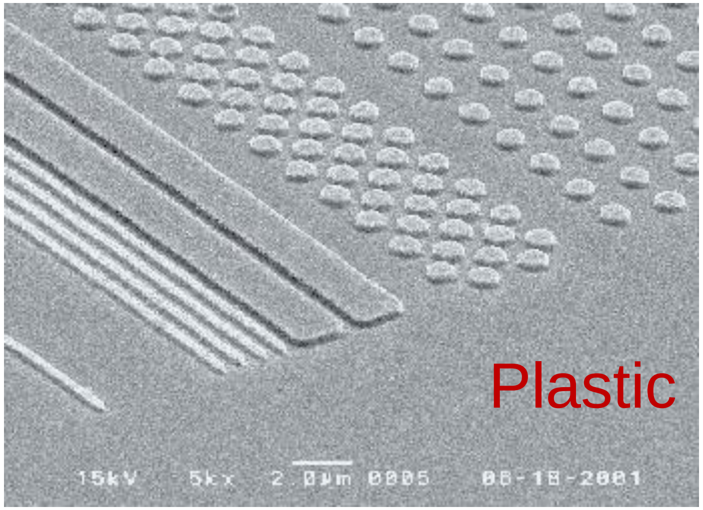
Motivation

What we want to do:

- Solve **hyper large deformation** analyses accurately and stably.
- Treat complex geometries with **tetrahedral meshes**.
- Consider **nearly incompressible materials** $(\nu \simeq 0.5)$.
- Support **contact** problems.
- Handle **auto re-meshing**.



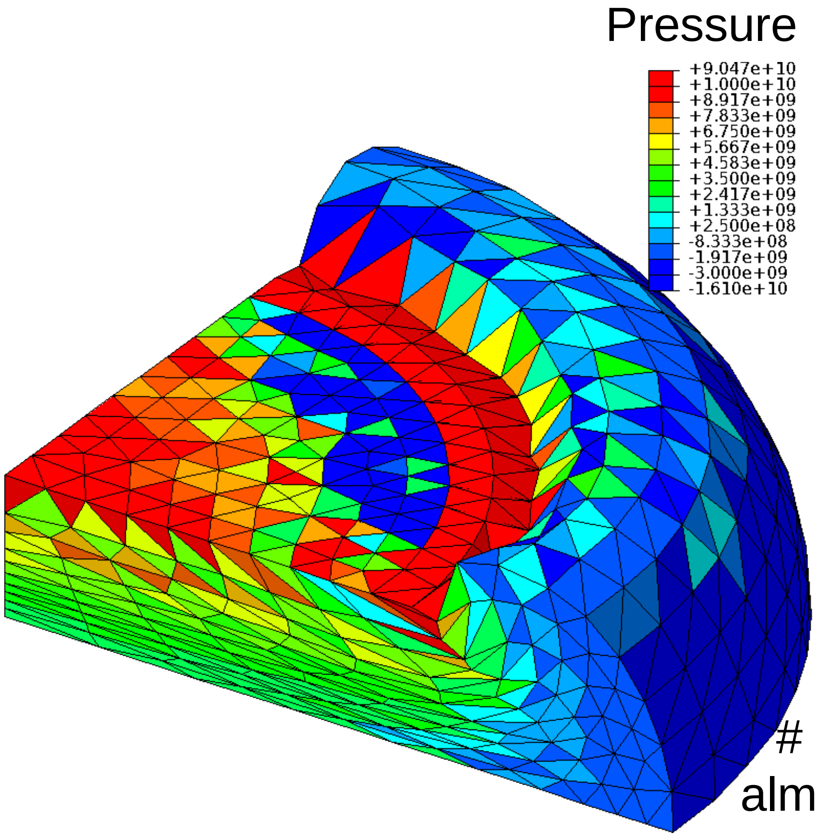
Issues

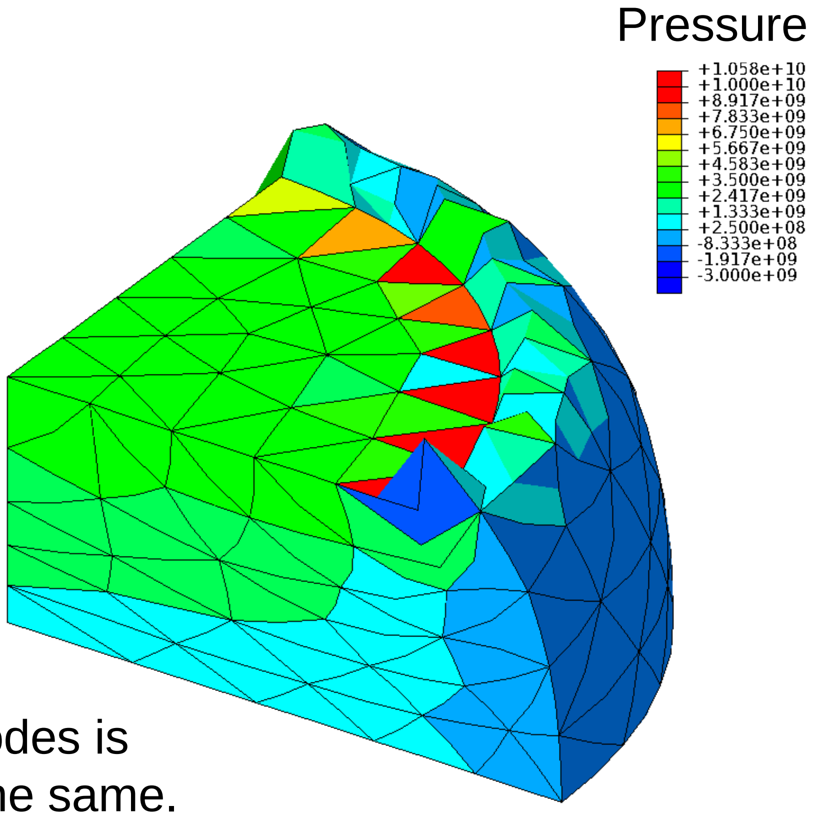
Conventional tetrahedral (T4/T10) FE formulations still have issues in accuracy or stability especially in nearly incompressible cases.

- 2nd or higher order elements:
  - ✗ Volumetric locking.
    Accuracy loss in large strain due to intermediate nodes.
- Enhanced assumed strain method (EAS):
  - ✗ Spurious low-energy modes.
- B-bar method, F-bar method, Selective reduced integration:
  - ✗ Not applicable to tetrahedral element directly.
- F-bar-Patch method:
  - ✗ Difficulty in building good-quality patches.
- **u/p mixed (hybrid) method:**
  (e.g., ABAQUS/Standard **C3D4H** and **C3D10MH**)
  - ✗ Pressure checkerboarding etc..

Issues (cont.)

E.g.) Compression of neo-Hookean **hyperelastic** body with $\nu_{\mathrm{ini}} = 0.49$





# of Nodes is almost the same.

1st order hybrid T4 (C3D4H)

- ✓ No shear/volumetric locking
- ✗ Pressure checkerboarding
- ✗ Corner locking

2nd order modified hybrid T10 (C3D10MH)

- ✓ No shear/volumetric locking
- ✗ Early convergence failure
- ✗ Low interpolation accuracy

A Recent Solution

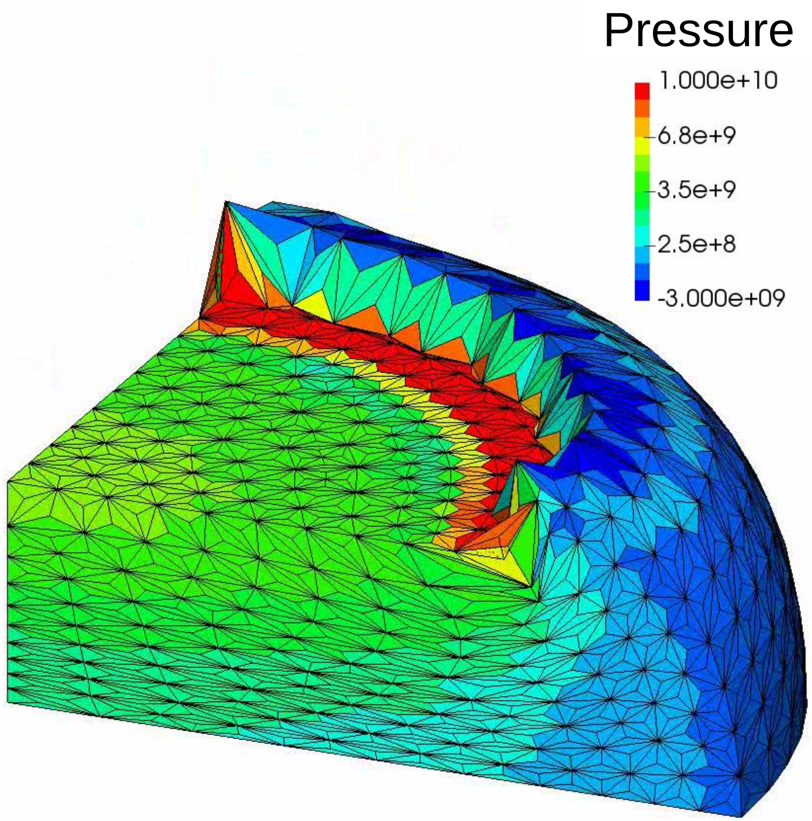
A new idea of FE formulation called “Smoothed Finite Element Method (**S-FEM**)” was recently proposed and is in researching today widely.

Our group has proposed a latest S-FEM named “**F-barES-FEM-T4**” (detailed later):

- No intermediate node
  (i.e., 4-node tetrahedral (T4) mesh),
- Free from shear, volumetric and corner locking,
- No pressure checkerboarding,
- Long lasting in large deformation.

A Recent Solution (cont.)

E.g.) Compression of neo-Hookean **hyperelastic** body with $\nu_{\mathrm{ini}} = 0.49$



Same mesh as C3D4H case.

F-barES-FEM-T4 (One of the latest S-FEM)

- ✓ No shear/volumetric locking
- ✓ No corner locking
- ✓ No pressure checkerboarding

Topic of Today's Talk

Introduce and demonstrate one of the latest S-FEM called **F-barES-FEM-T4** with explaining the **classical S-FEM-T4**s.

Keywords: Incompressibility, Tetrahedral mesh, Large deformation, Smoothed FEM

Table of Body Contents

- Introduction of **3 classical S-FEM-T4s**
- Introduction of **F-barES-FEM-T4**
- Demonstration of **F-barES-FEM-T4**
  (hyperelastic, elastoplastic, dynamic, remeshing, contact)
- Summary

Introduction of
3 classical S-FEM-T4s

What is S-FEM?

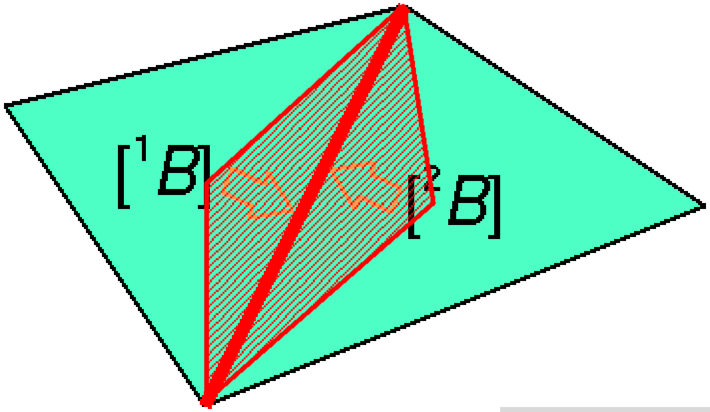
- ***Smoothed*** finite element method (**S-FEM**) is a relatively new FE formulation proposed by G. R. Liu in 2007.
- S-FEM is one of the **strain smoothing** techniques.
- There are several types of S-FEMs depending on the location of strain smoothing: edge, node, face, cell, etc..
- Mainly, there are the following **3 classical S-FEMs** with 4-node tetrahedral (T4) meshes:
  1. Edge-based S-FEM (ES-FEM)
  2. Node-based S-FEM (NS-FEM)
  3. Selective ES/NS-FEM

For simplicity,
these S-FEMs in 2D
are explained.

1. Outline of Edge-based S-FEM (ES-FEM)

- Calculate $[B]$ at each element as usual.
- Distribute $[B]$ to the connecting edges with area weight and build $[{}^{\text{Edge}}B]$.
- Calculate $\boldsymbol{F}, \boldsymbol{T}, \{f^{\text{int}}\}$ etc. in each edge smoothing domain.

As if putting
an integration point
on each edge center



$[{}^{\text{Edge}}B]$
↓
${}^{\text{Edge}}\boldsymbol{T}$
↓
$\{f^{\text{int}}\}$

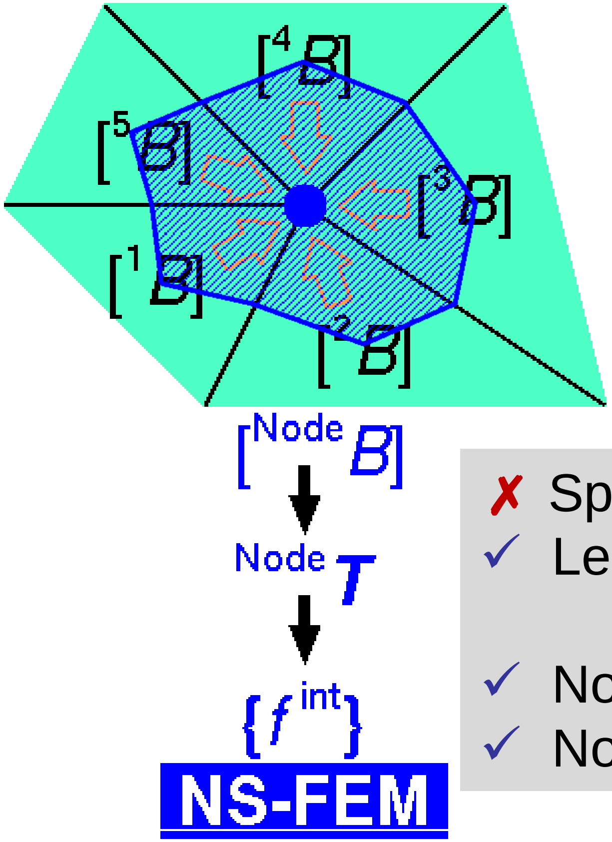
ES-FEM

- ✗ Volumetric locking
- ✗ Pressure checkerboarding
- ✓ No shear locking
- ✓ No spurious modes

2. Outline of Node-based S-FEM (NS-FEM)

- Calculate $[B]$ at each element as usual.
- Distribute $[B]$ to the connecting nodes with area weight and build $[{}^{\mathrm{Node}}B]$.
- Calculate $\boldsymbol{F}, \boldsymbol{T}, \{f^{\mathrm{int}}\}$ etc. in each node smoothing domain.

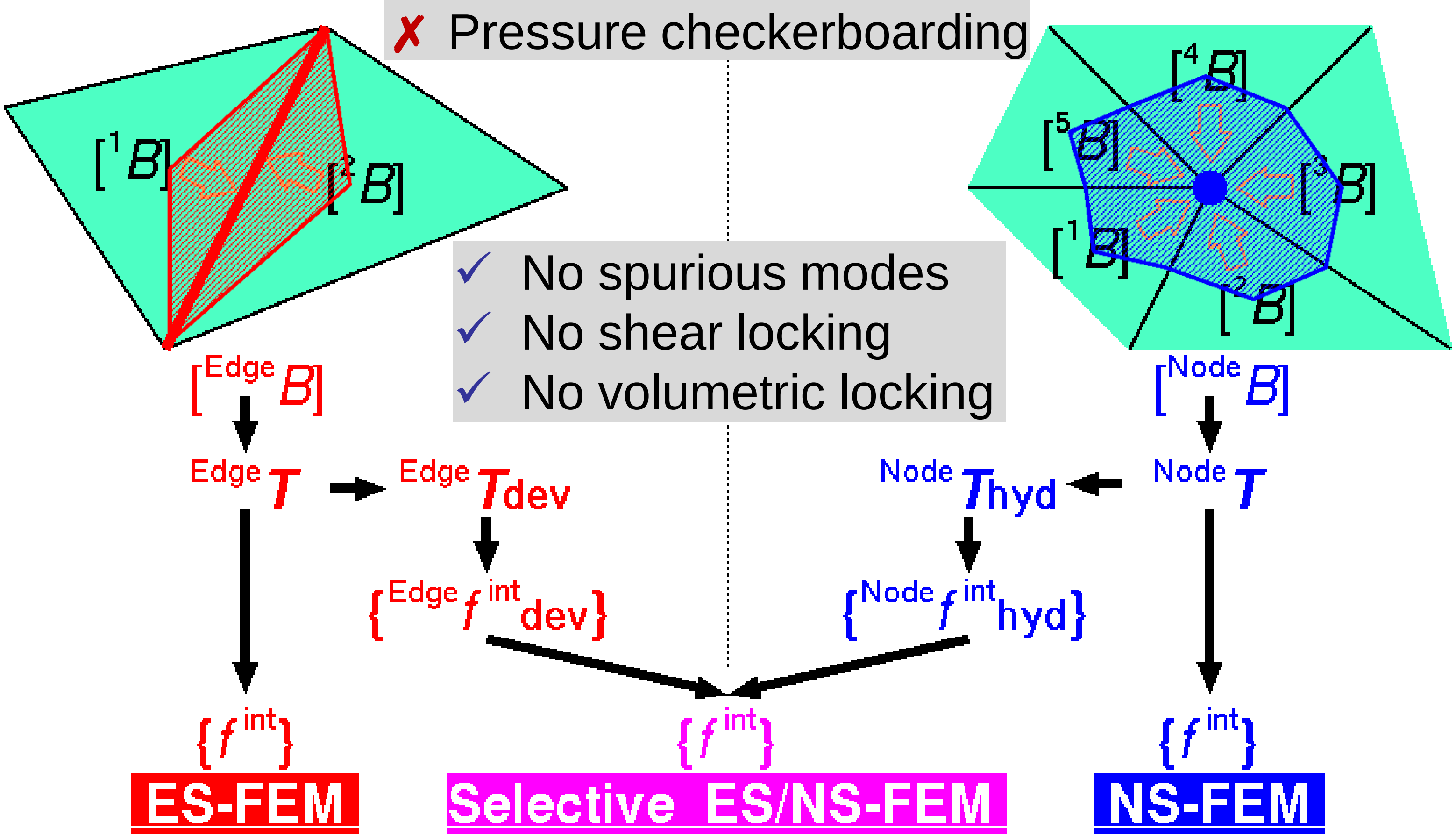
As if putting
an integration point
on each node



✗ Spurious low-energy mode
✓ Less pressure checkerboarding
✓ No shear locking
✓ No volumetric locking

3. Outline of Selective ES/NS-FEM

- Calculate $\boldsymbol{F}$ and $\boldsymbol{T}$ in both edge & node smoothing domains.
- Split $\boldsymbol{T}$ into deviatoric part $\boldsymbol{T}^{\mathrm{dev}}$ and hydrostatic part $\boldsymbol{T}^{\mathrm{hyd}}$.
- Calculate $\{{}^{\mathrm{Edge}}f^{\mathrm{int}}_{\mathrm{dev}}\}$ and $\{{}^{\mathrm{Node}}f^{\mathrm{int}}_{\mathrm{hyd}}\}$ and merge them.



✗ Pressure checkerboarding

✓ No spurious modes
✓ No shear locking
✓ No volumetric locking

$[{}^{\mathrm{Edge}}B]$ → ${}^{\mathrm{Edge}}\boldsymbol{T}$ → ${}^{\mathrm{Edge}}\boldsymbol{T}^{\mathrm{dev}}$ → $\{{}^{\mathrm{Edge}}f^{\mathrm{int}}_{\mathrm{dev}}\}$

$[{}^{\mathrm{Node}}B]$ → ${}^{\mathrm{Node}}\boldsymbol{T}$ → ${}^{\mathrm{Node}}\boldsymbol{T}^{\mathrm{hyd}}$ → $\{{}^{\mathrm{Node}}f^{\mathrm{int}}_{\mathrm{hyd}}\}$

${}^{\mathrm{Edge}}\boldsymbol{T}$ → $\{f^{\mathrm{int}}\}$ **ES-FEM**

$\{{}^{\mathrm{Edge}}f^{\mathrm{int}}_{\mathrm{dev}}\}$ + $\{{}^{\mathrm{Node}}f^{\mathrm{int}}_{\mathrm{hyd}}\}$ → $\{f^{\mathrm{int}}\}$ **Selective ES/NS-FEM**

${}^{\mathrm{Node}}\boldsymbol{T}$ → $\{f^{\mathrm{int}}\}$ **NS-FEM**

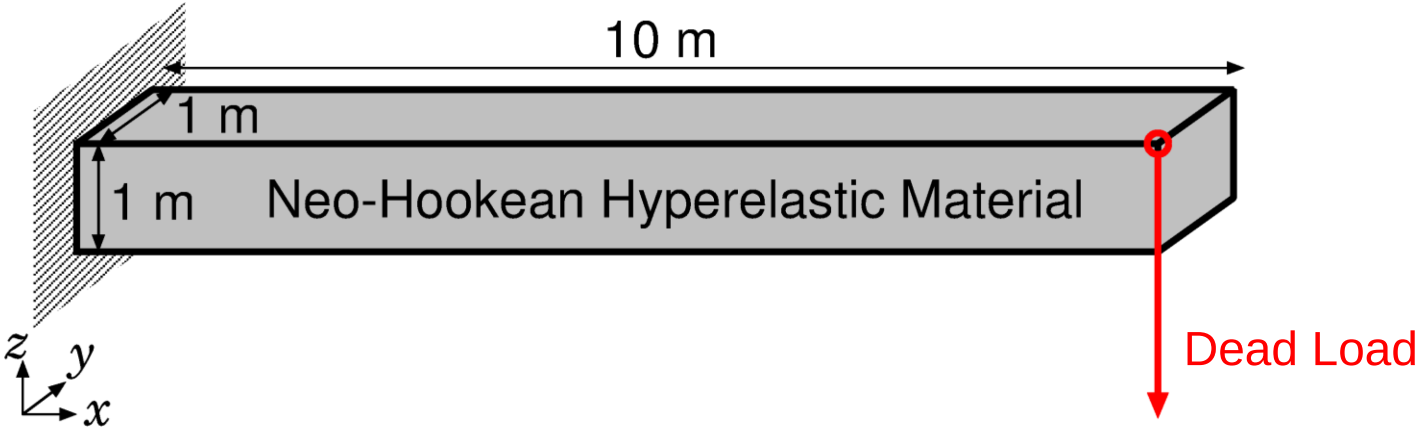
Characters of 3 Classical S-FEMs

- All S-FEMs have an unique **benefit**:
  No increase in DOF.
  - Purely displacement-based formulation.
  - The nodal displacement vector $\{u\}$ is only the unknown.
    (No pressure p or volumetric strain $\varepsilon^{\mathrm{vol}}$ unknowns.)
  - Static condensation is unnecessary.

- All S-FEMs have a common **drawback**:
  Increase in bandwidth of the stiffness matrix $[K]$.
  - [Bandwidth of ES-FEM-T4]
    $\simeq \mathbf{2} \times$ [Bandwidth of standard FEM-T4])
  - [Bandwidth of NS-FEM-T4]
    = [Bandwidth of Selective ES/NS-FEM-T4]
    $\simeq \mathbf{4} \times$ [Bandwidth of standard FEM-T4])

Bending of Cantilever

Outline

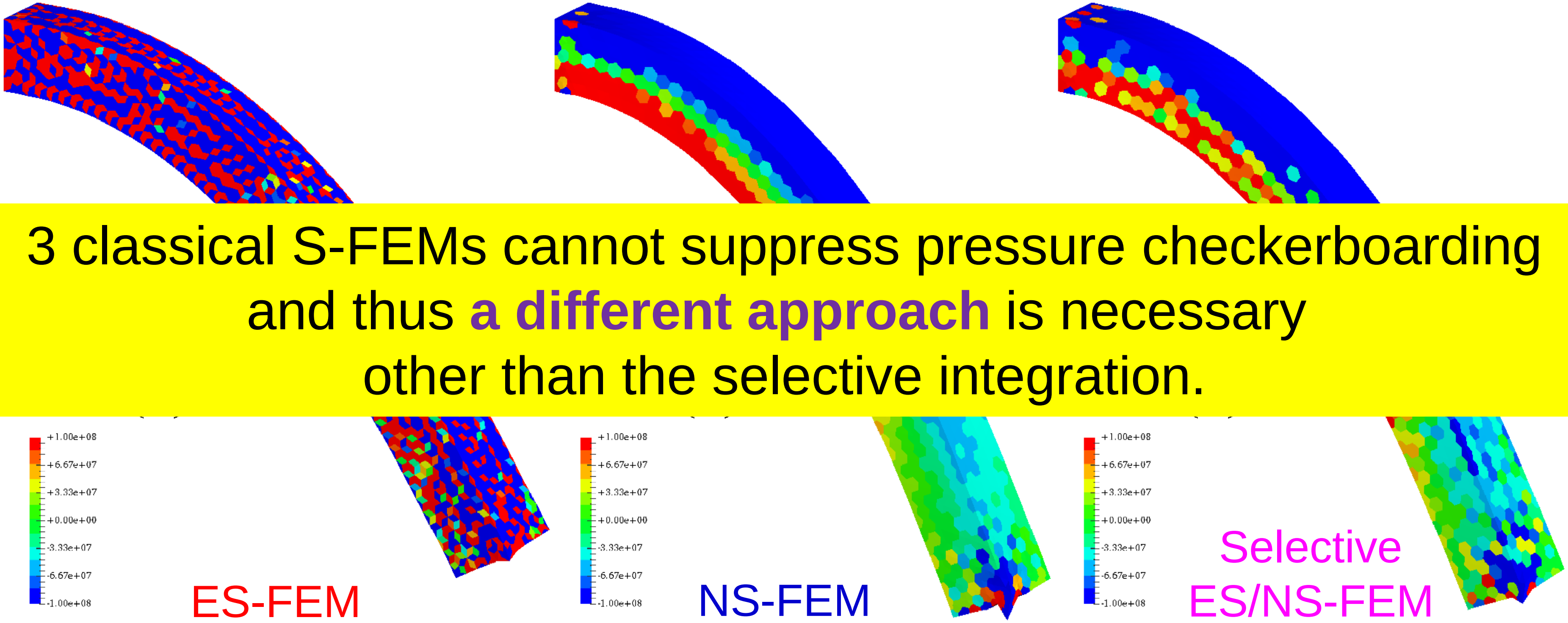


- Neo-Hookean hyperelastic material
- Initial Poisson's ratio: 0.499.
- Bending dead load is applied to the cantilever tip.
- Results of ES-FEM, NS-FEM and Selective ES/NS-FEM are compared.

Static Implicit

Bending of Cantilever

Pressure dist.



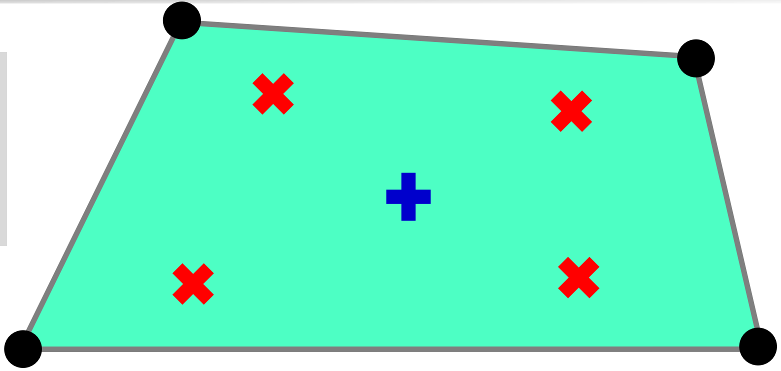
3 classical S-FEMs cannot suppress pressure checkerboarding and thus **a different approach** is necessary other than the selective integration.

- ES-FEM has locking and major checkerboarding.
- NS-FEM has spurious modes and minor checkerboarding.
- Selective ES/NS-FEM has medium checkerboarding.

Introduction of
F-barES-FEM-T4

Quick Review of F-bar Method

For quadrilateral (Q4) or hexahedral (H8) elements



Algorithm

1. Calculate deformation gradient $\boldsymbol{F}$ at the element center, and then make the relative volume change $\bar{J}\ (= \det(\boldsymbol{F}))$.
2. Calculate deformation gradient $\boldsymbol{F}$ at each gauss point as usual, and then make $\boldsymbol{F}^{\mathrm{iso}}\ (= \boldsymbol{F} \,/\, J^{1/3})$.
3. Modify $\boldsymbol{F}$ at each gauss point to obtain $\bar{\boldsymbol{F}}$ as
$$\bar{\boldsymbol{F}} = \bar{J}^{\,1/3}\ \boldsymbol{F}^{\mathrm{iso}}.$$
4. Use $\bar{\boldsymbol{F}}$ to calculate the stress $\boldsymbol{T}$, nodal force $\{f^{\mathrm{int}}\}$ etc..

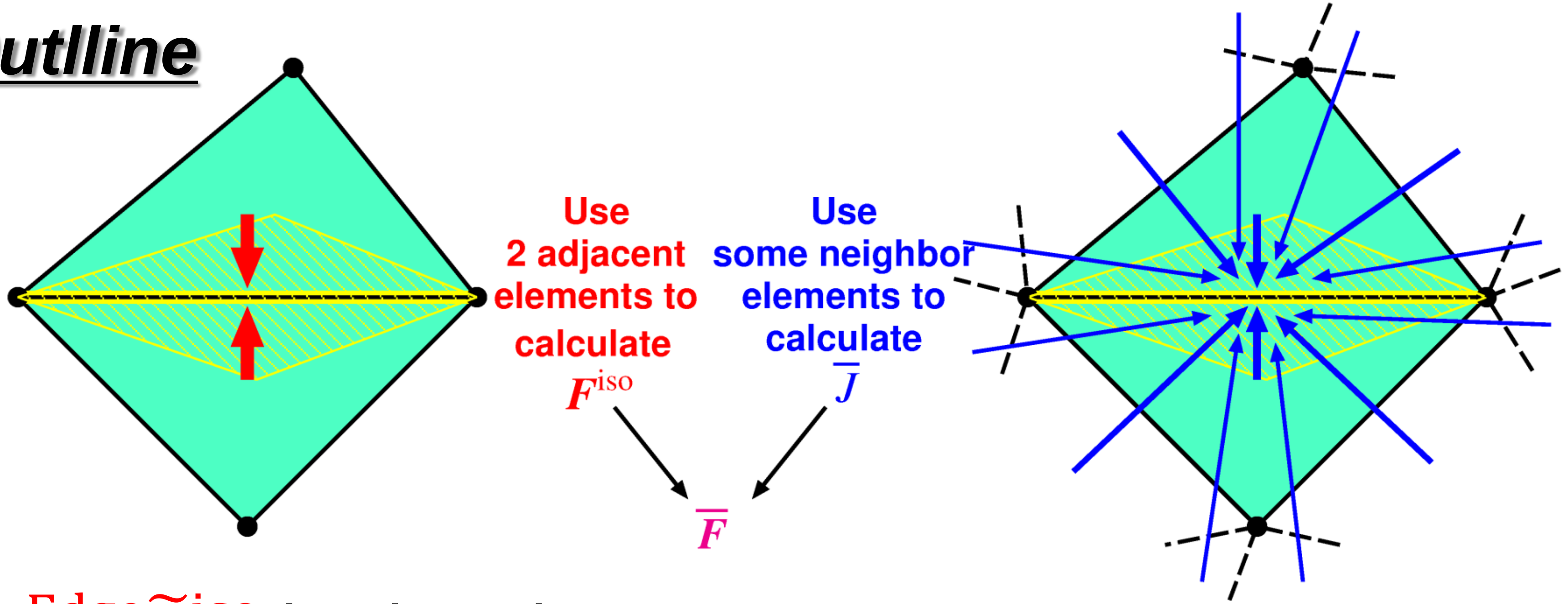
A kind of low-pass filter for J

F-bar method is used to **avoid volumetric locking** in Q4 or H8 elements. Yet, it **cannot avoid shear locking**.

Outline of F-barES-FEM

Concept: combine ES-FEM and NS-FEM using F-bar method

Outlline



Use 2 adjacent elements to calculate $\boldsymbol{F}^{\mathrm{iso}}$

Use some neighbor elements to calculate $\bar{J}$

$\bar{\boldsymbol{F}}$

- ${}^{\mathrm{Edge}}\tilde{\boldsymbol{F}}^{\mathrm{iso}}$ is given by ES-FEM.
- ${}^{\mathrm{Edge}}\bar{J}$ is given by cyclically applied NS-FEM.
- ${}^{\mathrm{Edge}}\bar{\boldsymbol{F}}$ is calculated in the manner of F-bar method:

$$ {}^{\mathrm{Edge}}\bar{\boldsymbol{F}} = {}^{\mathrm{Edge}}\bar{J}^{1/3}\ {}^{\mathrm{Edge}}\tilde{\boldsymbol{F}}^{\mathrm{iso}} . $$

Outline of F-barES-FEM (cont.)

Brief Formulation

1. Make ${}^{\mathrm{Elem}}\boldsymbol{F}$ as usual and calculate ${}^{\mathrm{Elem}}J$.

2. Smooth ${}^{\mathrm{Elem}}J$ at nodes and get ${}^{\mathrm{Node}}\tilde{J}$.
3. Smooth ${}^{\mathrm{Node}}\tilde{J}$ at elements and get ${}^{\mathrm{Elem}}\tilde{J}$.
4. Repeat 2. and 3. as necessary (c times).

Cyclic Smoothing of J

A kind of low-pass filter

5. Smooth ${}^{\mathrm{Elem}}\overset{\vdots}{\tilde{\tilde{J}}}$ (c layers of ~) at edges and get ${}^{\mathrm{Edge}}\bar{J}$.
6. Combine ${}^{\mathrm{Edge}}\bar{J}$ and ${}^{\mathrm{Edge}}\boldsymbol{F}^{\mathrm{iso}}$ of ES-FEM as

$$ {}^{\mathrm{Edge}}\bar{\boldsymbol{F}} = {}^{\mathrm{Edge}}\bar{J}^{\,1/3}\ {}^{\mathrm{Edge}}\boldsymbol{F}^{\mathrm{iso}}. $$

Hereafter, F-barES-FEM-T4 with $\boldsymbol{c}$ cycles of smoothing is called "F-barES-FEM-T4($\boldsymbol{c}$)".

Additional Point of F-barES-FEM

Typical Flow of FE Solver

Displace-ment $\boldsymbol{u}$ → **deformation evaluation** → Deformation Gradient $\boldsymbol{F}$ → constitutive model → Cauchy Stress $\boldsymbol{T}$ → **domain integration** → Internal Force $\{f^{\text{int}}\}$

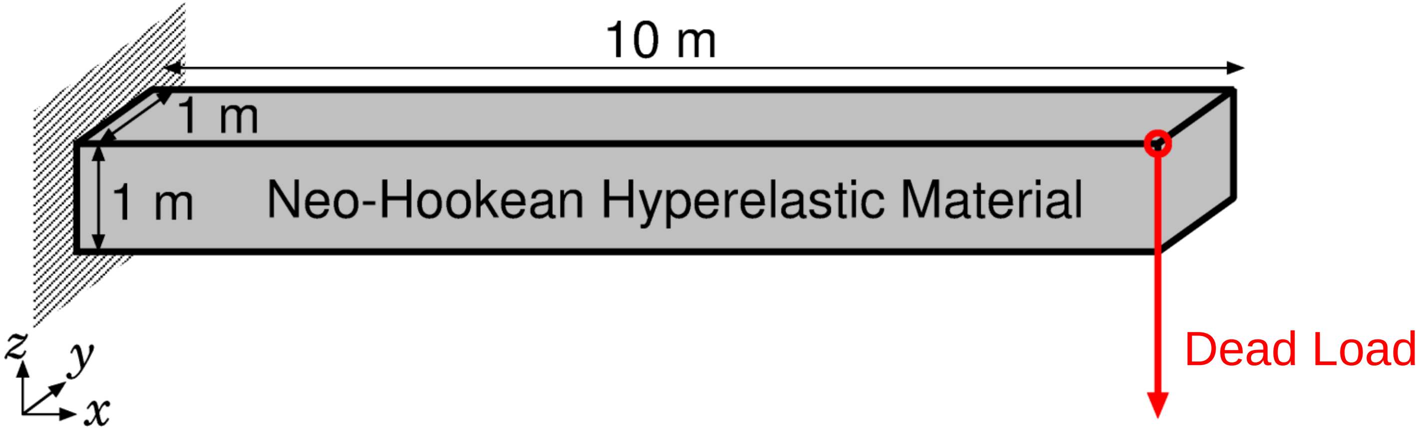
- **Selective ES/NS-FEM**
  splits $\boldsymbol{T}$ into $\boldsymbol{T}^{\text{hyd}}$ and $\boldsymbol{T}^{\text{dev}}$
  and merges $\{f^{\text{int}}_{\text{hyd}}\}$ and $\{f^{\text{int}}_{\text{dev}}\}$ into $\{f^{\text{int}}\}$.

- **F-barES-FEM**
  builds $\boldsymbol{F}^{\text{vol}}$ and $\boldsymbol{F}^{\text{iso}}$ separately
  and combines $\boldsymbol{F}^{\text{vol}}$ and $\boldsymbol{F}^{\text{iso}}$ into $\boldsymbol{F}$.

F-barES-FEM can handle
any kind of material constitutive model.

Bending of Cantilever

Outline



- Neo-Hookean hyperelastic material
- Initial Poisson’s ratio: 0.49 or 0.499.
- Two types of T4 meshes:
  a **structured** mesh and an **unstructured** mesh.
- Compared to ABAQUS C3D4H (hybrid T4 element).

Static Implicit

Bending of Cantilever

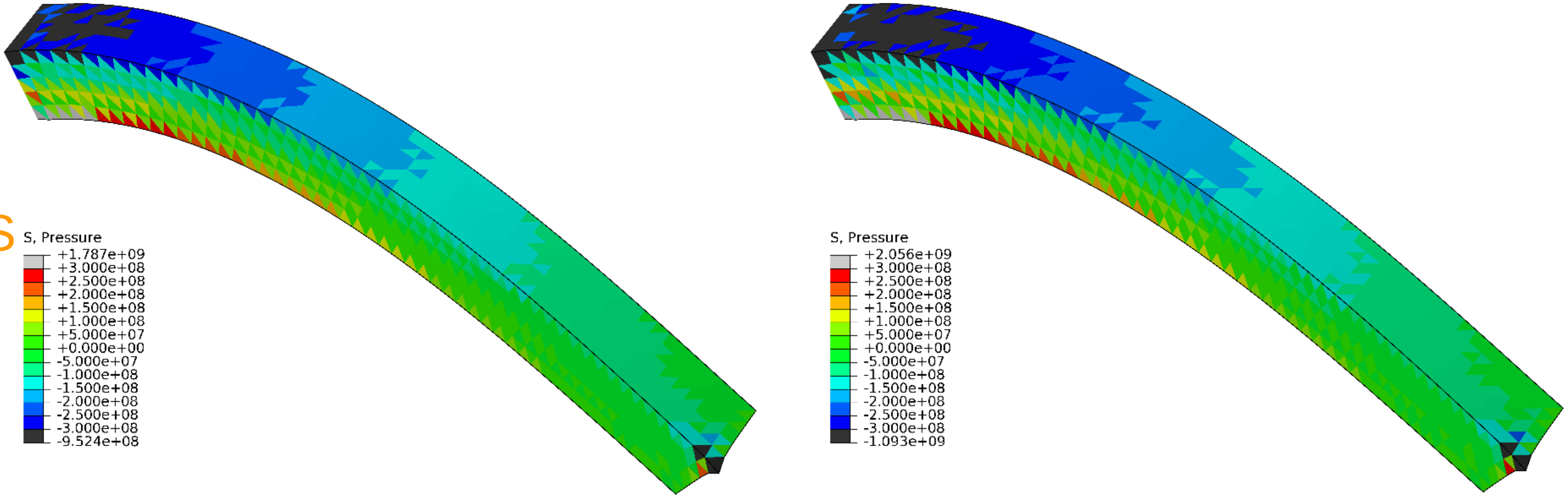
Pressure dist.

$\nu^{\mathrm{ini}} = 0.49$

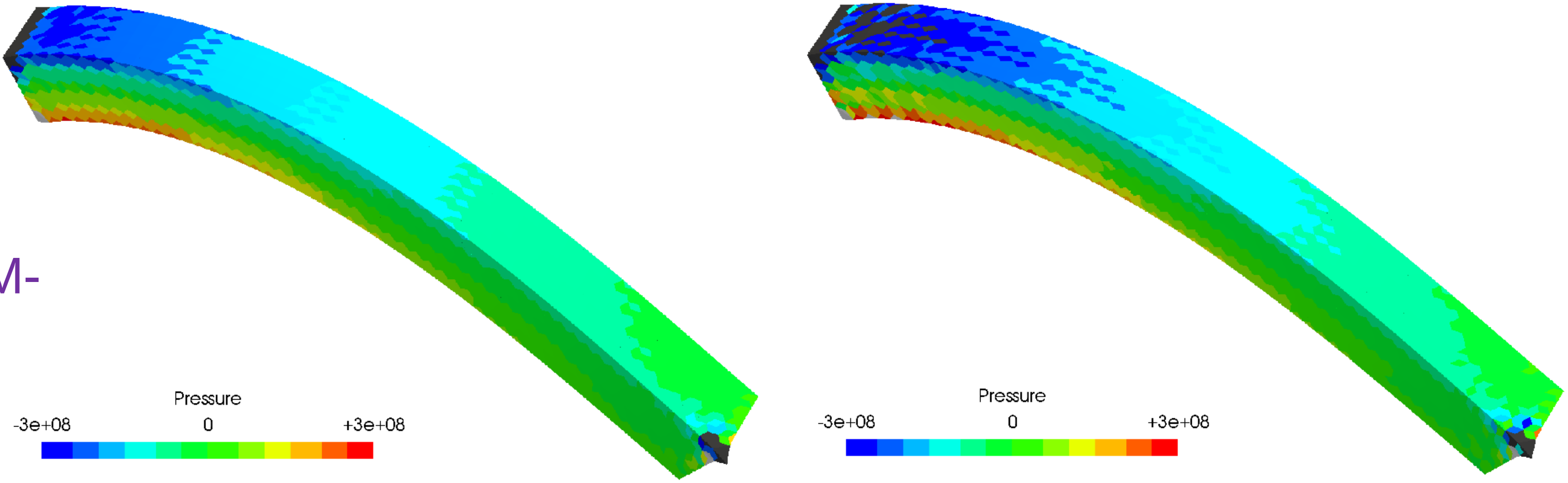
Structured Mesh

$\nu^{\mathrm{ini}} = 0.499$

ABAQUS C3D4H



F-bar ES-FEM-T4(1)



Static Implicit

Bending of Cantilever

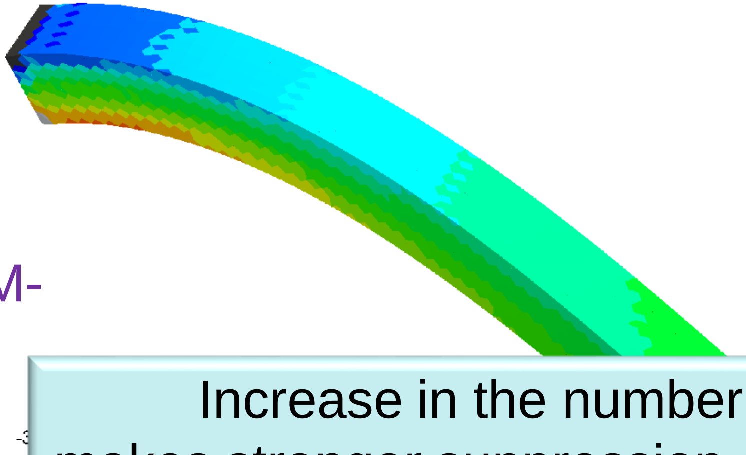
Pressure dist.

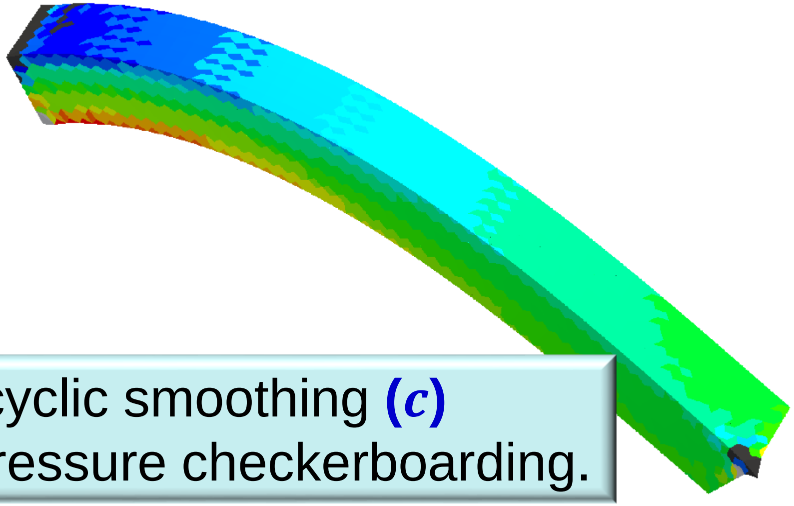
$\nu^{\mathrm{ini}} = 0.49$

Structured Mesh

$\nu^{\mathrm{ini}} = 0.499$

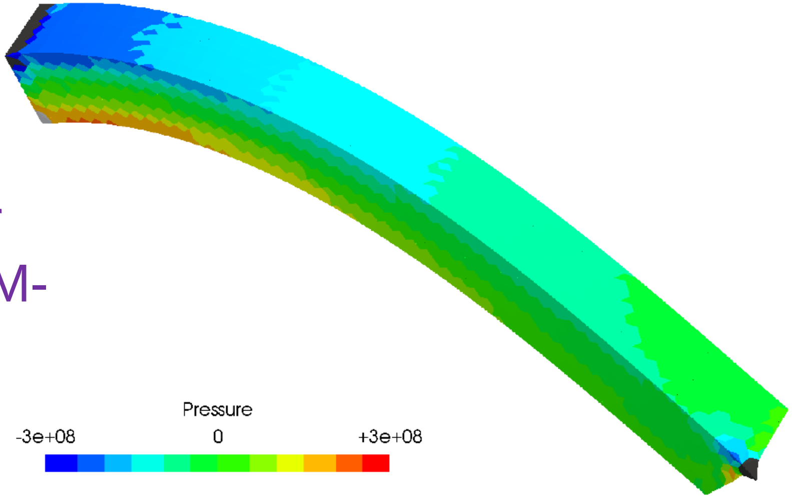
F-bar ES-FEM-T4(2)

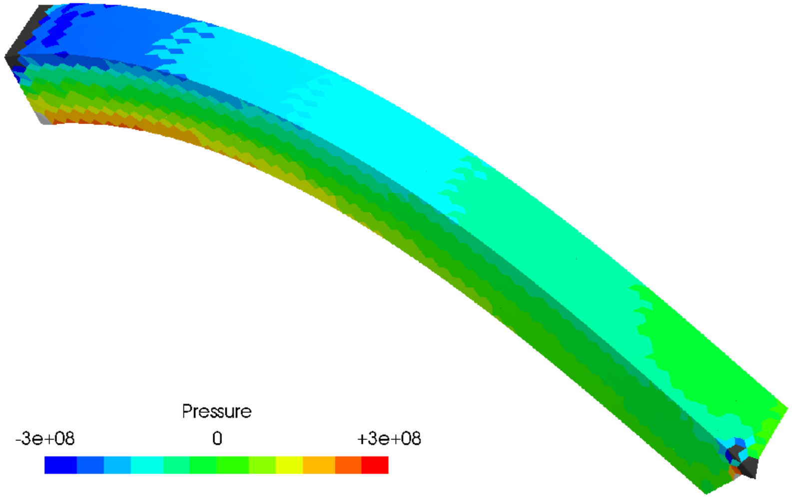




Increase in the number of cyclic smoothing (c) makes stronger suppression of pressure checkerboarding.

F-bar ES-FEM-T4(3)





Static Implicit

Bending of Cantilever

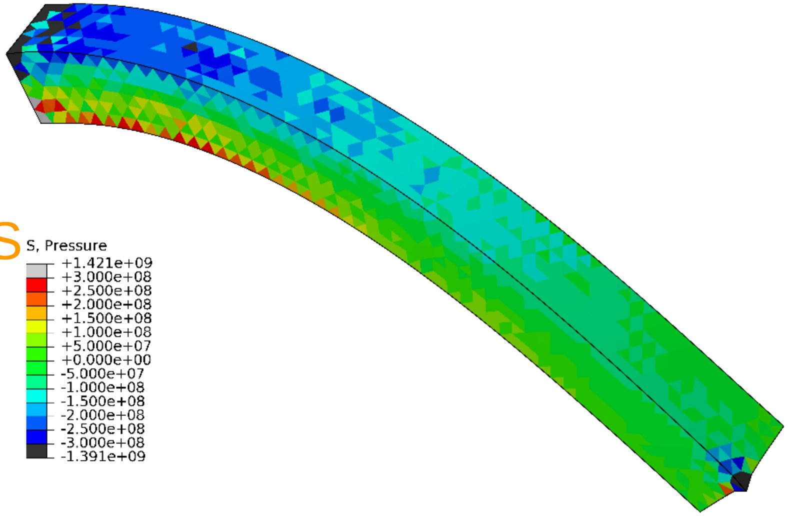
Pressure dist.

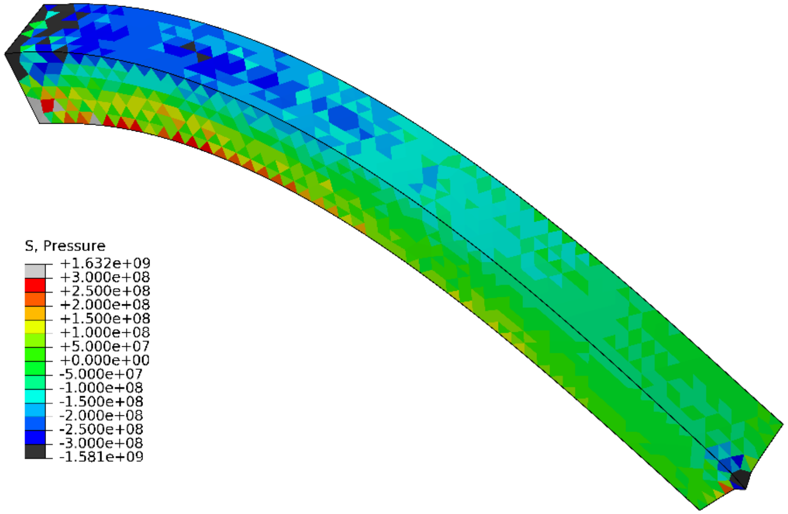
$\nu^{\text{ini}} = 0.49$

Unstructured Mesh

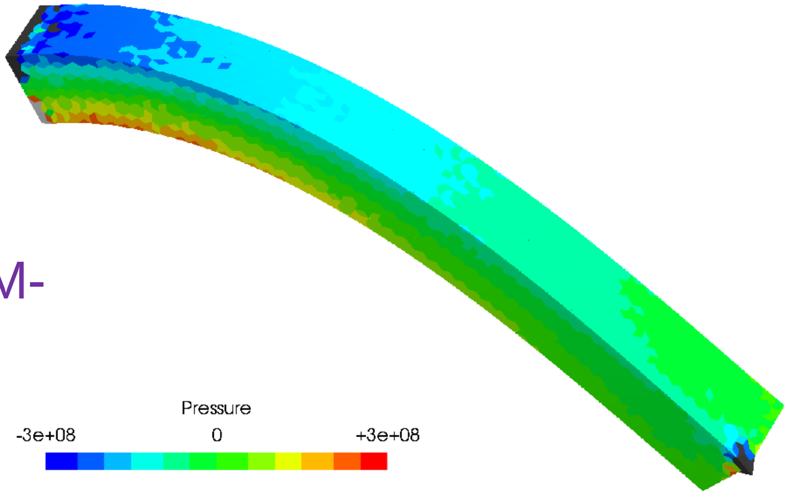
$\nu^{\text{ini}} = 0.499$

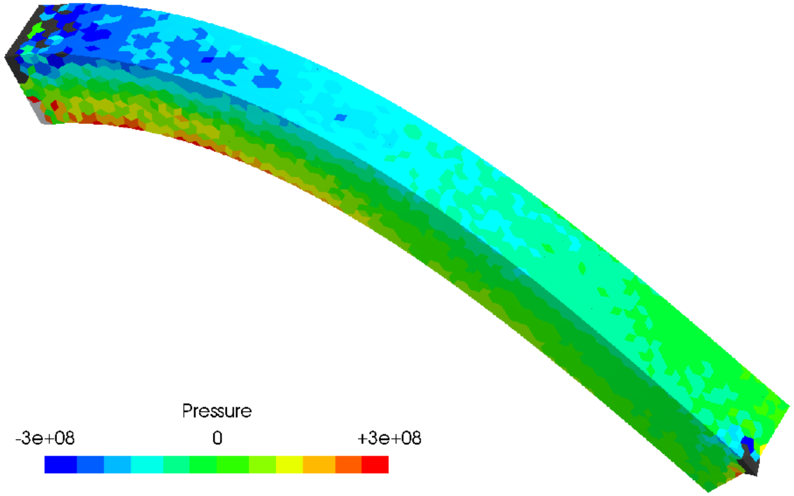
ABAQUS C3D4H





F-bar ES-FEM-T4(1)





Static Implicit

Bending of Cantilever

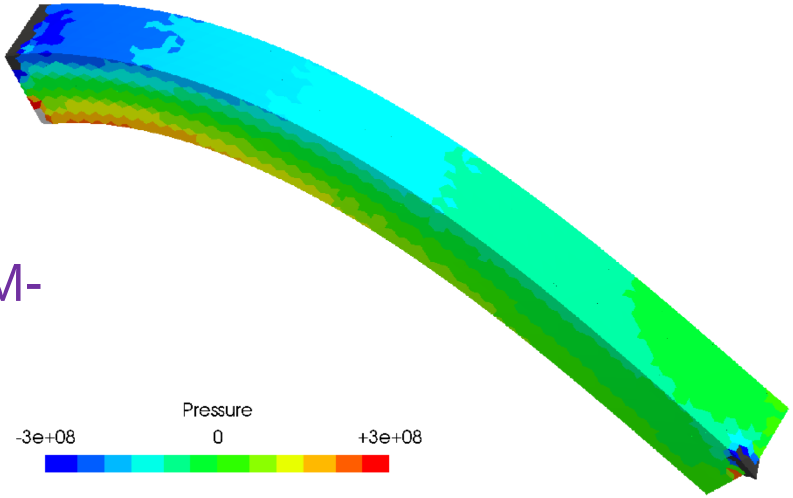
Pressure dist.

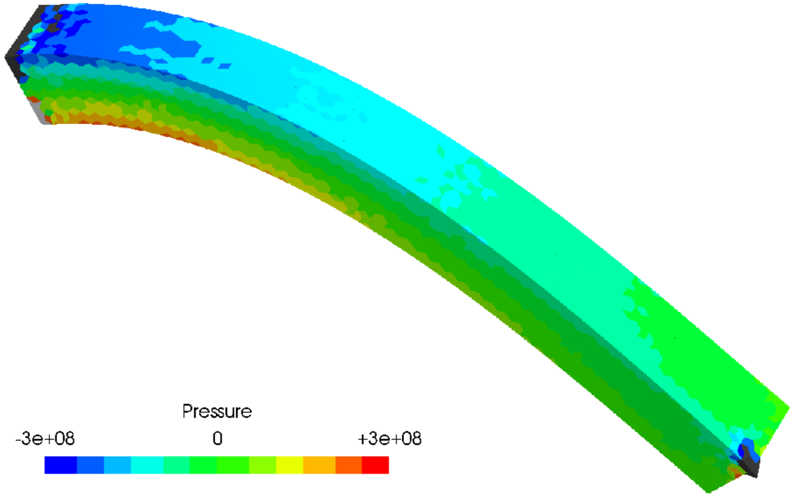
$\nu^{\mathrm{ini}} = 0.49$

Unstructured Mesh

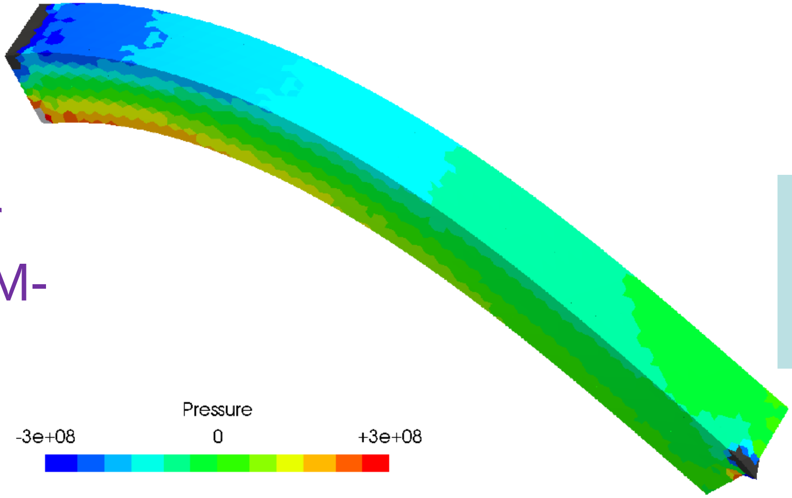
$\nu^{\mathrm{ini}} = 0.499$

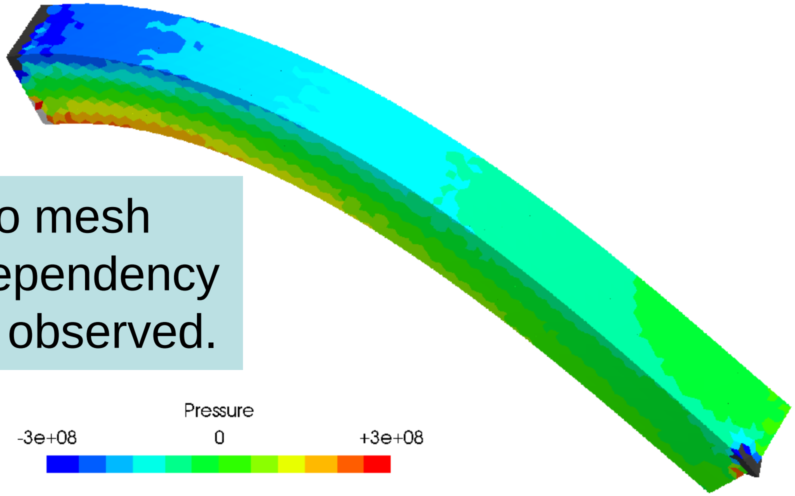
F-bar ES-FEM-T4(2)





F-bar ES-FEM-T4(3)



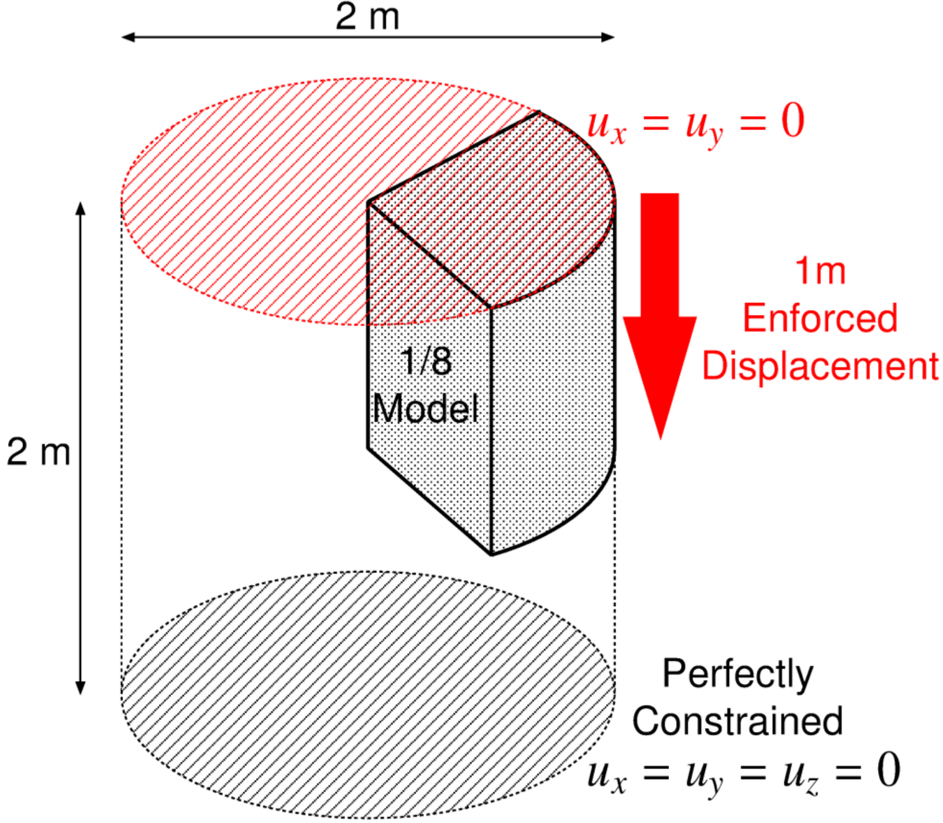


No mesh dependency is observed.

Static Implicit

Barreling of 1/8 Cylinder

Outline



- Neo-Hookean hyperelastic material ($\nu_{\mathrm{ini}} = 0.499$).
- Enforced displacement is applied to the top surface.
- Compared to ABAQUS C3D4H with the same unstructured T4 mesh.

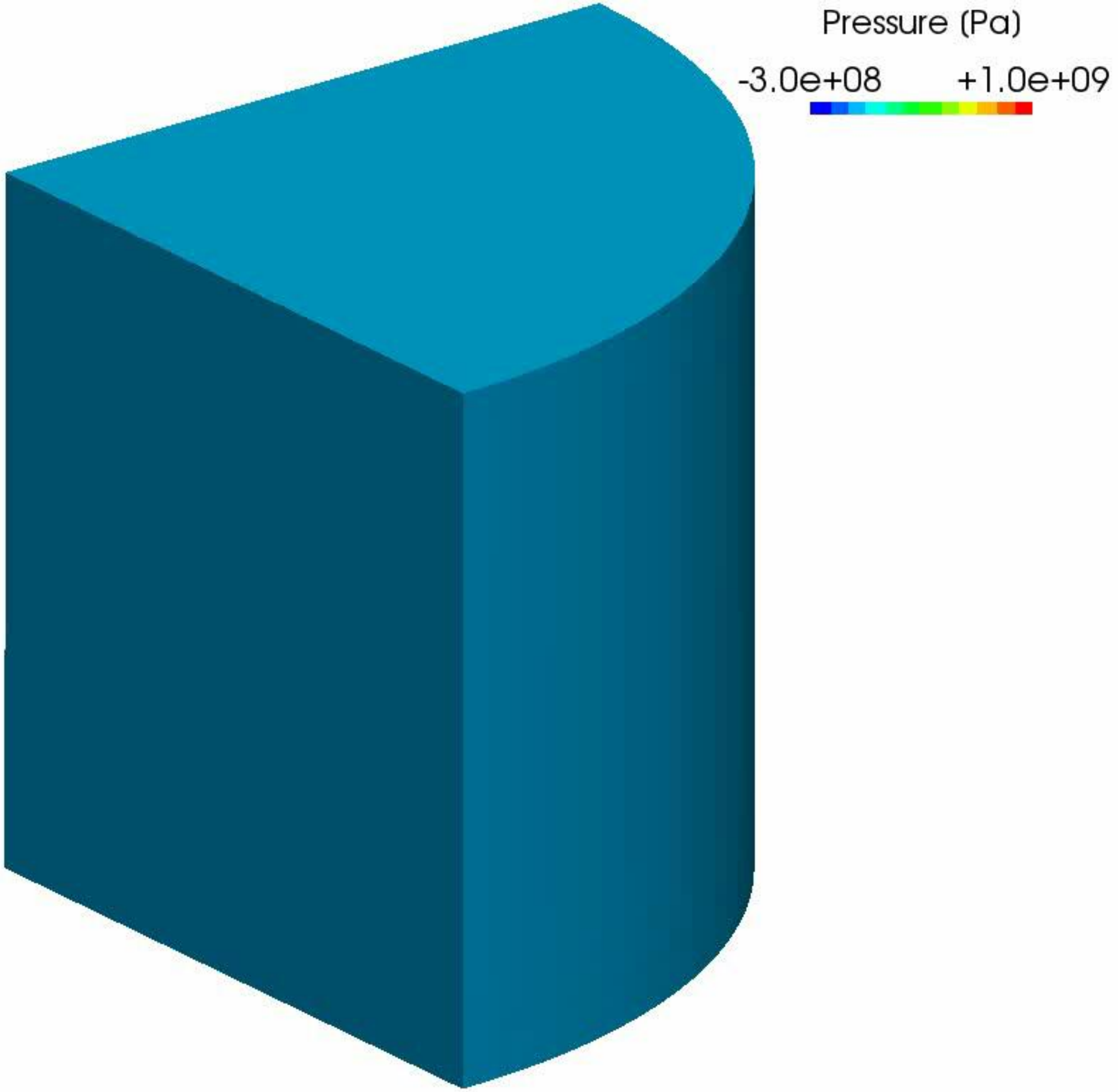
Static
Implicit

Barreling of 1/8 Cylinder

Result of
F-bar
ES-FEM(2)
(Pressure)

50% nominal
compression

Almost smooth pressure distribution is obtained except just around the rim.



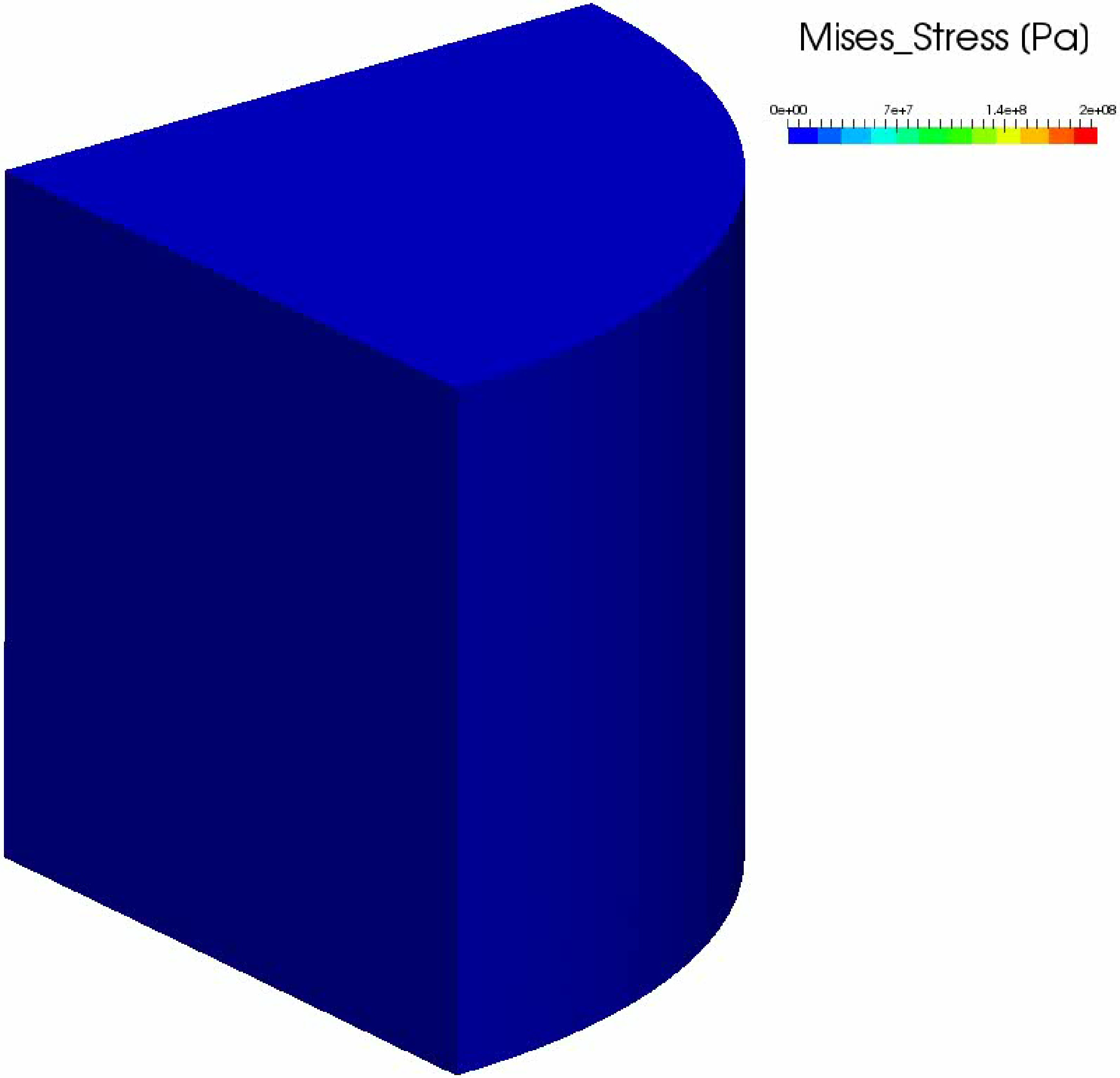
Static
Implicit

Barreling of 1/8 Cylinder

Result of
F-bar
ES-FEM(2)
(Mises Stress)

50% nominal compression

Smooth Mises stress distribution is obtained except just around the rim.



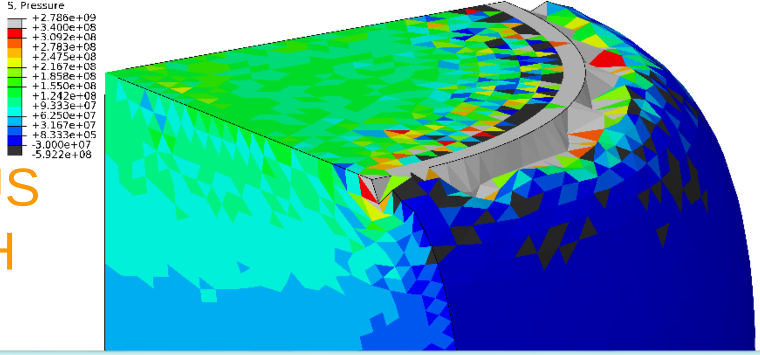
Static
Implicit

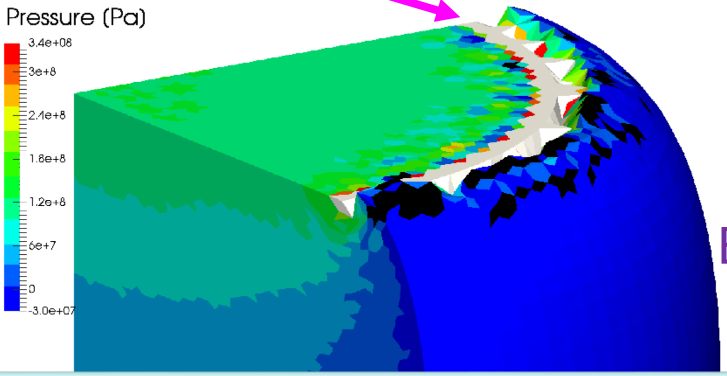
Barreling of 1/8 Cylinder

Pressure dist.

Strange deformation (corner locking) around the rim

ABAQUS
C3D4H

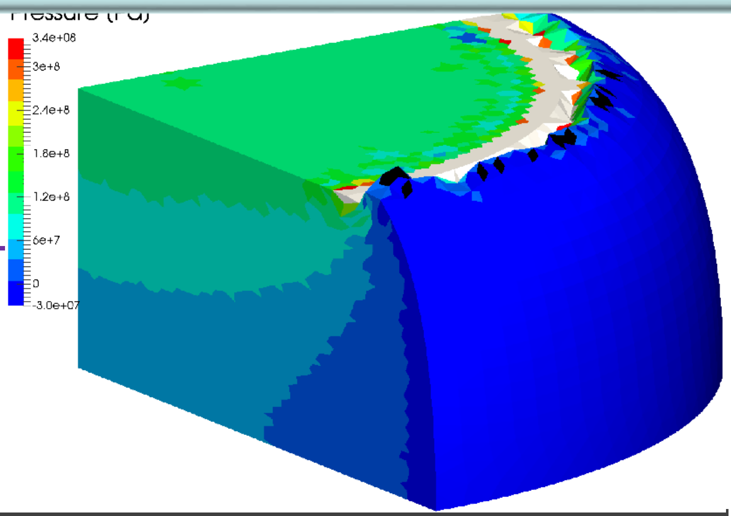


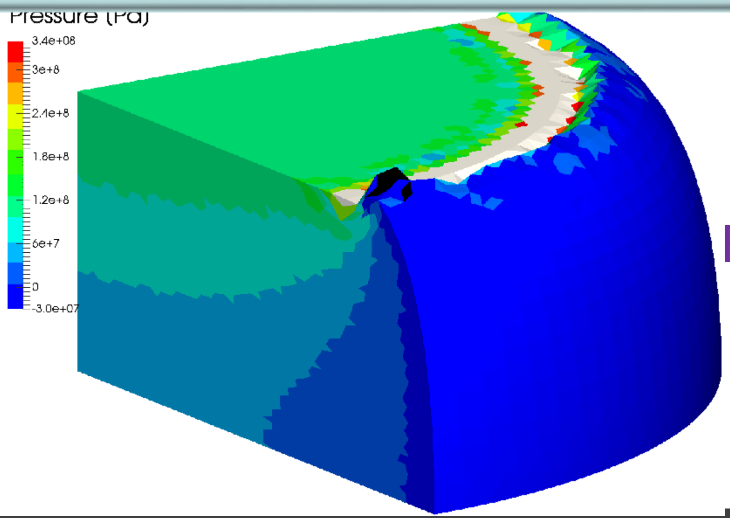


F-bar
ES-FEM-
T4(2)

F-barES-FEM-T4 with a sufficient cyclic smoothing can resolve the corner locking issue.

F-bar
ES-FEM-
T4(3)





F-bar
ES-FEM-
T4(4)

Characteristics of FEM-T4s

	Shear & Volumetric Locking	Zero-Energy Mode	Dev/Vol Coupled Material	Pressure Oscillation	Corner Locking	Severe Strain
Standard FEM-T4	✗	✓	✓	✗	✗	✓
ABAQUS C3D4H	✓	✓	✓	✗	✗	✓
Selective S-FEM-T4	✓	✓	✗	✗	✗	✓
F-bar ES-FEM-T4	✓	✓	✓	✓*	✓*	✓

*) when the num. of cyclic smoothings is sufficiently large.

Characteristics of [K] in F-barES-FEM-T4

✓ No increase in DOF.
(No Lagrange multiplier. No static condensation.)

✓ Positive definite.

✗ Wider bandwidth.
In case of standard unstructured T4 meshes,

Method	Approx. Bandwidth	Approx. Ratio
Standard FEM-T4	40	1
F-barES-FEM-T4(1)	390	x10
F-barES-FEM-T4(2)	860	x20
F-barES-FEM-T4(3)	1580	x40
F-barES-FEM-T4(4)	2600	x65

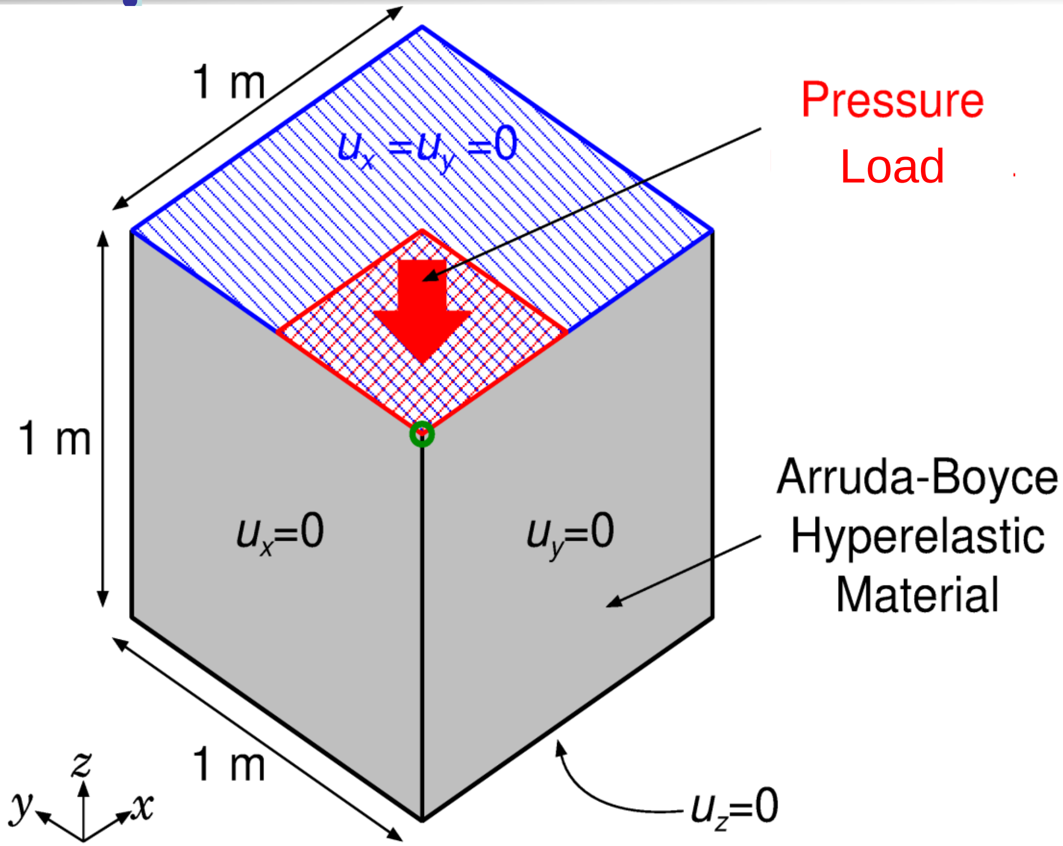
✗ Ill-posedness in nearly incompressible cases.
(No improvement in condition number.)

Demonstration of
F-barES-FEM-T4

Static Implicit

Compression of a Block

Outline



- Arruda-Boyce hyperelastic material ($\nu_{\text{ini}} = 0.499$).
- Applying pressure on ¼ of the top face.
- Compared to ABAQUS C3D4H with the same unstructured T4 mesh.

Static
Implicit

Compression of a Block

Pressure dist.

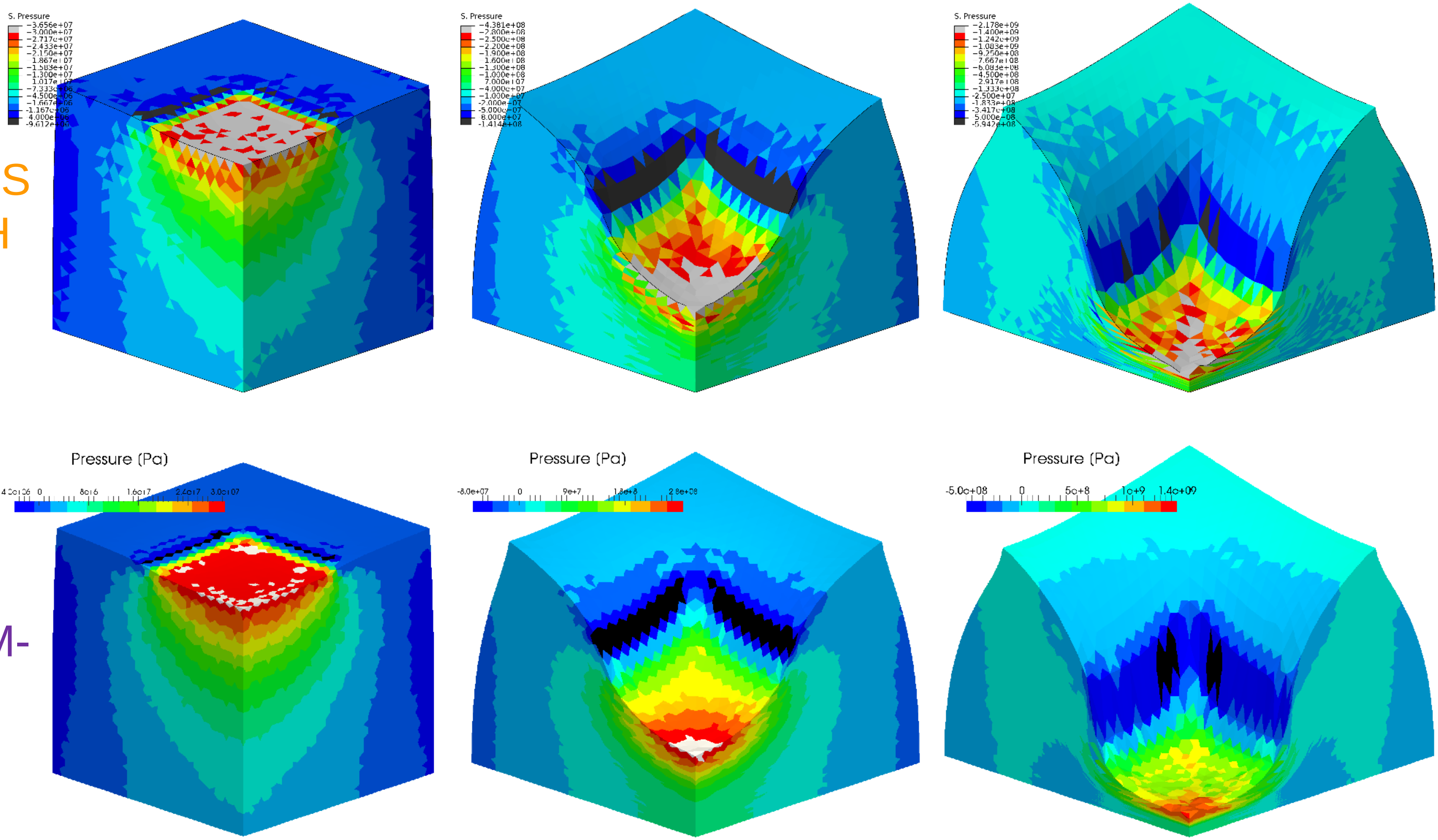
Early stage

Middle stage

Later stage

ABAQUS
C3D4H

F-bar
ES-FEM-
T4(2)



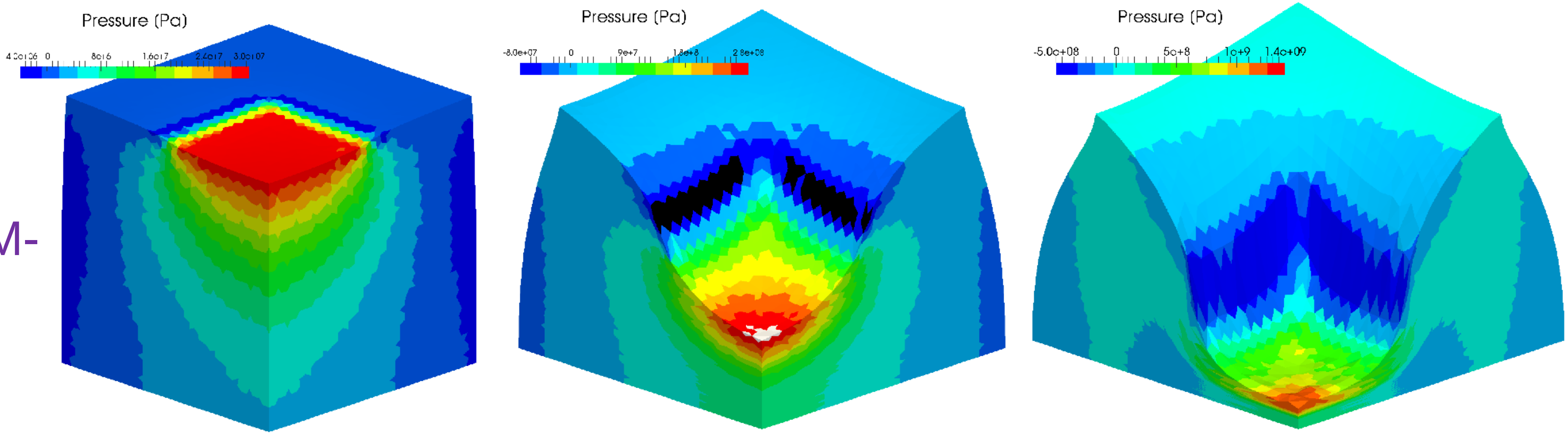
Static
Implicit

Compression of a Block

Pressure dist.

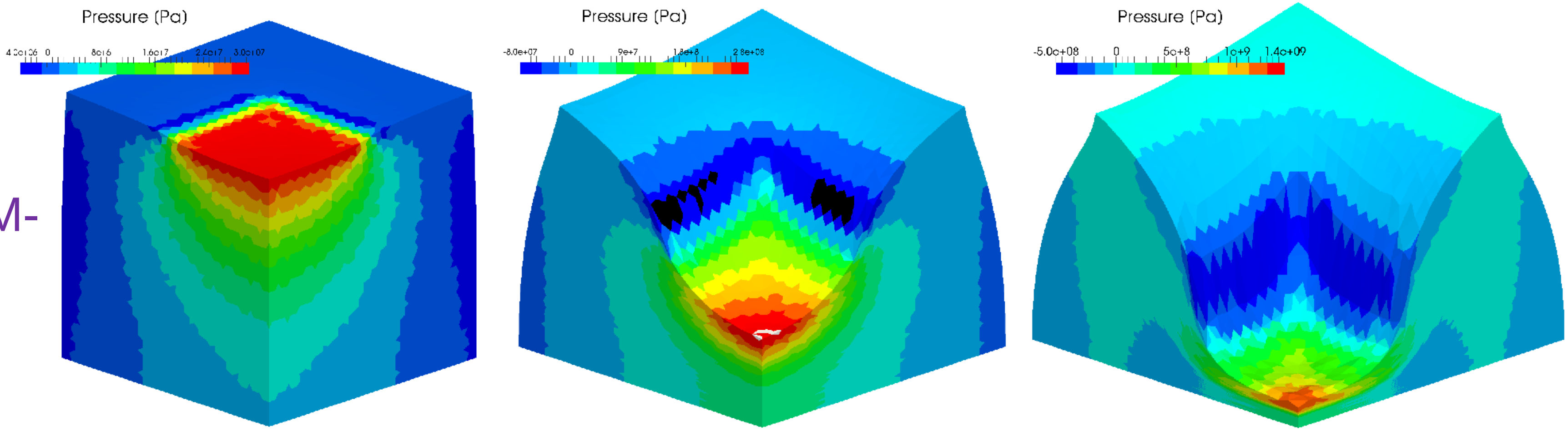
Early stage Middle stage Later stage

F-bar
ES-FEM-
T4(3)



Smooth pressure distributions are obtained.

F-bar
ES-FEM-
T4(4)

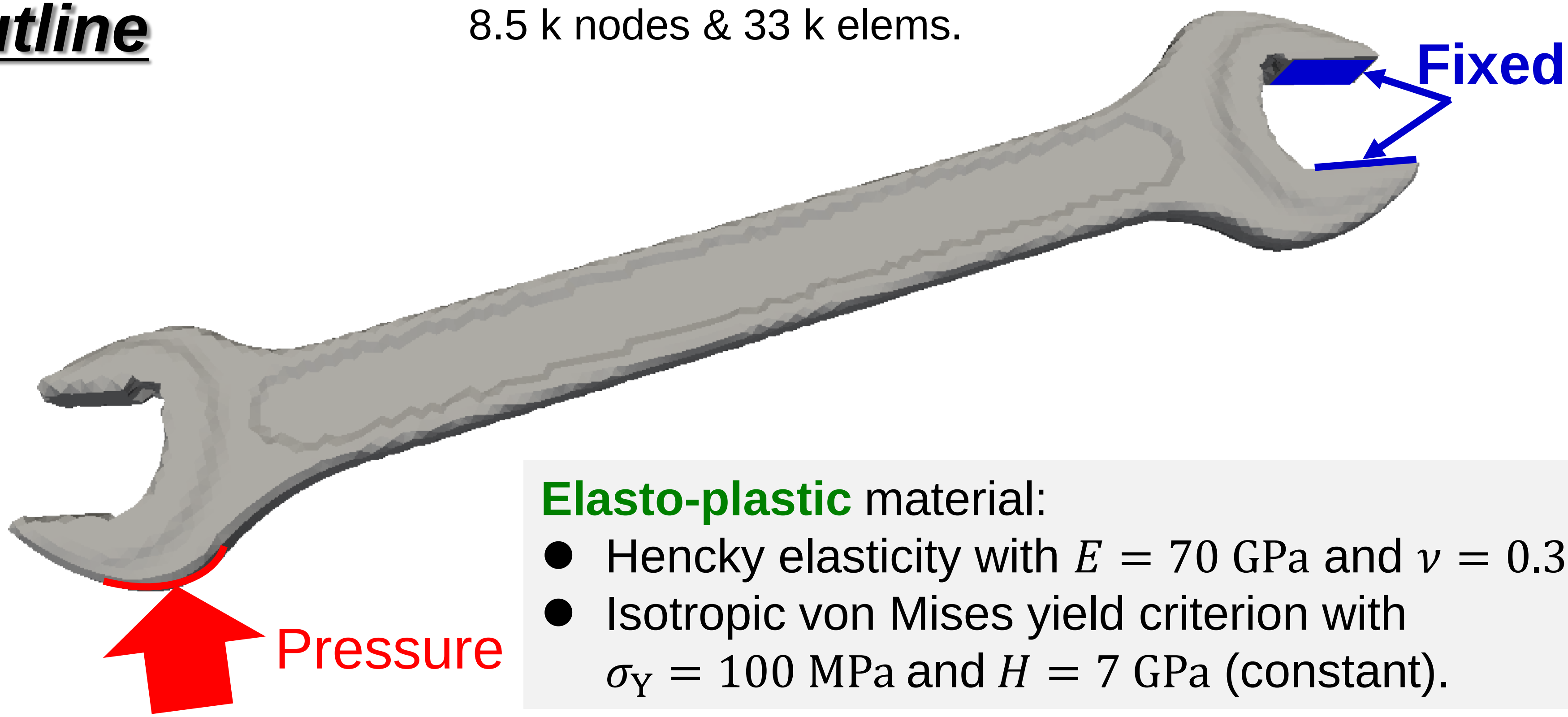


Static Implicit

Bending of Elasto-plastic Spanner

Outline

8.5 k nodes & 33 k elems.



Elasto-plastic material:

- Hencky elasticity with $E = 70\ \mathrm{GPa}$ and $\nu = 0.3$.
- Isotropic von Mises yield criterion with $\sigma_\mathrm{Y} = 100\ \mathrm{MPa}$ and $H = 7\ \mathrm{GPa}$ (constant).

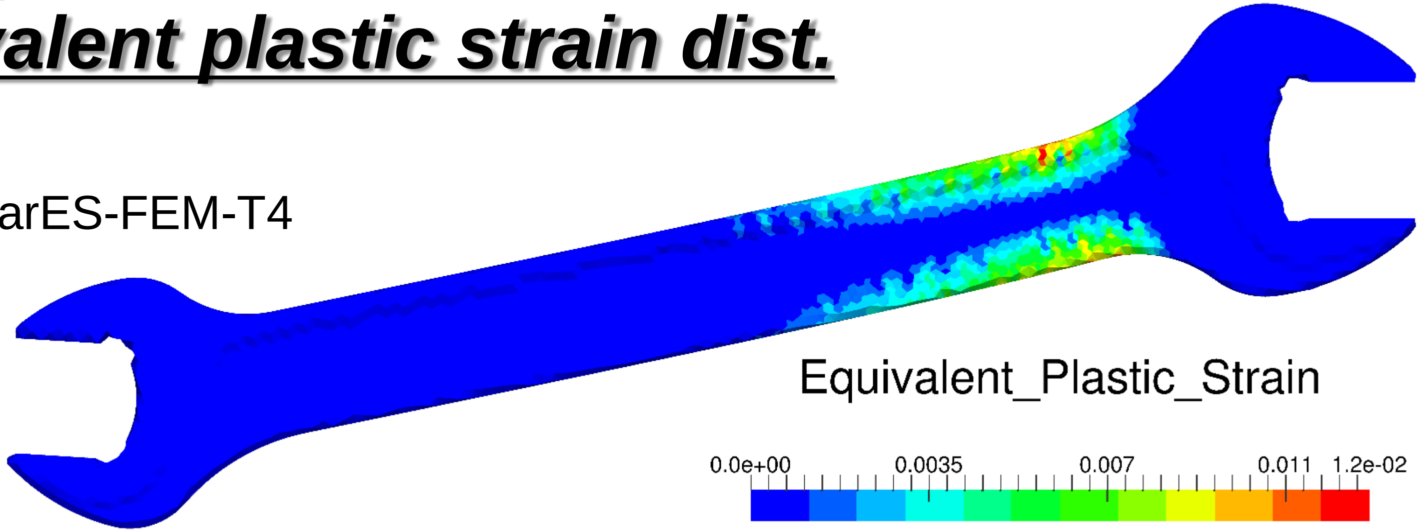
- 2 faces are perfectly constrained.
- Pressure is applied to a side part of the spanner.
- Compared to ABAQUS C3D4H with the same unstructured T4 mesh.

Static
Implicit

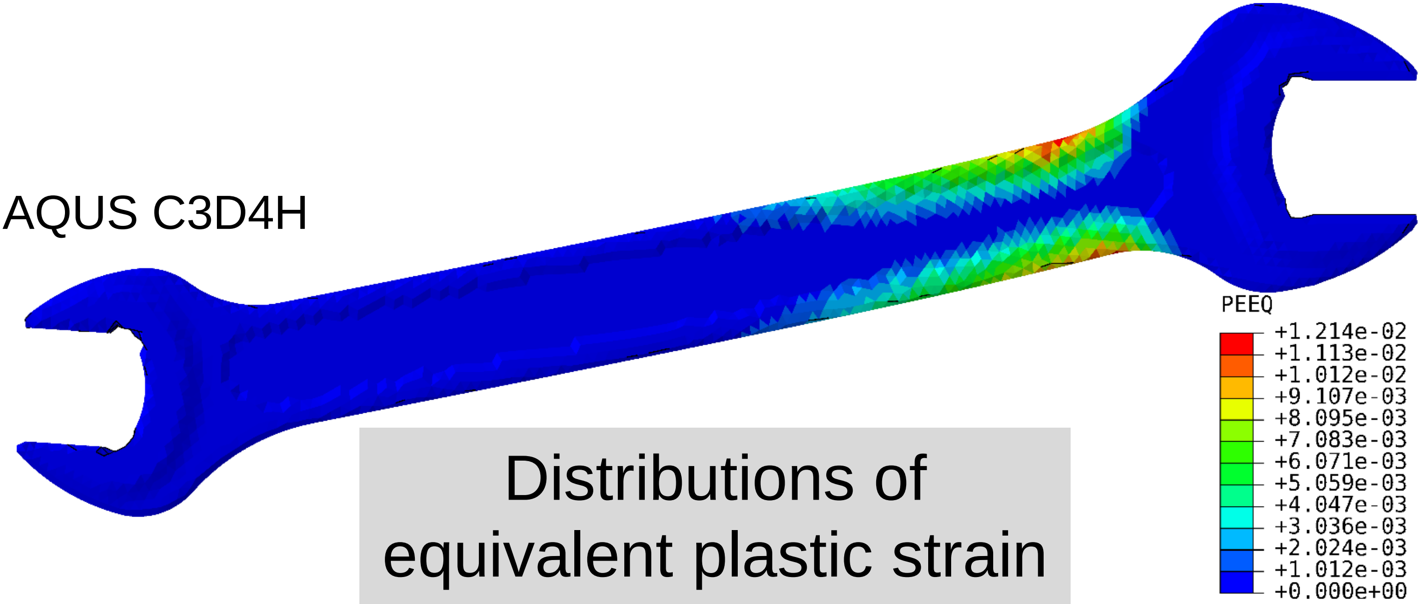
Bending of Elasto-plastic Spanner

Equivalent plastic strain dist.

F-barES-FEM-T4



ABAQUS C3D4H



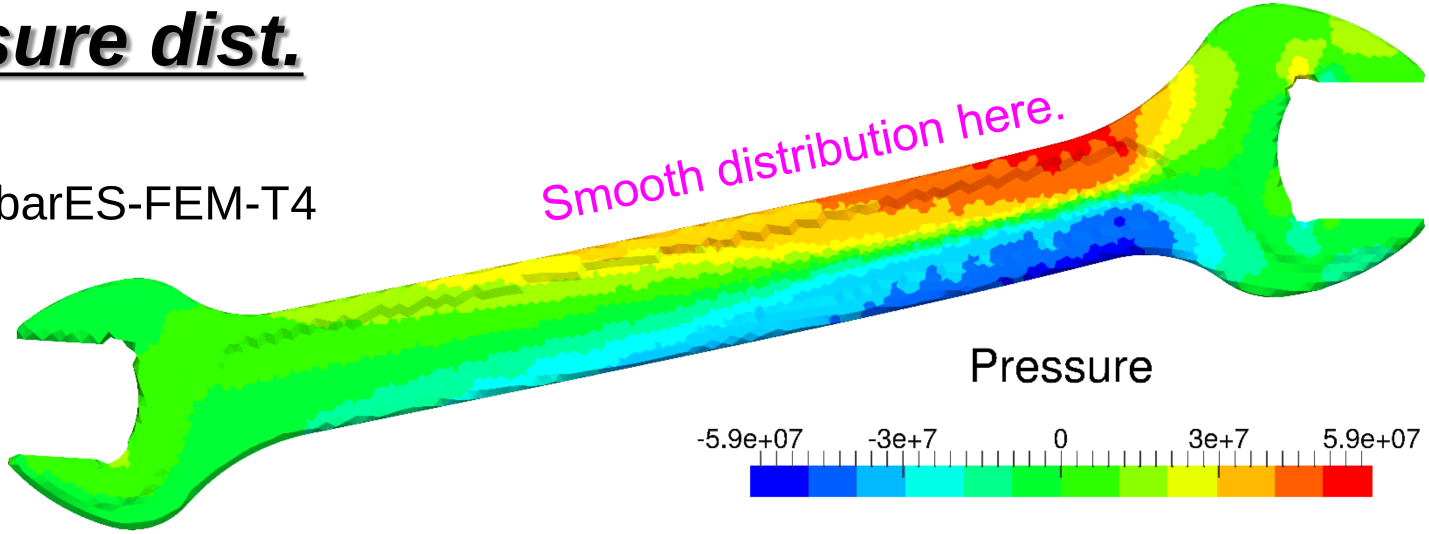
Distributions of equivalent plastic strain are about the same.

Static Implicit

Bending of Elasto-plastic Spanner

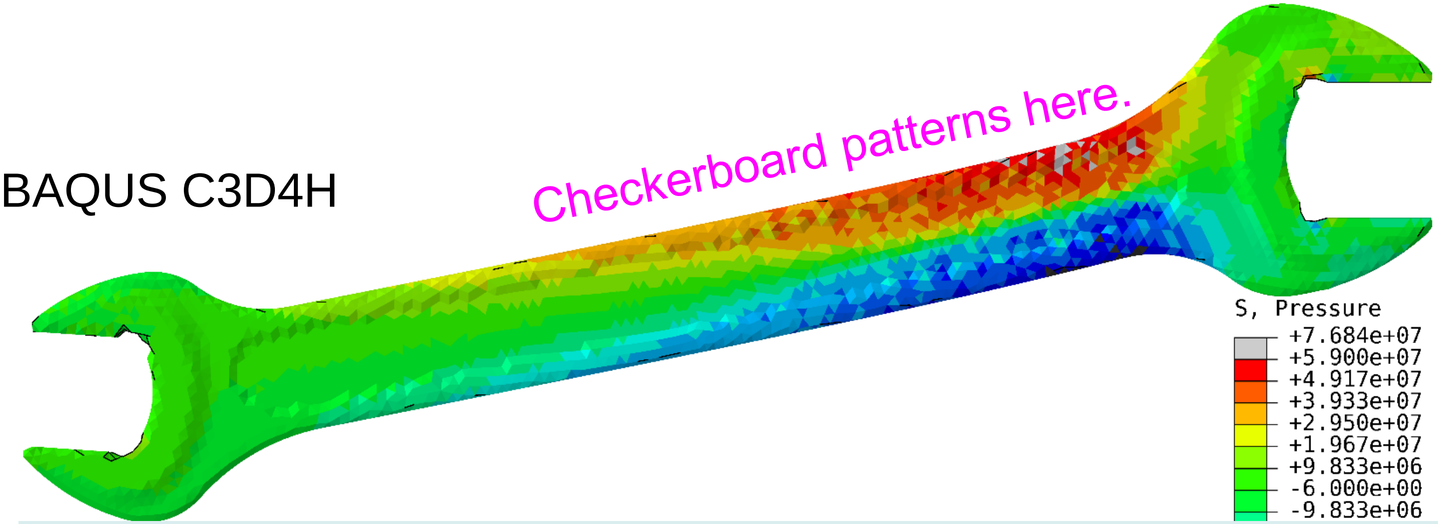
Pressure dist.

F-barES-FEM-T4



Smooth distribution here.

ABAQUS C3D4H



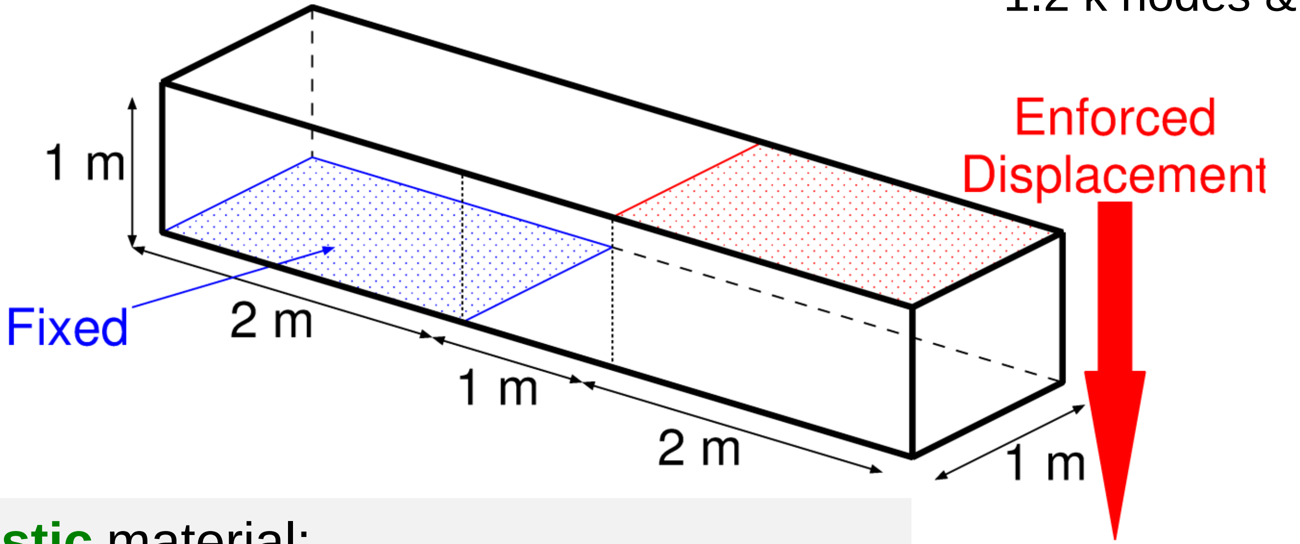
Checkerboard patterns here.

ABAQUS C3D4H suffers from pressure checkerboarding even in a small deformation elasto-plastic case.

Shear-tensioning of Elasto-plastic Bar

Outline

1.2 k nodes & 4.8 k elems.



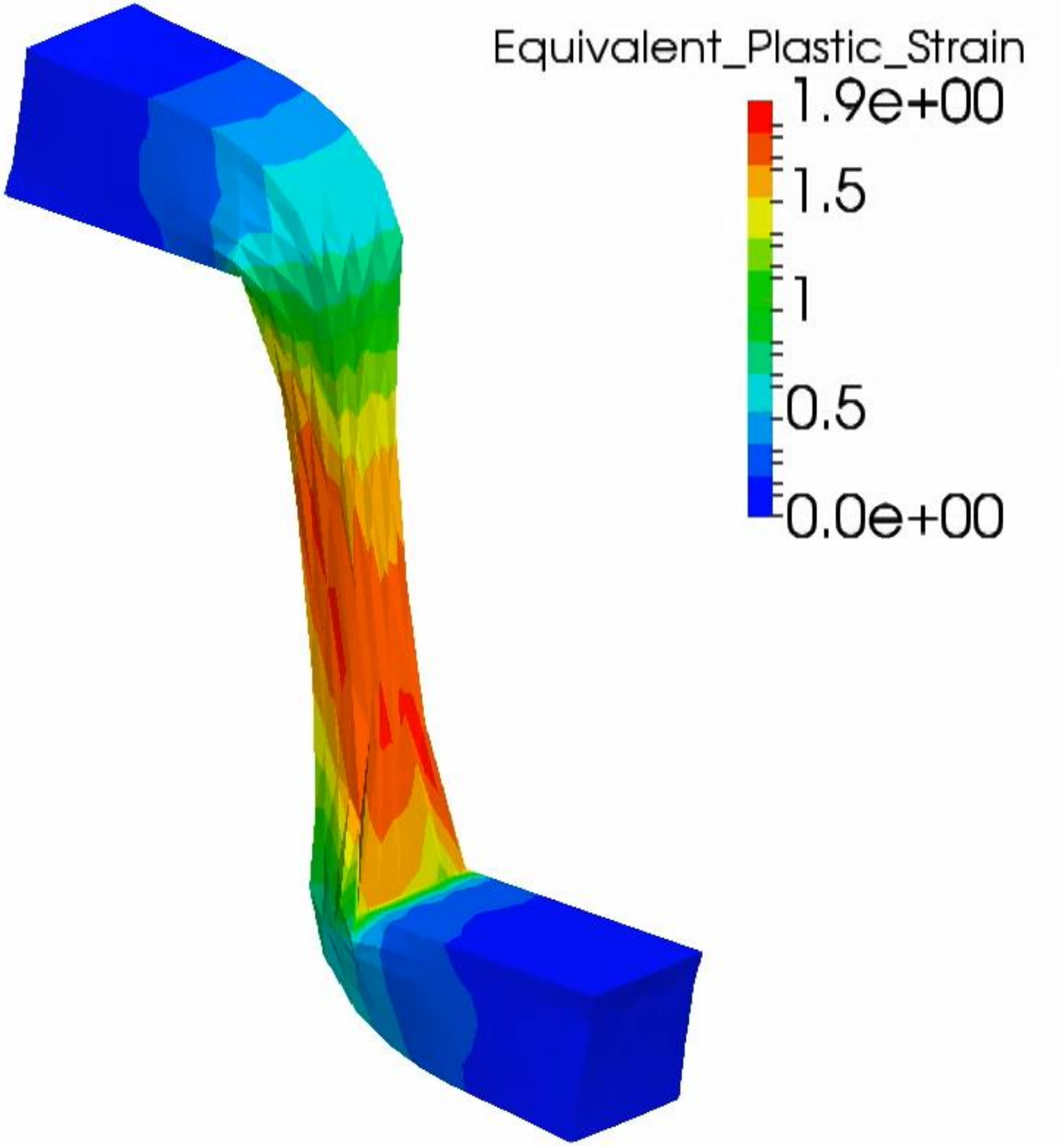
Elasto-plastic material:
- Hencky elasticity with $E = 1\ \mathrm{GPa}$ and $\nu = 0.3$.
- Isotropic von Mises yield criterion with $\sigma_\mathrm{Y} = 1\ \mathrm{MPa}$ and $H = 0.1\ \mathrm{GPa}$ (constant).

- Blue face is perfectly constrained.
- Red face is constrained in plane and pressed down.
- Compared to ABAQUS C3D4H with the same unstructured T4 mesh.

Static
Implicit

Shear-tensioning of Elasto-plastic Bar

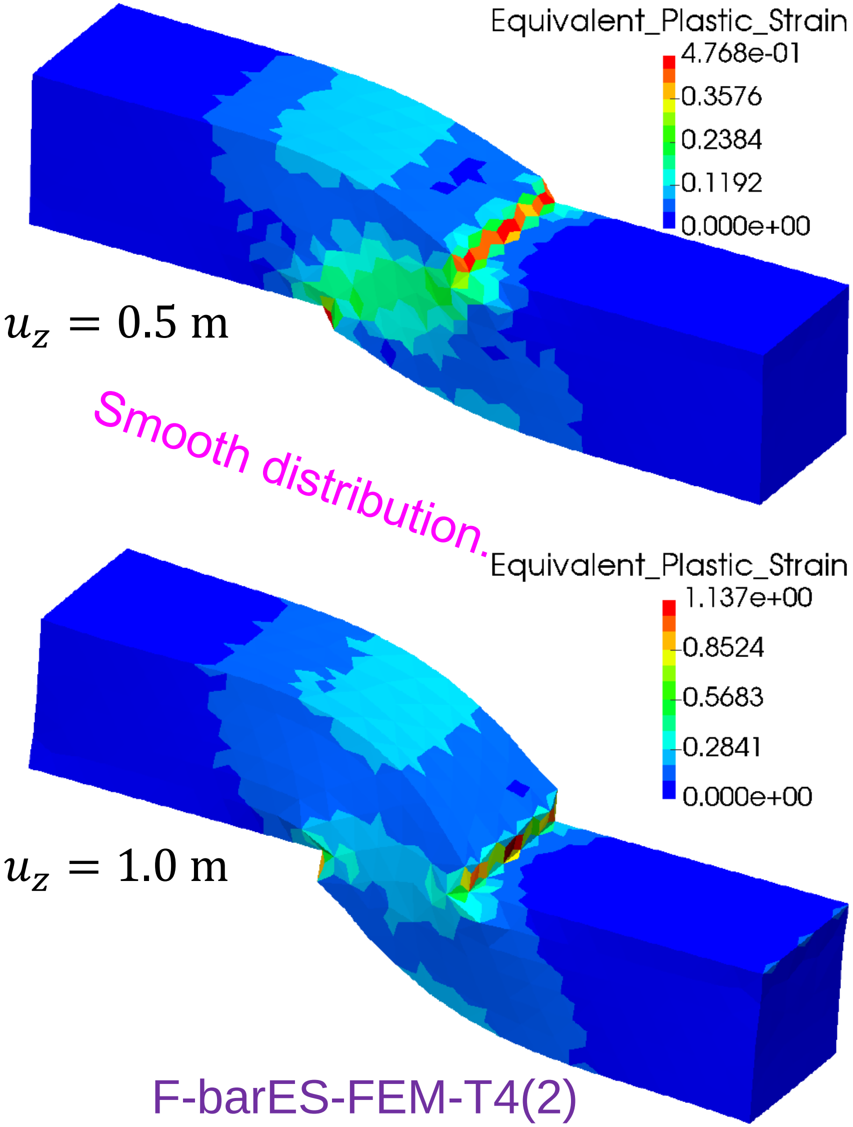
Result of
F-bar
ES-FEM(2)
(Equiv. plastic strain)



Static Implicit

Shear-tensioning of Elasto-plastic Bar

Equivalent plastic strain dist.

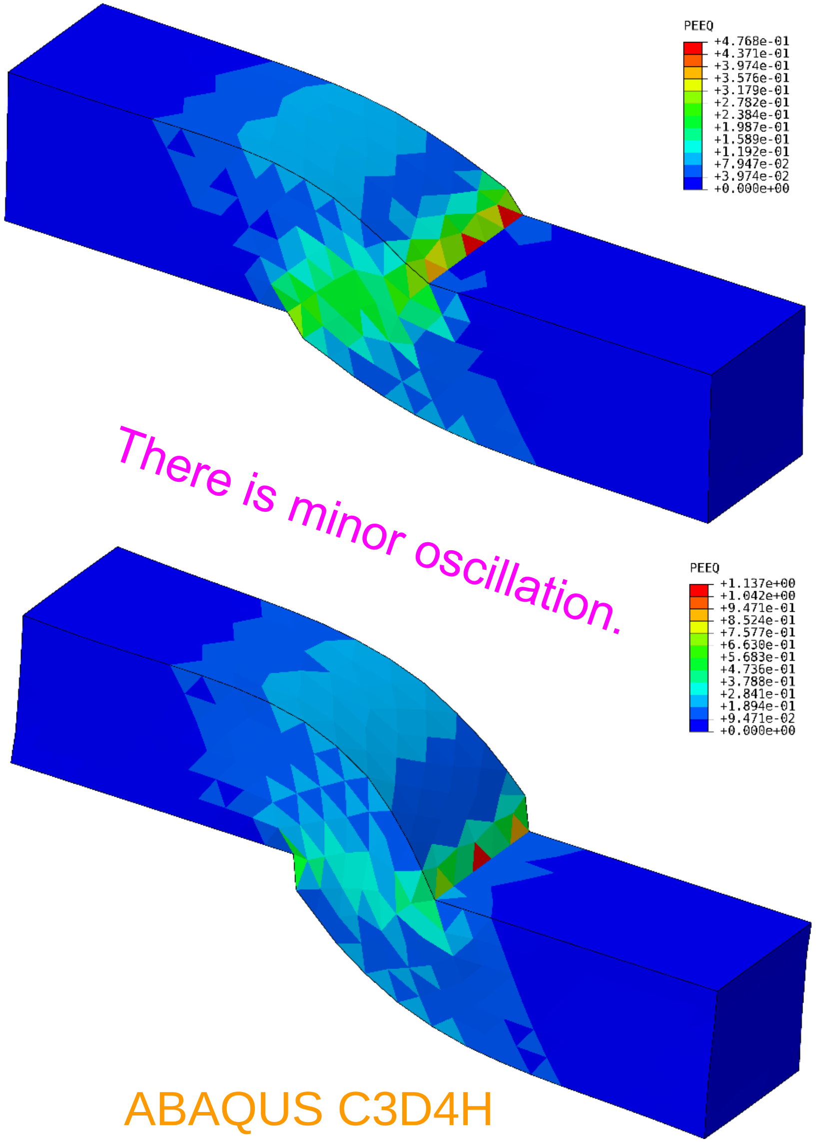


$u_z = 0.5\ \mathrm{m}$

$u_z = 1.0\ \mathrm{m}$

Smooth distribution.

F-barES-FEM-T4(2)



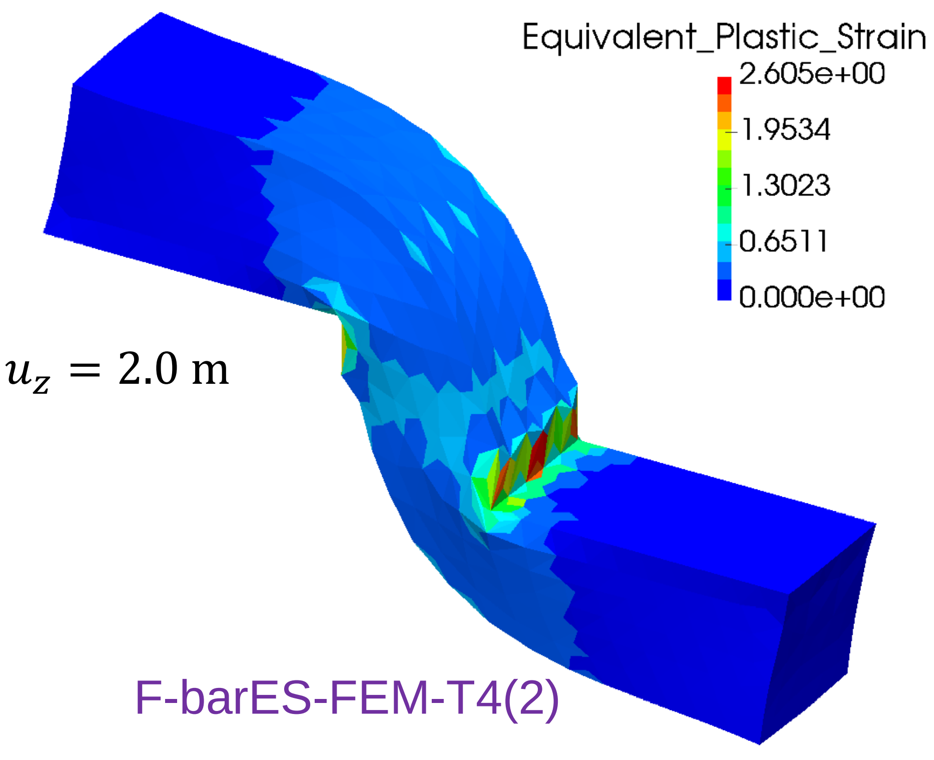
There is minor oscillation.

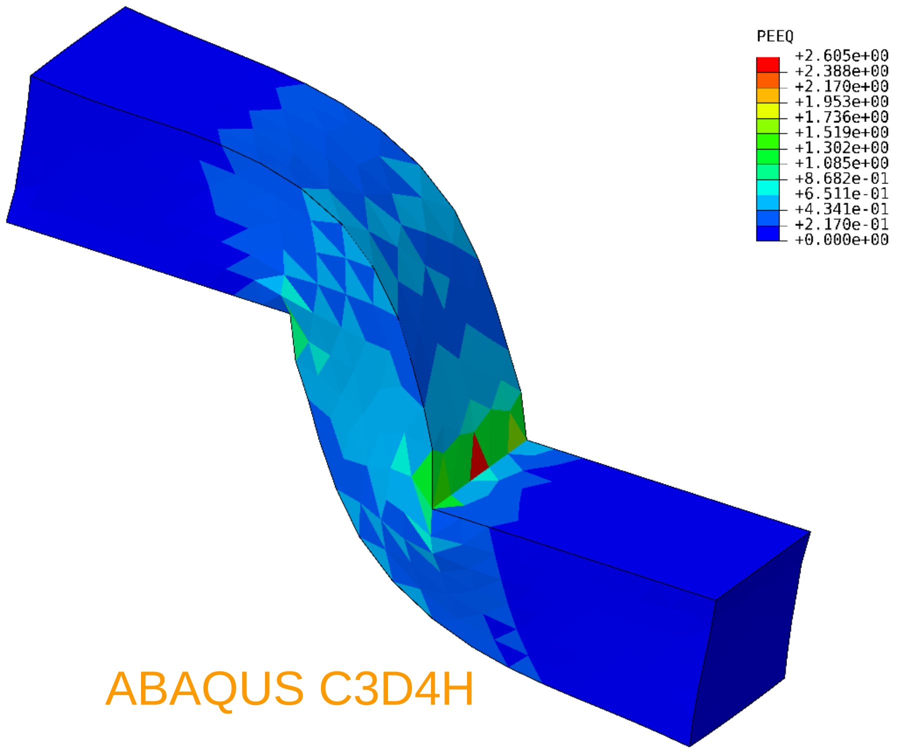
ABAQUS C3D4H

Static
Implicit

Shear-tensioning of Elasto-plastic Bar

Equivalent plastic strain dist.



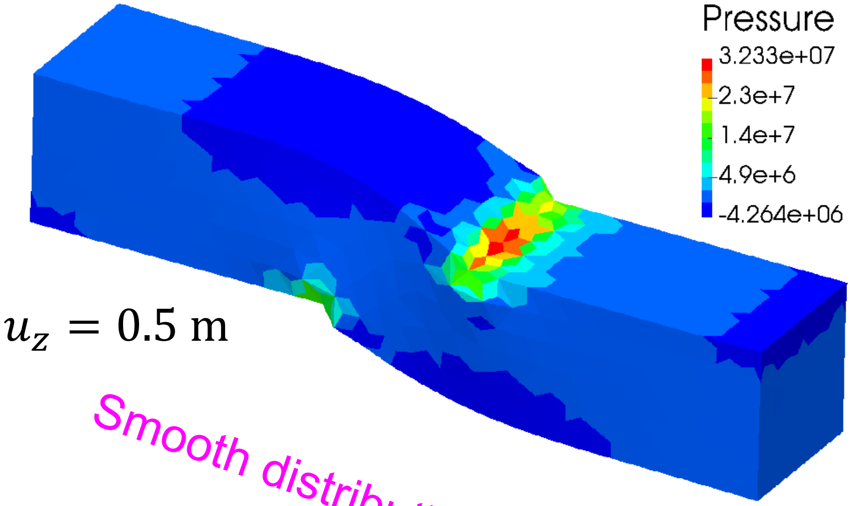


Accuracy of equivalent plastic strain seems no much different.

Static Implicit

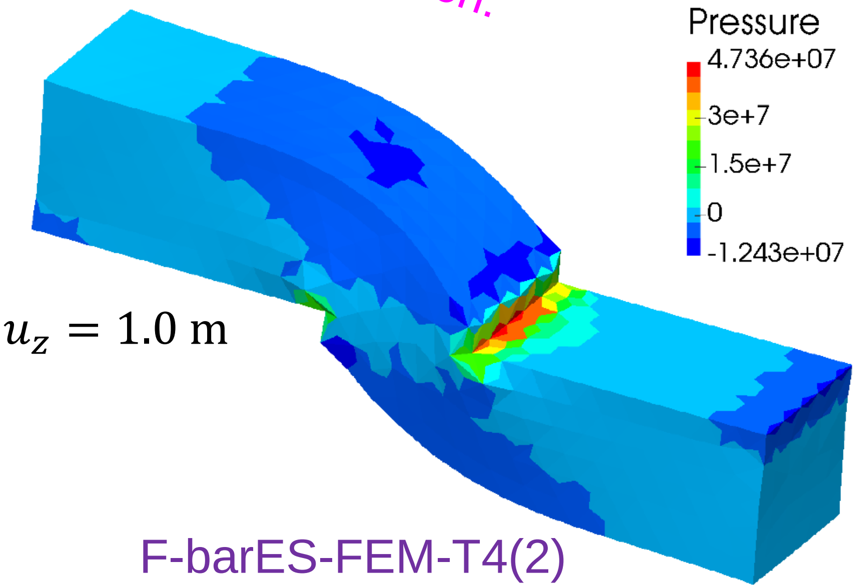
Shear-tensioning of Elasto-plastic Bar

Pressure



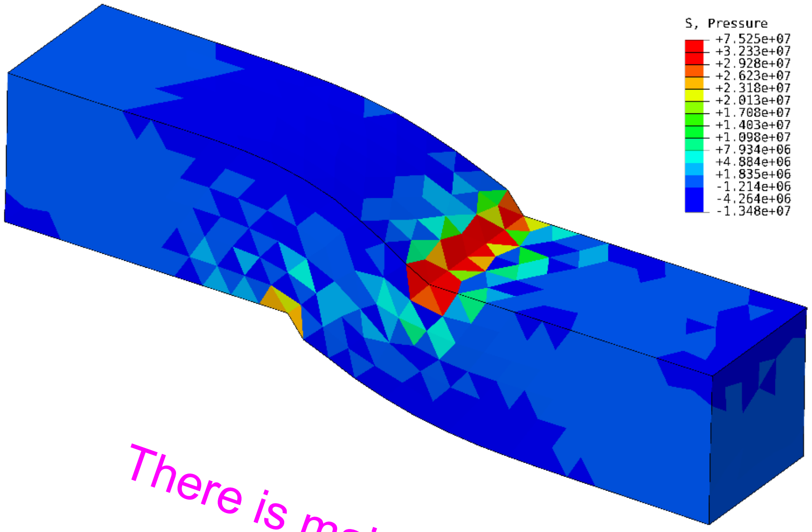
$u_z = 0.5\ \mathrm{m}$

Smooth distribution.

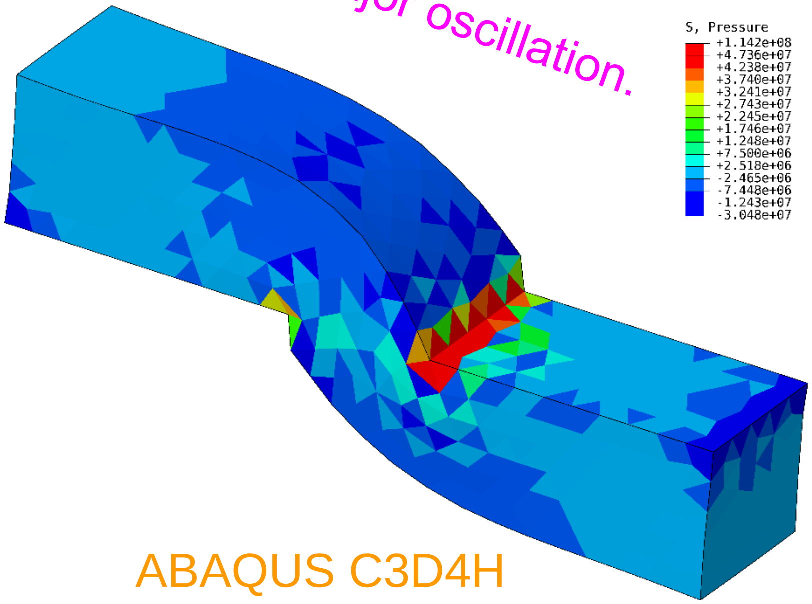


$u_z = 1.0\ \mathrm{m}$

F-barES-FEM-T4(2)



There is major oscillation.



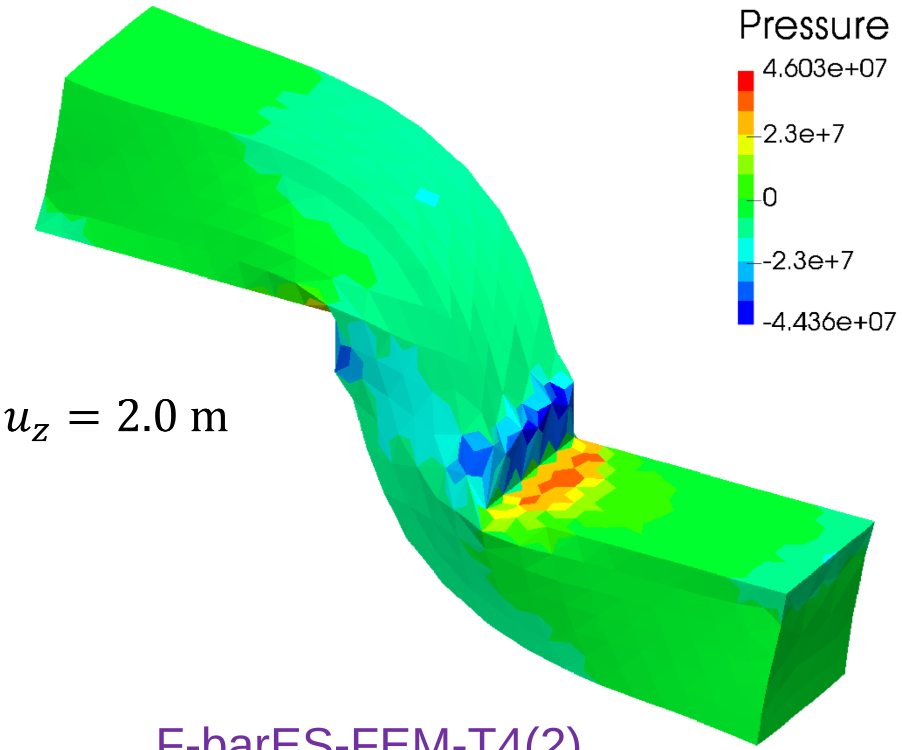
ABAQUS C3D4H

Static Implicit

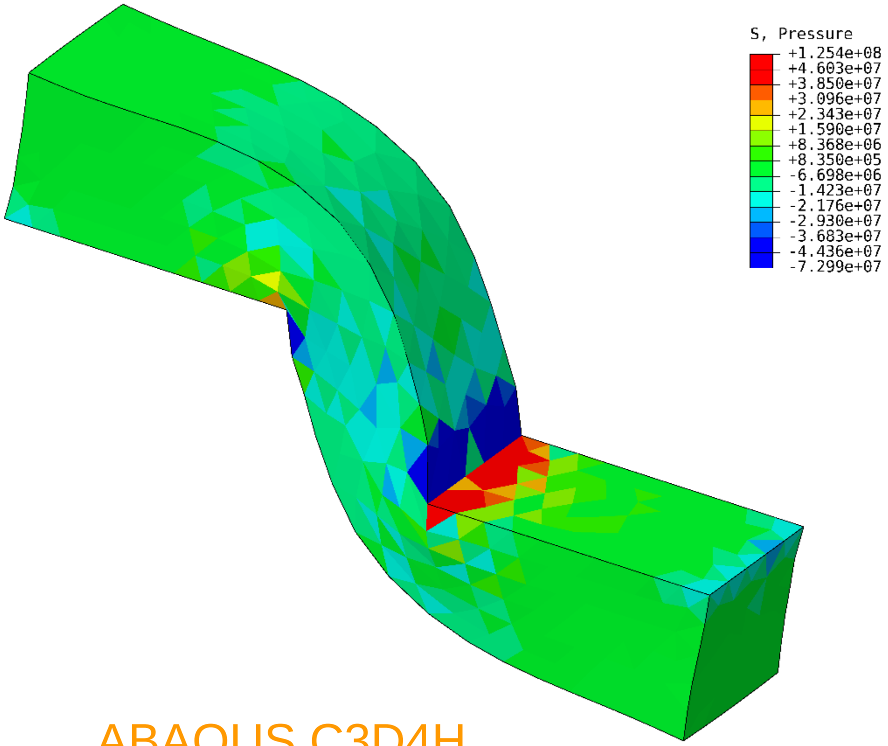
Shear-tensioning of Elasto-plastic Bar

Pressure

$u_z = 2.0\ \mathrm{m}$



F-barES-FEM-T4(2)



ABAQUS C3D4H

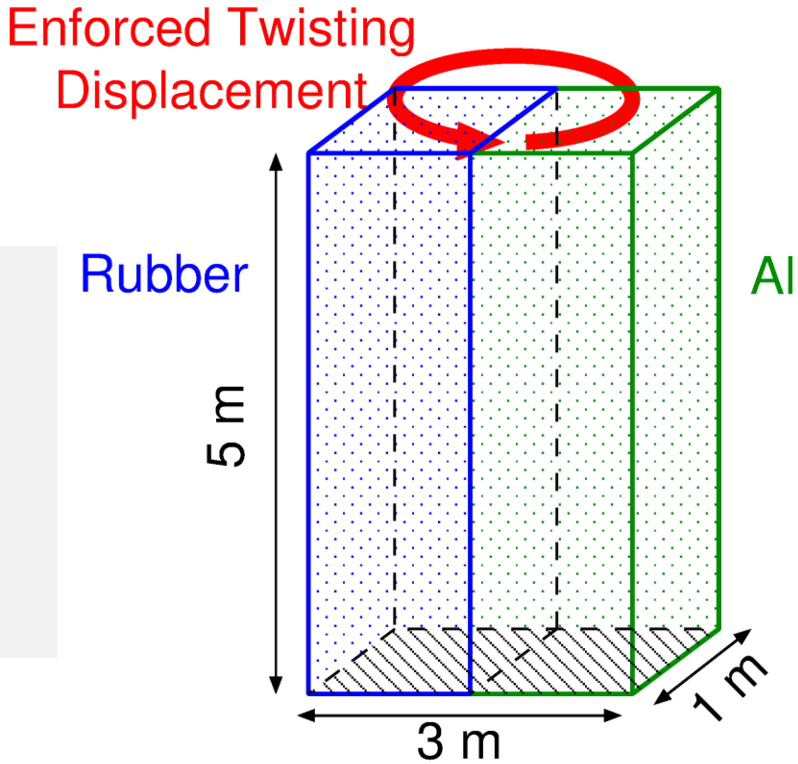
F-barES-FEM-T4 is free from pressure checkerboarding in elasto-plastic analysis.

Static Implicit

Twist of Rubber/Aluminium Composite

Outline

3 k nodes & 14 k elems.



[Rubber]
Neo-Hook Hyperelasticity:
$E^{\mathrm{ini}} = 5\ \mathrm{MPa}$,
$\nu^{\mathrm{ini}} = 0.49$,
$(c = 1)$

[Aluminium]
Hencky elasticity:
$E = 70\ \mathrm{GPa}$,
$\nu = 0.3$.
Isotropic von Mises plasticity:
$\sigma_{\mathrm{Y}} = 100\ \mathrm{MPa}$,
$H = 0.7\ \mathrm{GPa}$ (const.),
$(c = 2)$

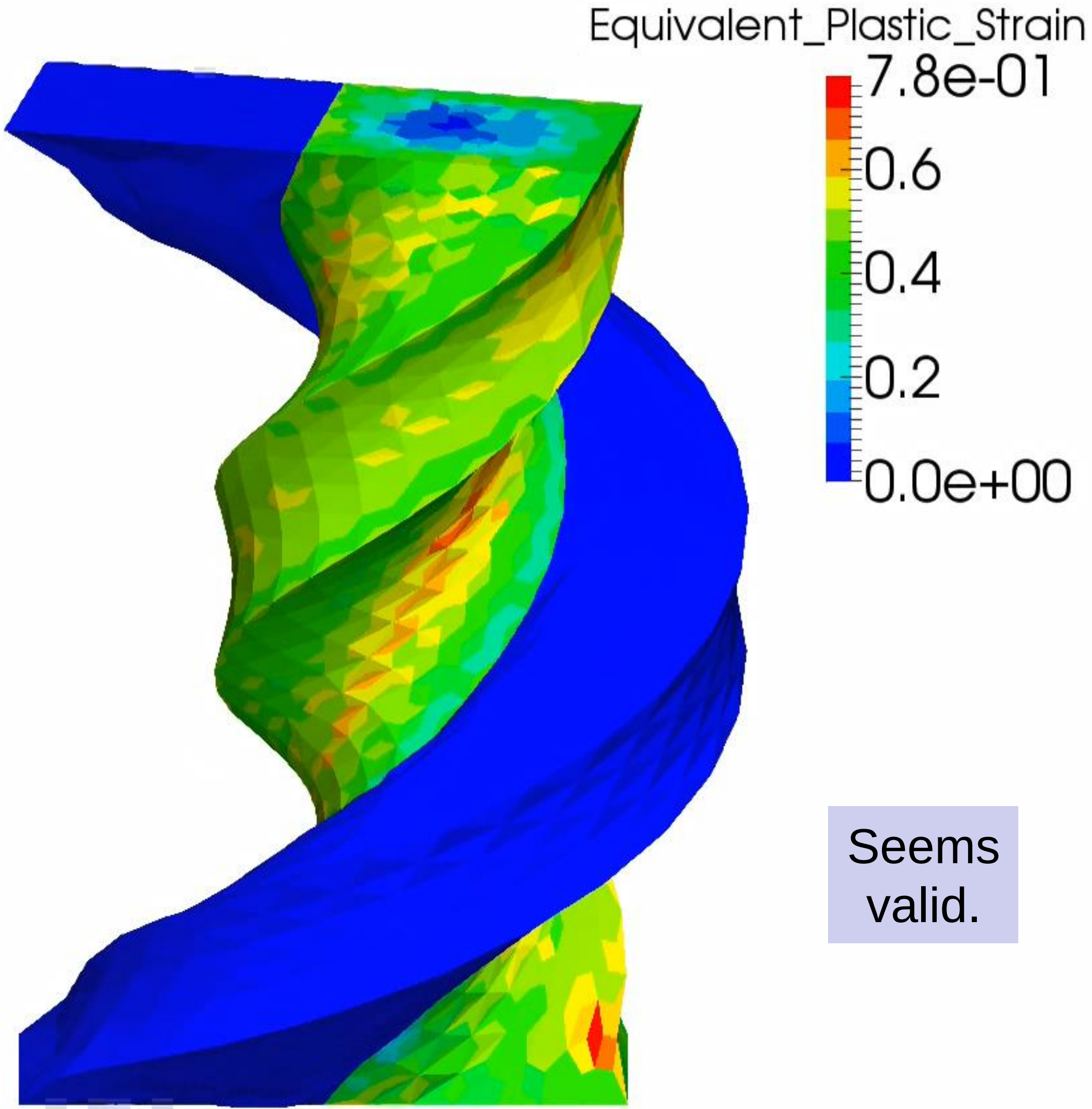
- Bottom face is perfectly constrained.
- Top face is constrained in the plane and twisted 360 deg. around the vertical axis.
- Compared to ABAQUS C3D4H with the same tet mesh.
- Multiple Fs at each edge on the material interface.

Static
Implicit

Twist of Rubber/Aluminium Composite

Result of
F-bar
ES-FEM-T4

Equivalent
Plastic
Strain



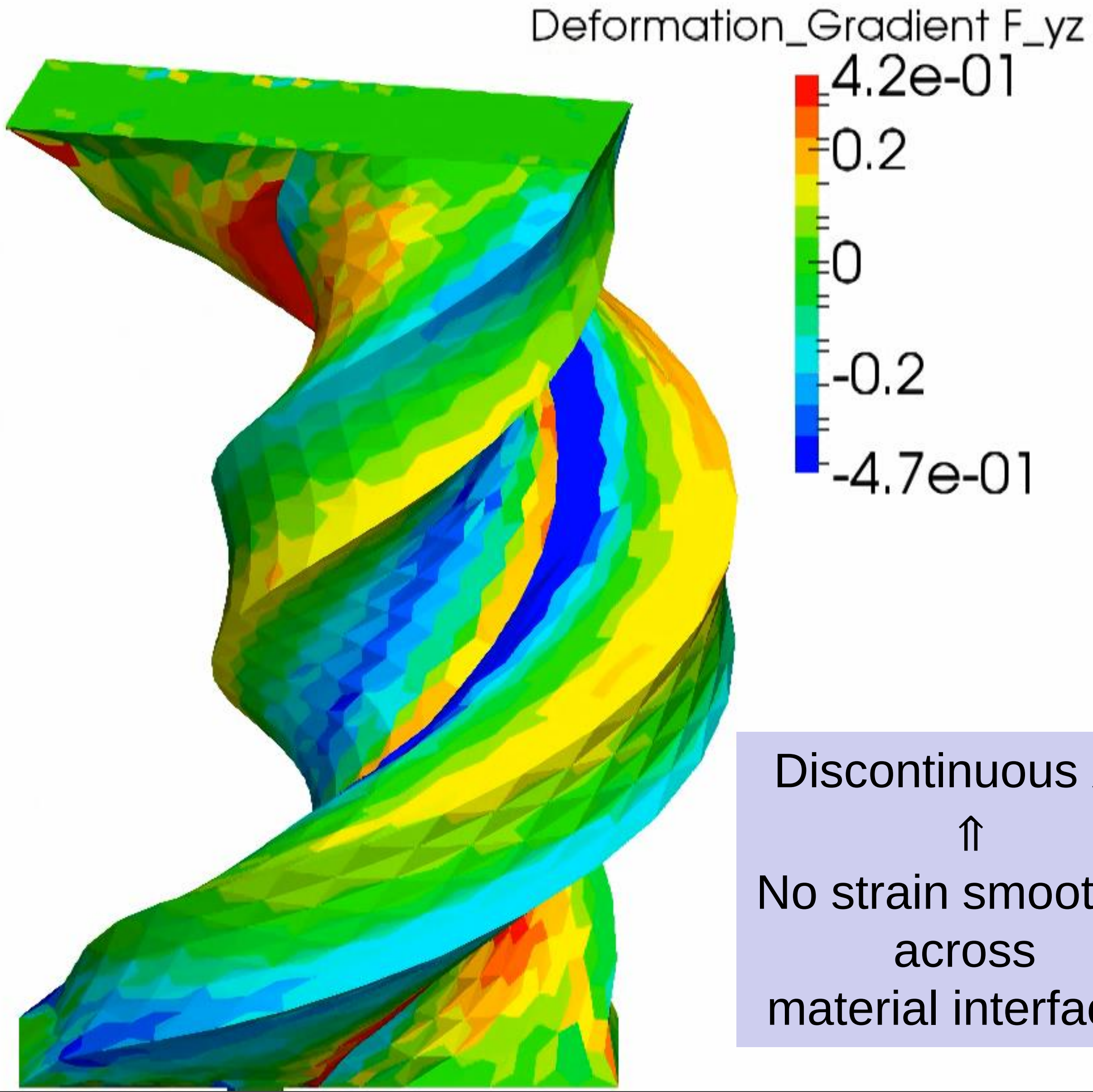
Seems
valid.

Static Implicit

Twist of Rubber/Aluminium Composite

Result of
F-bar
ES-FEM-T4

Deformation Gradient F_{yz}



Discontinuous F_{yz}.

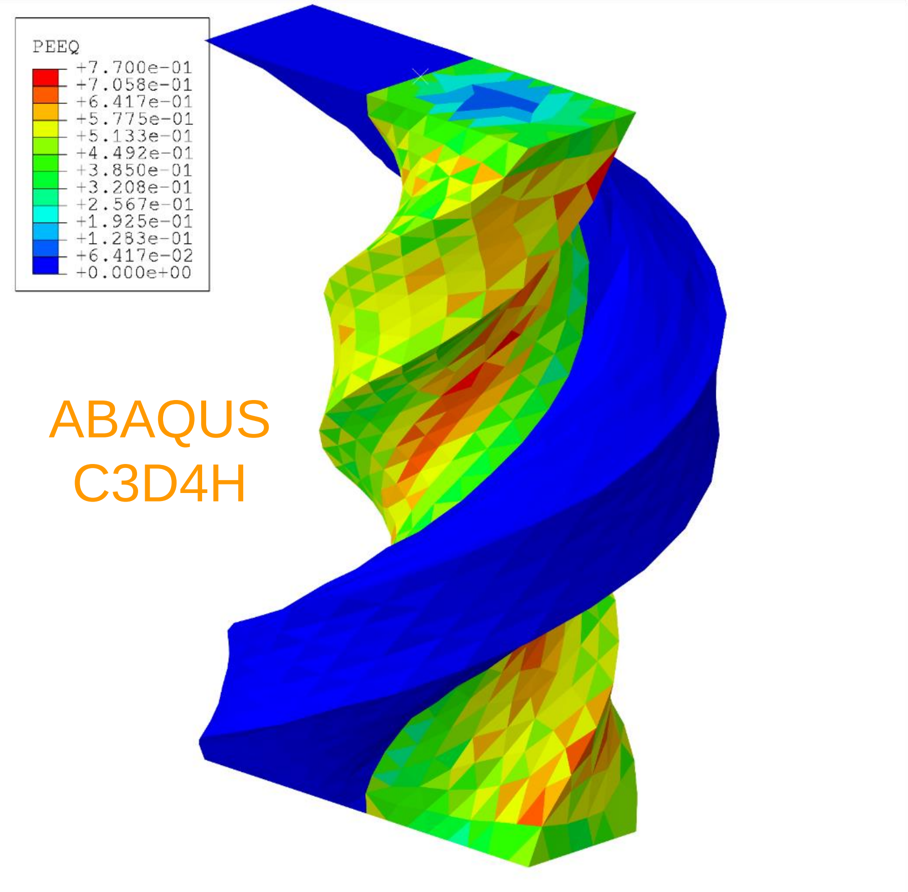
⇑

No strain smoothing across material interfaces.

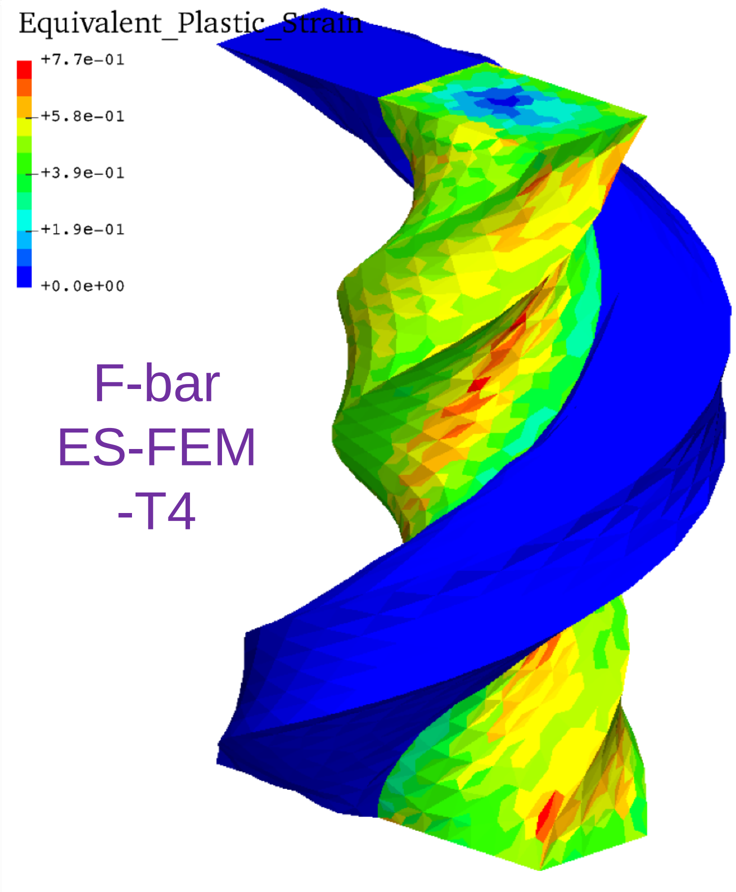
Static Implicit

Twist of Rubber/Aluminium Composite

Equivalemt plastic strain dist.



ABAQUS C3D4H



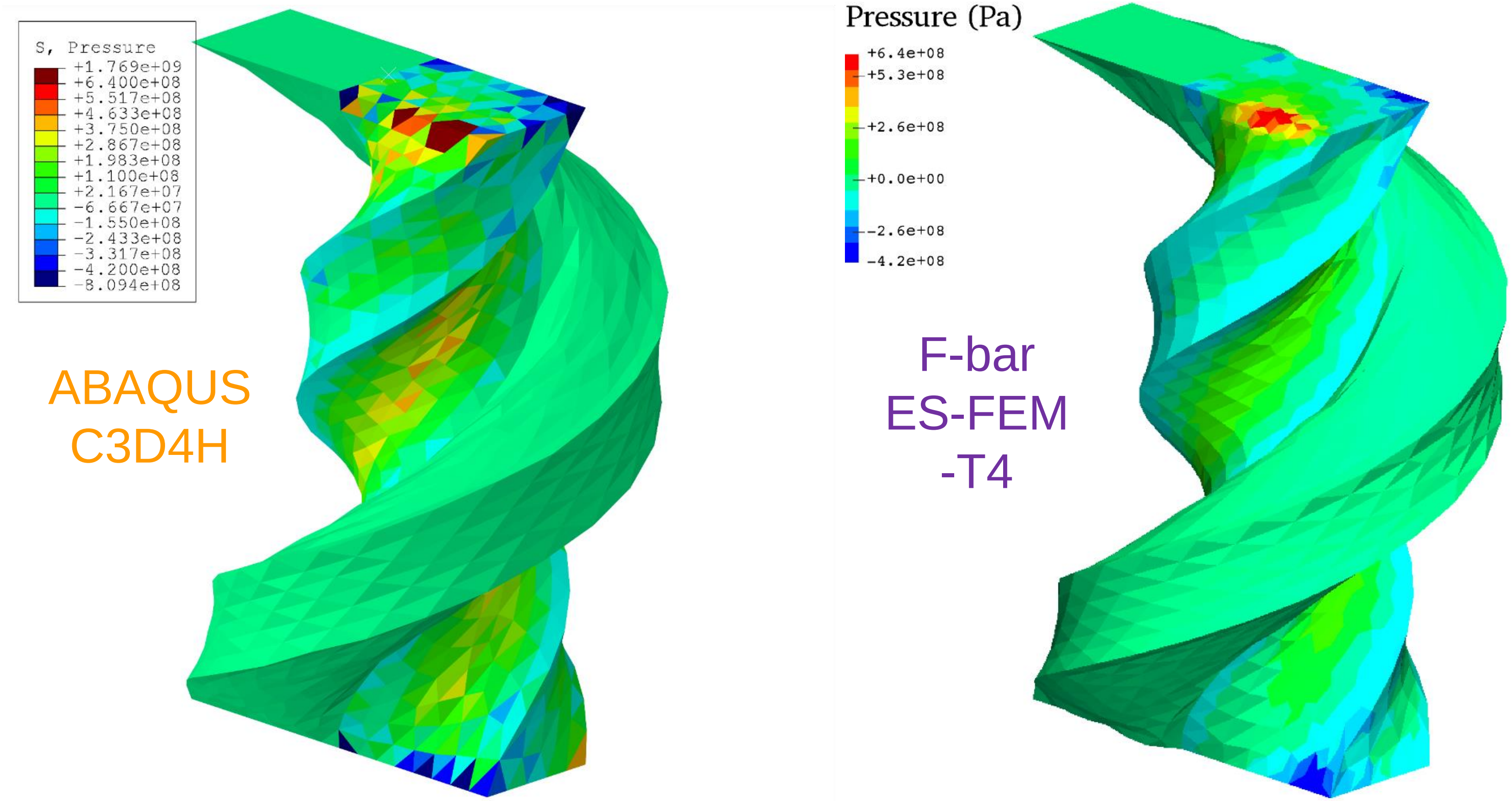
F-bar ES-FEM -T4

ABAQUS C3D4H has checkerboarding in plastic strains, meanwhile F-barES-FEM-T4 has smooth plastic strain dist..

Static Implicit

Twist of Rubber/Aluminium Composite

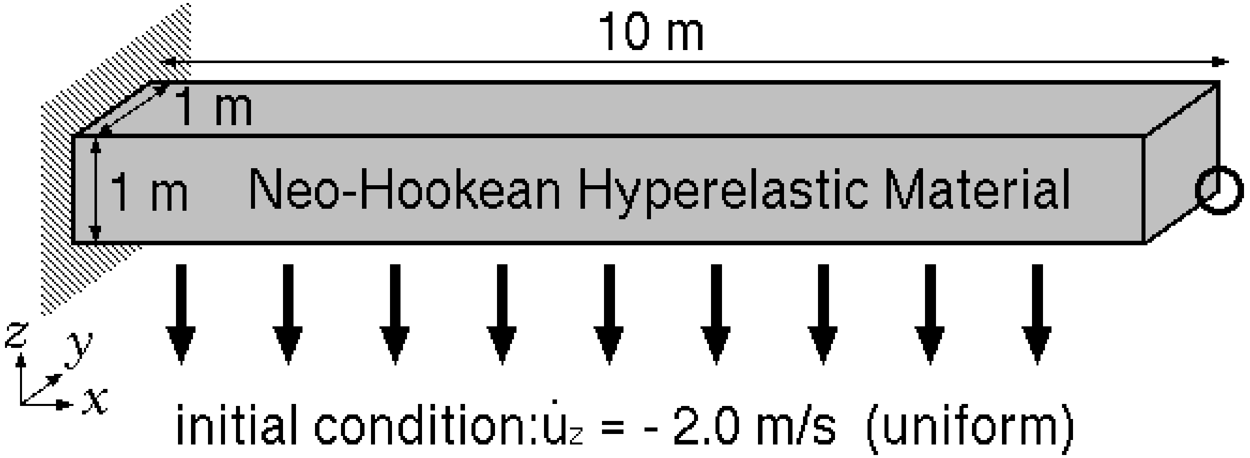
Pressure dist.



ABAQUS C3D4H has checkerboarding in stresses, meanwhile F-barES-FEM-T4 has smooth stress dist..

Dynamic Bend of Cantilever

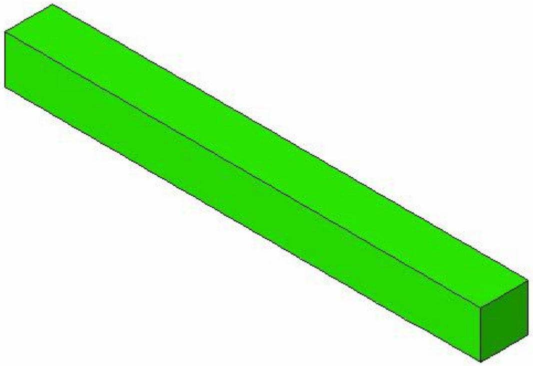
Outline



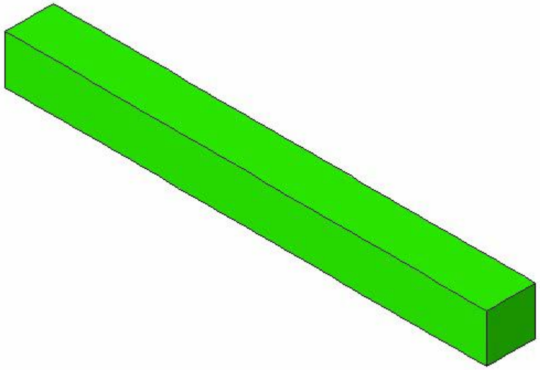
- Neo-Hookean hyperelastic material: $E_{\mathrm{ini}} = 6\ \mathrm{MPa},\ \nu_{\mathrm{ini}} = 0.499,\ \rho = 1000\ \mathrm{kg/m^3}.$
- Uniform initial velocity: $\dot{u}_z = -2\ \mathrm{m/s}.$
- Compared to ABAQUS/Explicit C3D4 (NOT C3D4H) & C3D8 (hexahedral selective reduced integration).

Dynamic Bend of Cantilever

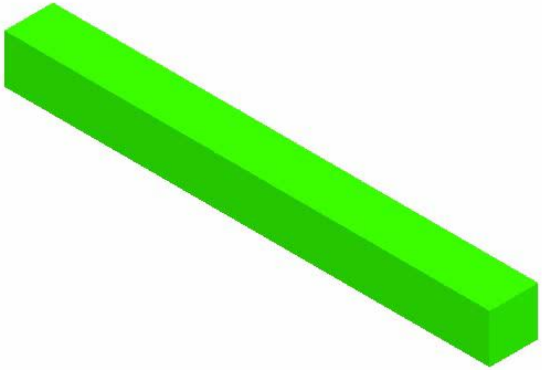
Pressure sign distributions



ABAQUS/Explicit C3D4
(✗ Locking &
Pressure oscillation)



ABAQUS/Explicit C3D8



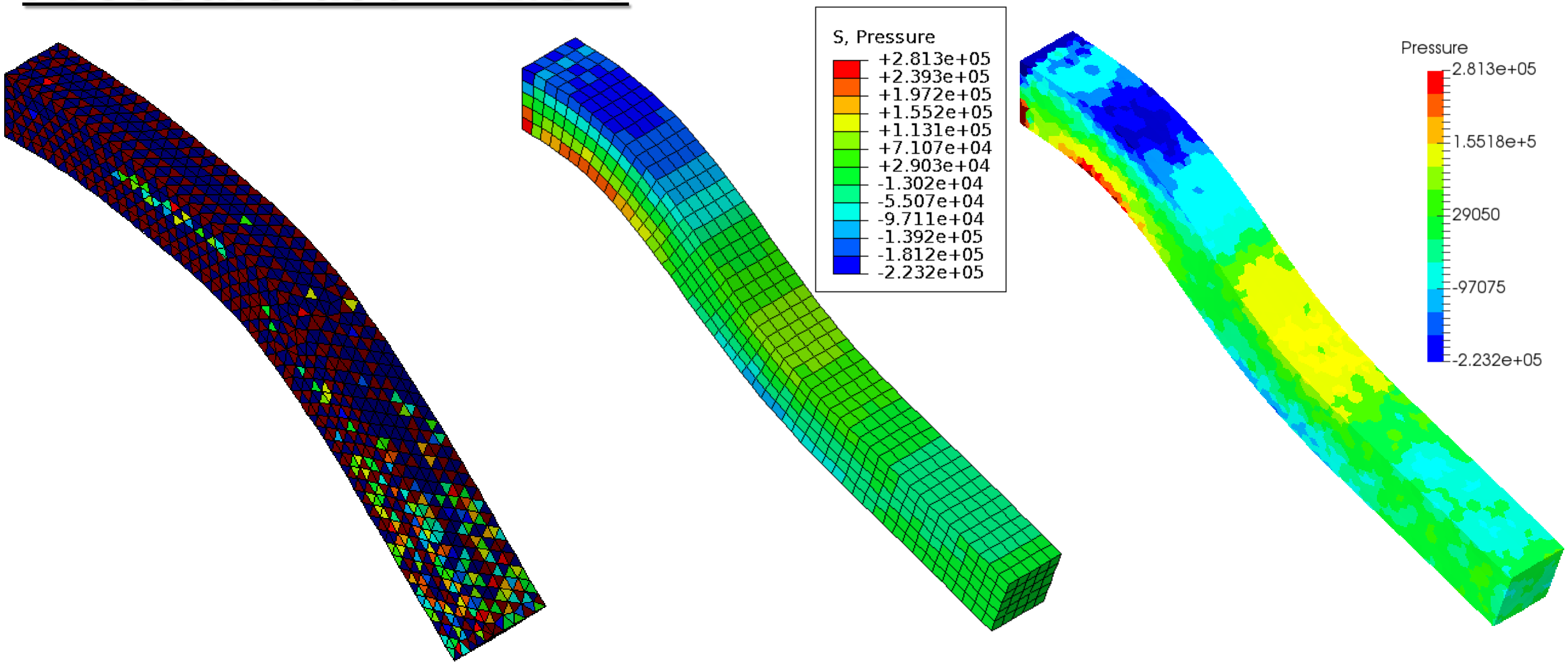
F-barES-FEM-T4(2)

F-barES-FEM-T4 has no locking & less pressure checkerboarding in dynamic explicit analysis.

Dynamic Explicit

Dynamic Bend of Cantilever

Pressure at $t = 1.5$ s



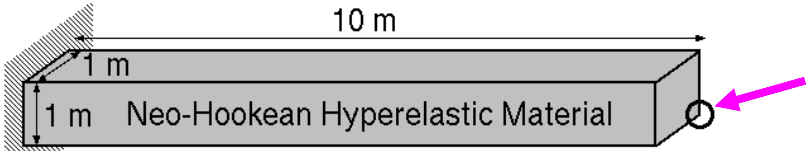
ABAQUS/Explicit C3D4 ABAQUS/Explicit C3D8 F-barES-FEM-T4(2)

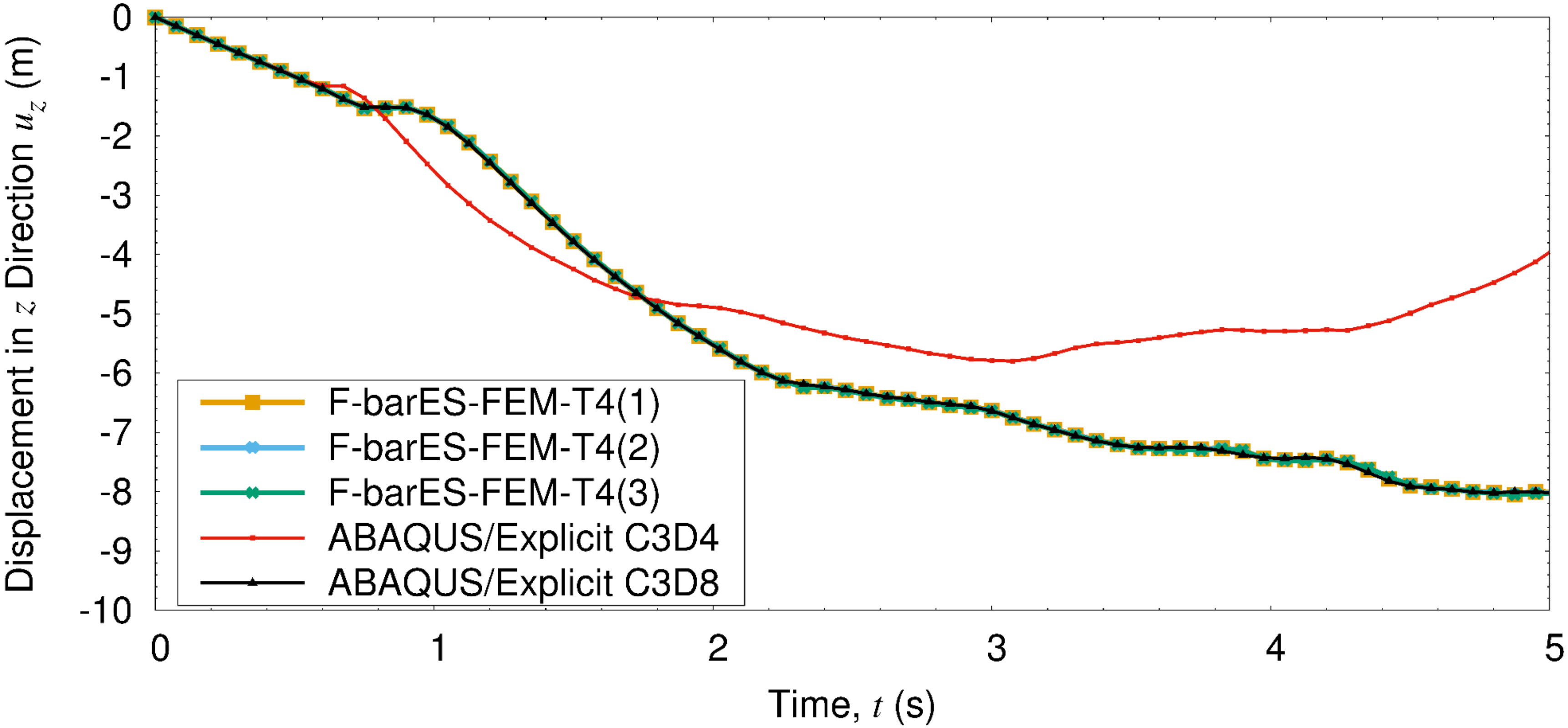
F-barES-FEM-T4 has good accuracy in pressure.

Dynamic Explicit

Dynamic Bend of Cantilever

Deflection

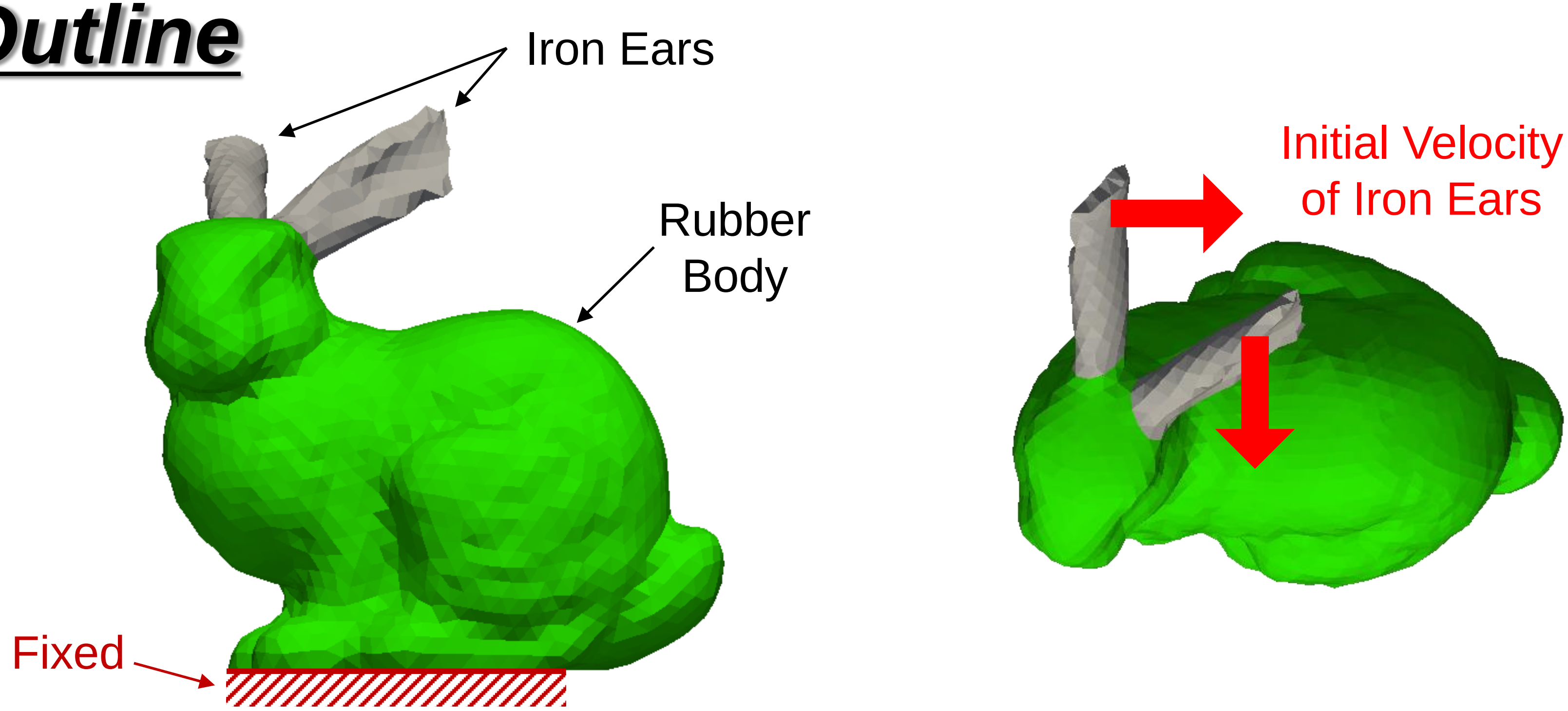




Displacement accuracy of F-barES-FEM is independent of the number of cyclic smoothings.

Swinging of Bunny Ears

Outline

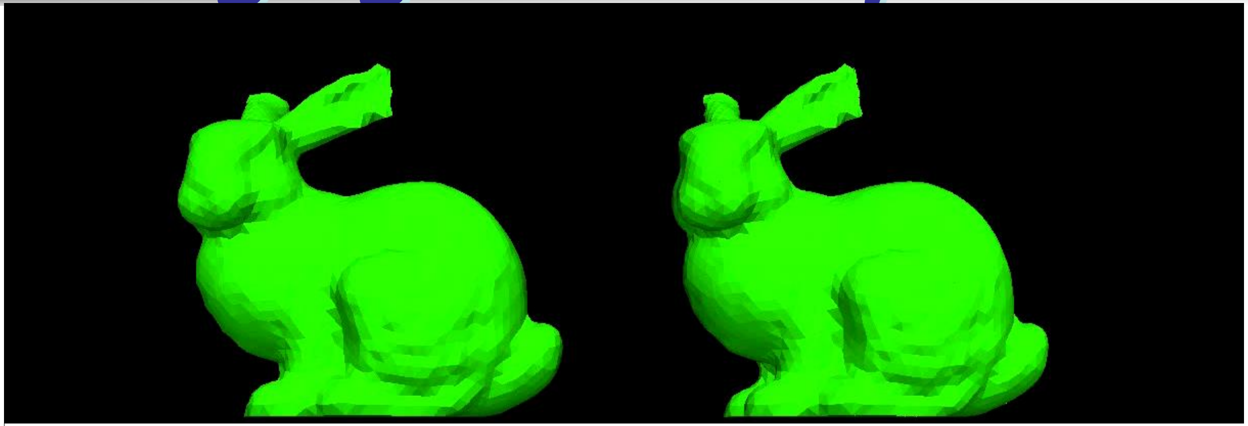


- Iron ears: $E_{\mathrm{ini}} = 200\ \mathrm{GPa},\ \nu_{\mathrm{ini}} = \mathbf{0.3},\ \rho = 7800\ \mathrm{kg/m^3}$, Neo-Hookean, **No cyclic smoothing.**
- Rubber body: $E_{\mathrm{ini}} = 6\ \mathrm{MPa},\ \nu_{\mathrm{ini}} = \mathbf{0.49},\ \rho = 920\ \mathrm{kg/m^3}$, Neo-Hookean, **1 cycle of smoothing.**
- Compared to ABAQUS/Explicit C3D4 etc..

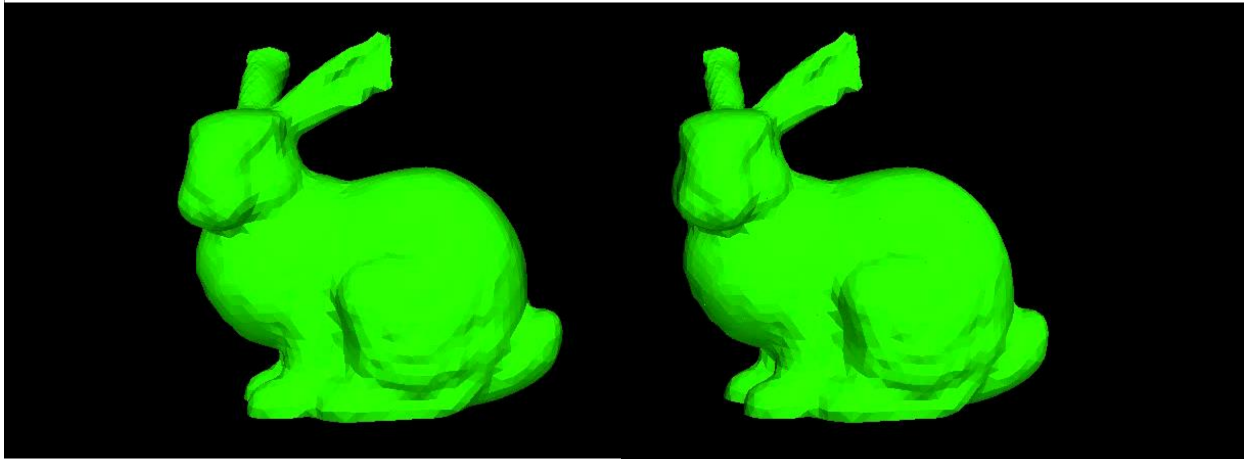
Dynamic
Explicit

Swinging of Bunny Ears

Pressure sign dist.



ABAQUS/Explicit C3D4 Selective ES/NS-FEM-T4

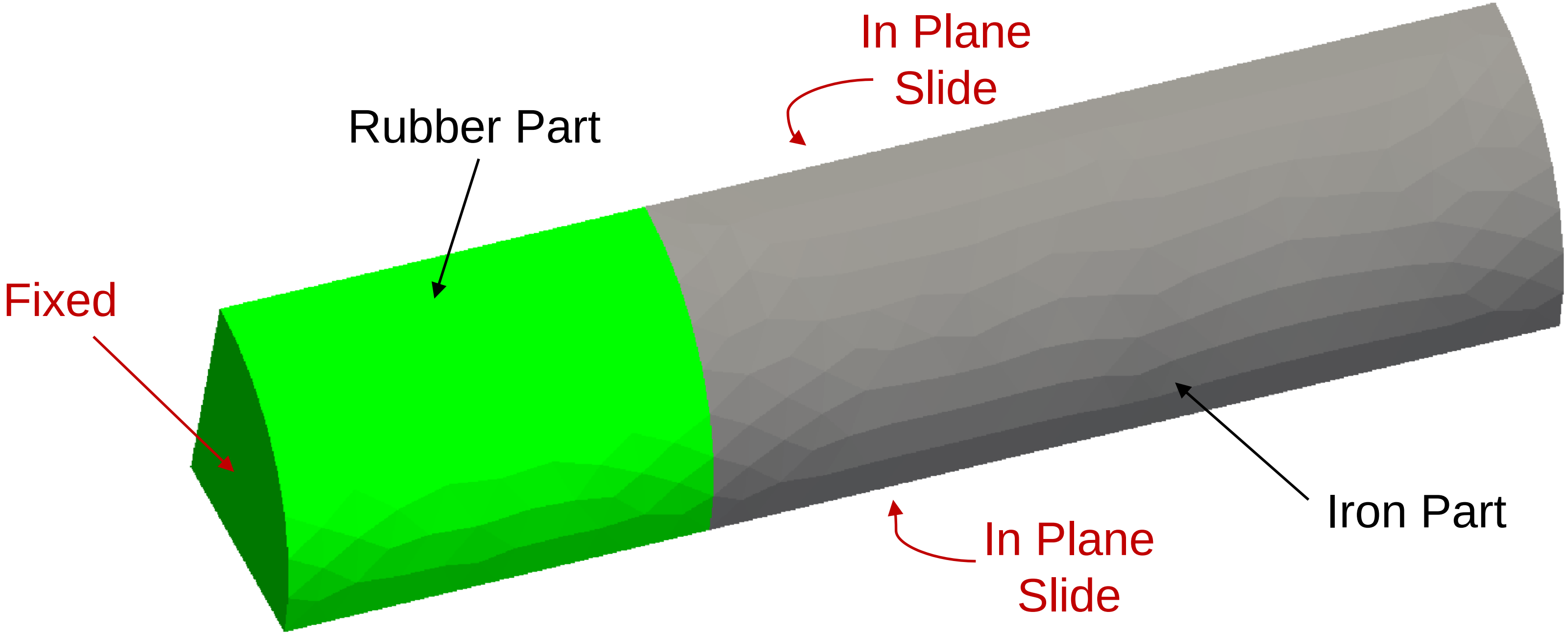


F-barES-FEM-T4 NS-FEM-T4

Only F-barES-FEM-T4 presents a valid result.

Natural Modes of ¼ Cylinder

Outline

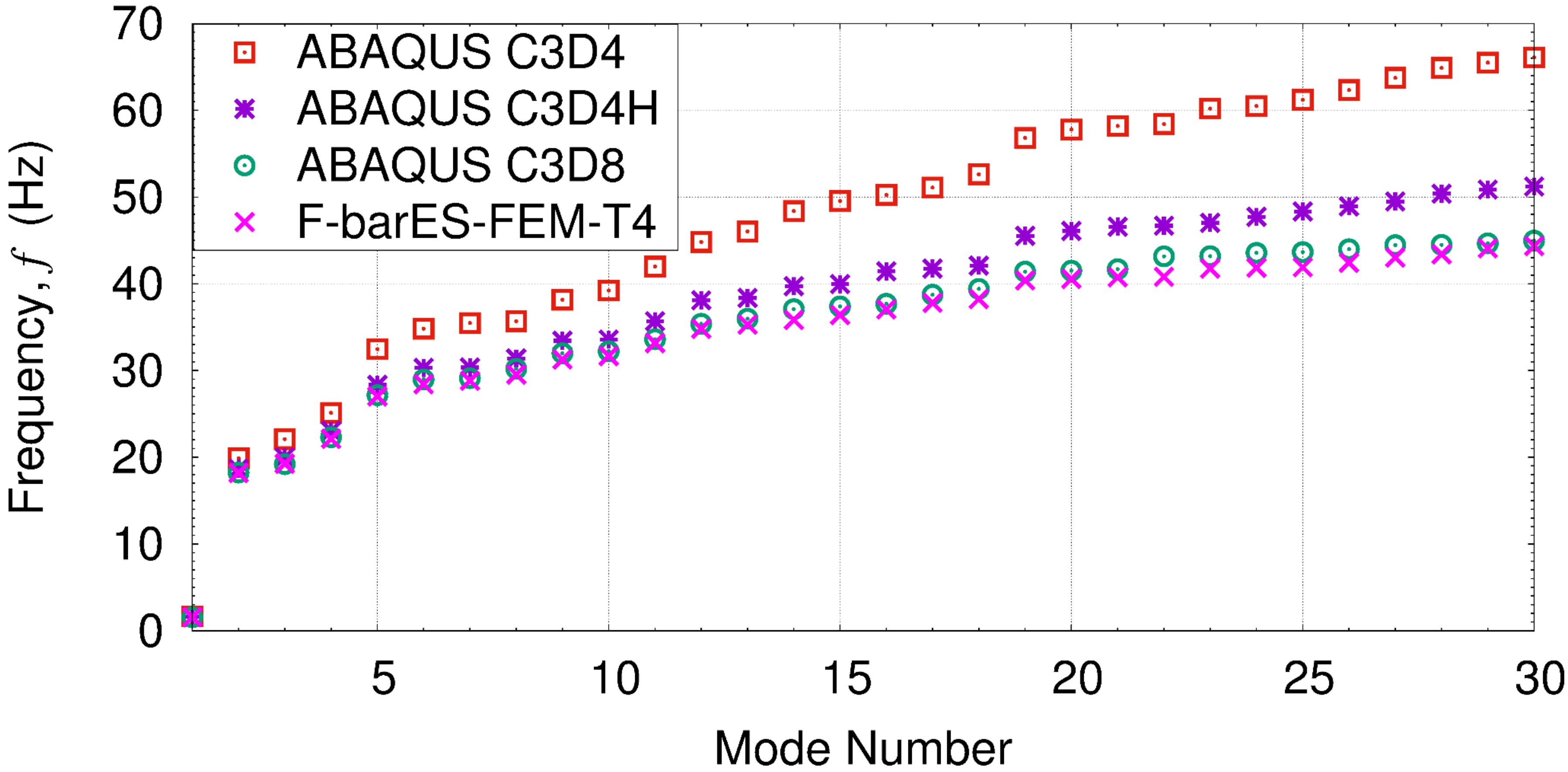


- Iron part: $E_{\text{ini}} = 200\ \text{GPa},\ \nu_{ini} = \mathbf{0.3},\ \rho = 7800\ \text{kg/m}^3$, Elastic, **No cyclic smoothing.**
- Rubber part: $E_{\text{ini}} = 6\ \text{MPa},\ \nu_{ini} = \mathbf{0.499},\ \rho = 920\ \text{kg/m}^3$, Elastic, **2 cycles of smoothing.**
- Compared to ABAQUS C3D4, C3D4H, and C3D8.

Eigen Mode

Natural Modes of ¼ Cylinder

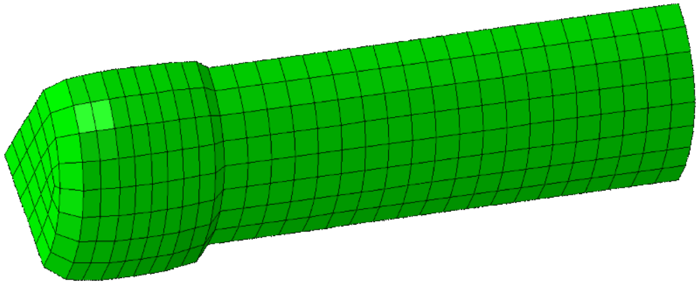
Eigen frequencies



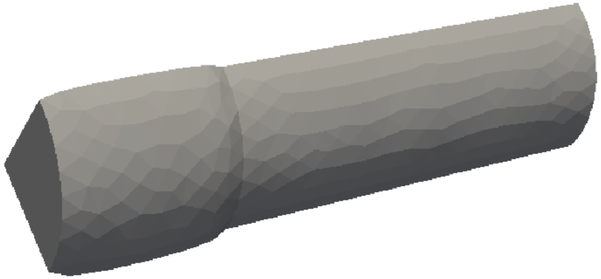
- C3D4 and C3D4H show higher frequencies (stiffer results).
- F-barES-FEM-T4 and C3D8 are in good agreement.

Natural Modes of ¼ Cylinder

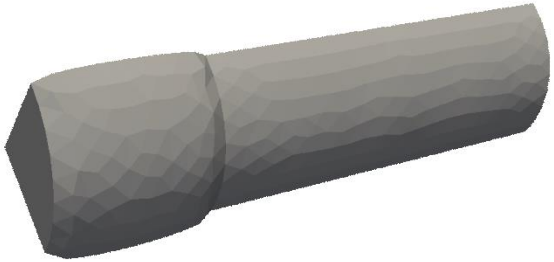
1st eigen mode



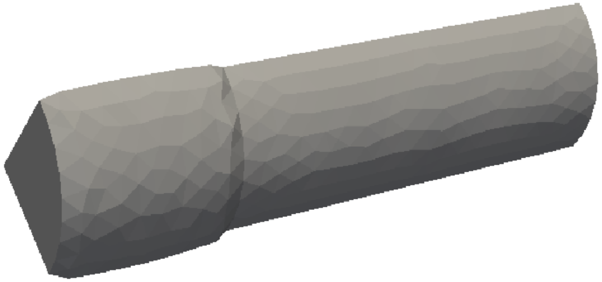
ABAQUS C3D8
(reference)



Selective ES/NS-FEM-T4



F-barES-FEM-T4

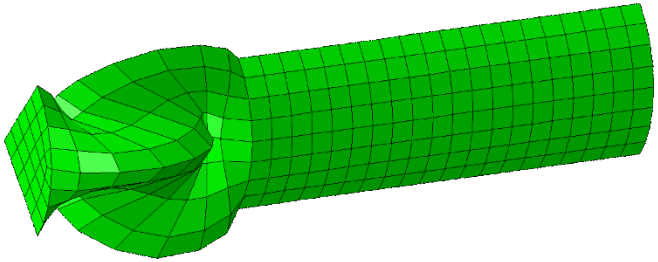


NS-FEM-T4

The 1st modes are all the same as the reference solution.

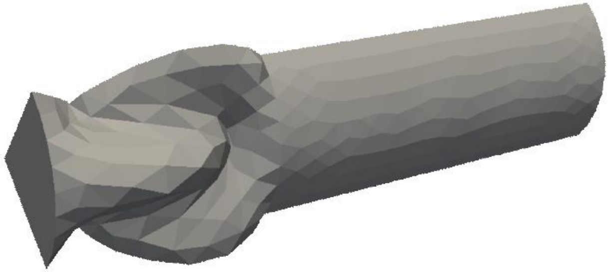
Natural Modes of ¼ Cylinder

11th eigen mode



ABAQUS C3D8
(reference)

Selective ES/NS-FEM-T4



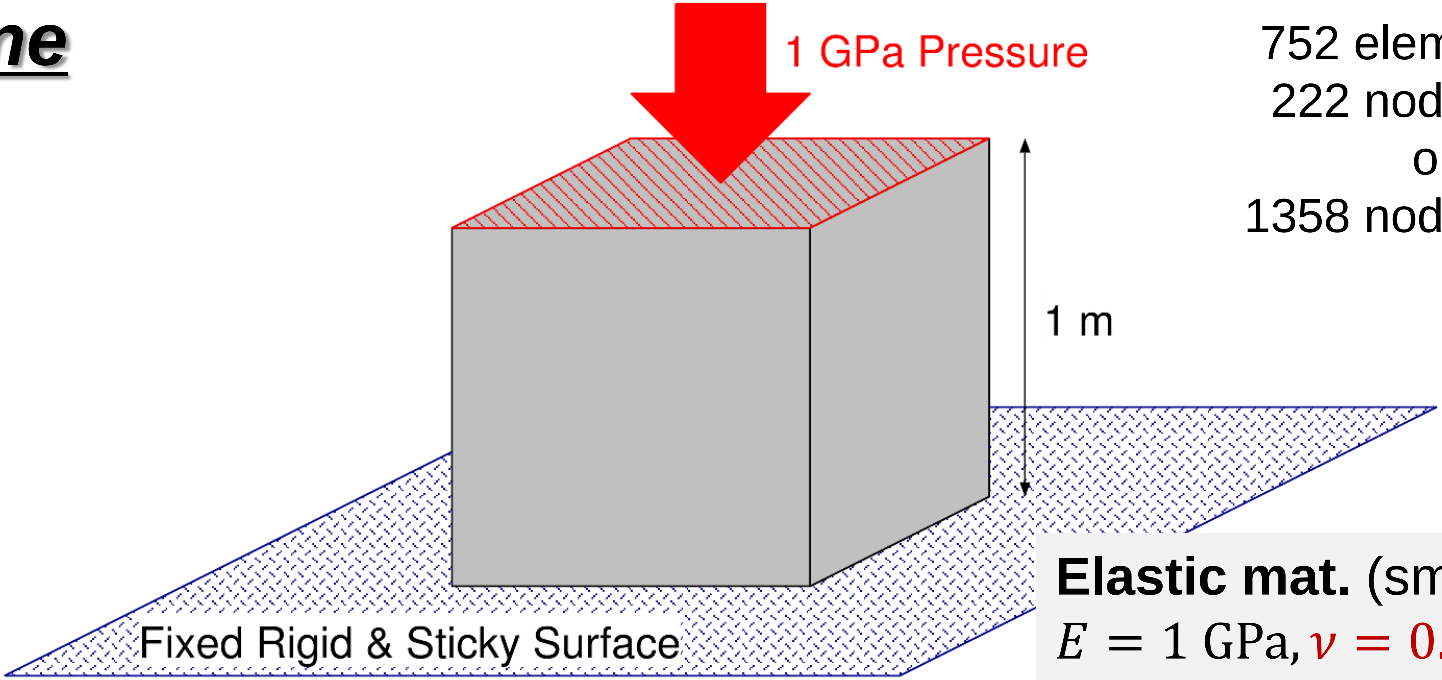
F-barES-FEM-T4

NS-FEM-T4

- NS-FEM-T4 shows strange results due to low-energy mode.
- Selective ES/NS-FEM-T4 & F-barES-FEM-T4 are valid.

Contact Press of Block

Outline



752 elements &
222 nodes (T4)
or
1358 nodes (T10)

Elastic mat. (small def.)
$E = 1\,\mathrm{GPa}, \nu = 0.49$

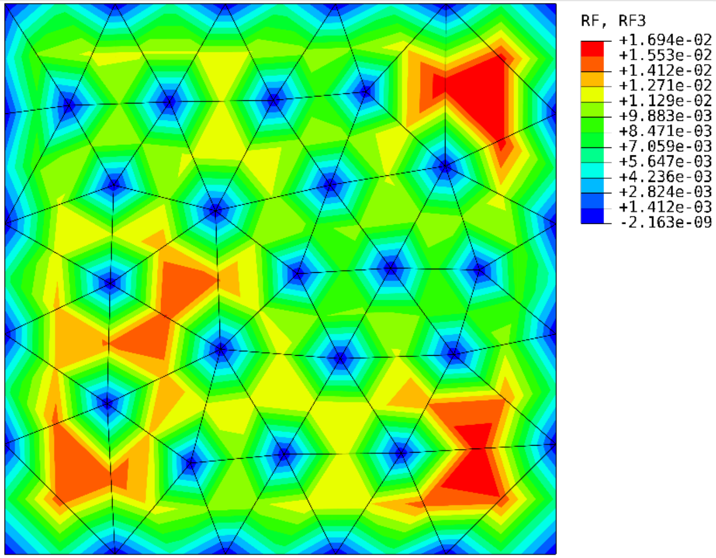
- Sticky contact between a rigid surface & a block.
- Weak pressure on the top of the block.
- Compare the contact force distributions among ABAQUS C3D10H, C3D10HS, C3D10MH and C3D4H with the same Tet mesh.

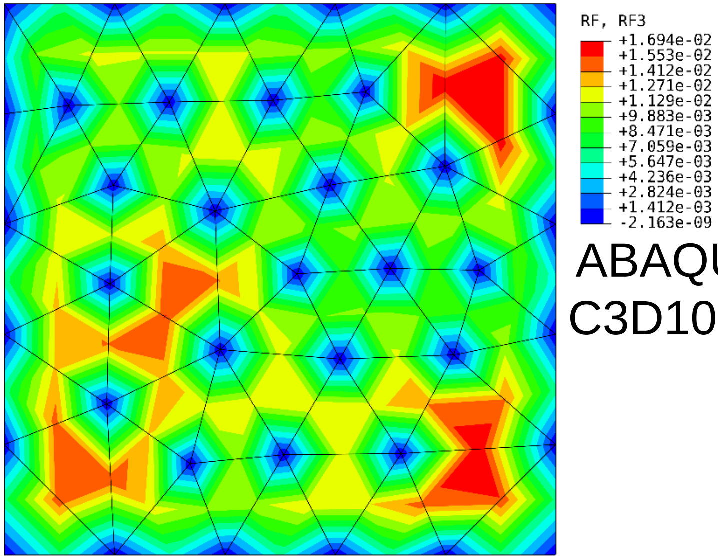
Static
Implicit

Contact Press of Block

Nodal normal contact force dist.

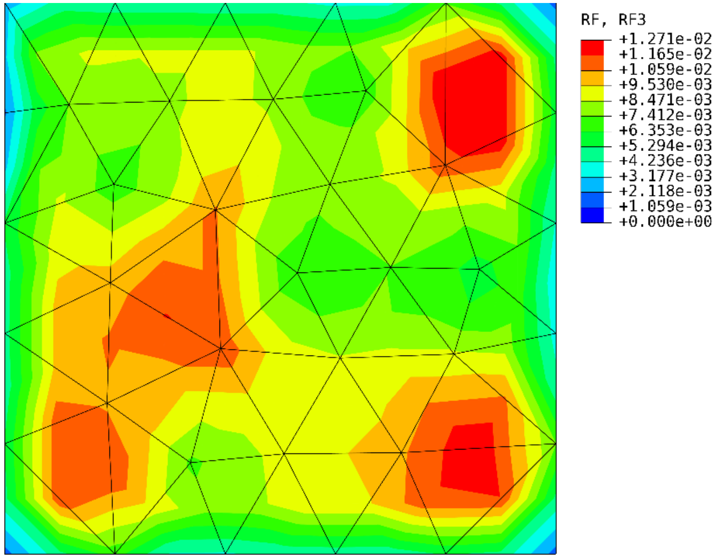
ABAQUS
C3D10H





ABAQUS
C3D10HS

ABAQUS
C3D10MH



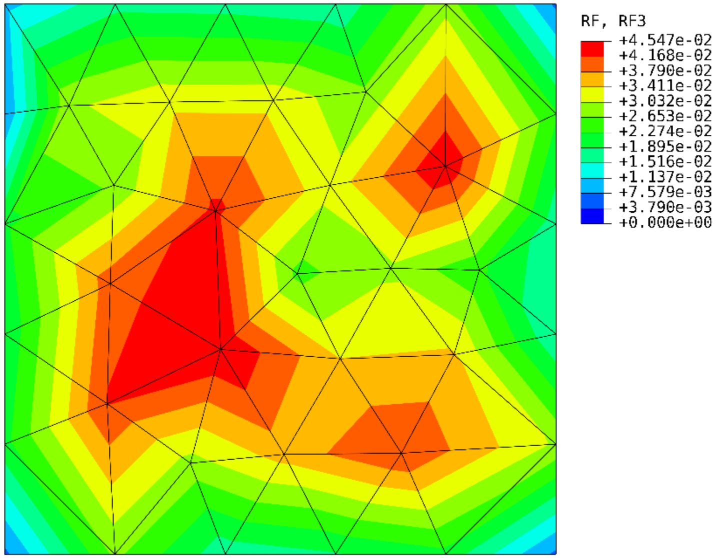
T10 elements present contact force oscillation except for C3D10MH.

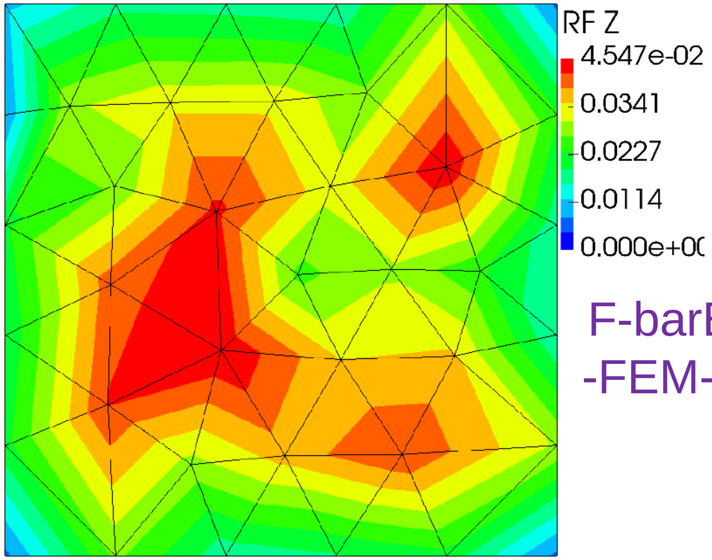
Static
Implicit

Contact Press of Block

Nodal normal contact force dist.

ABAQUS
C3D4H



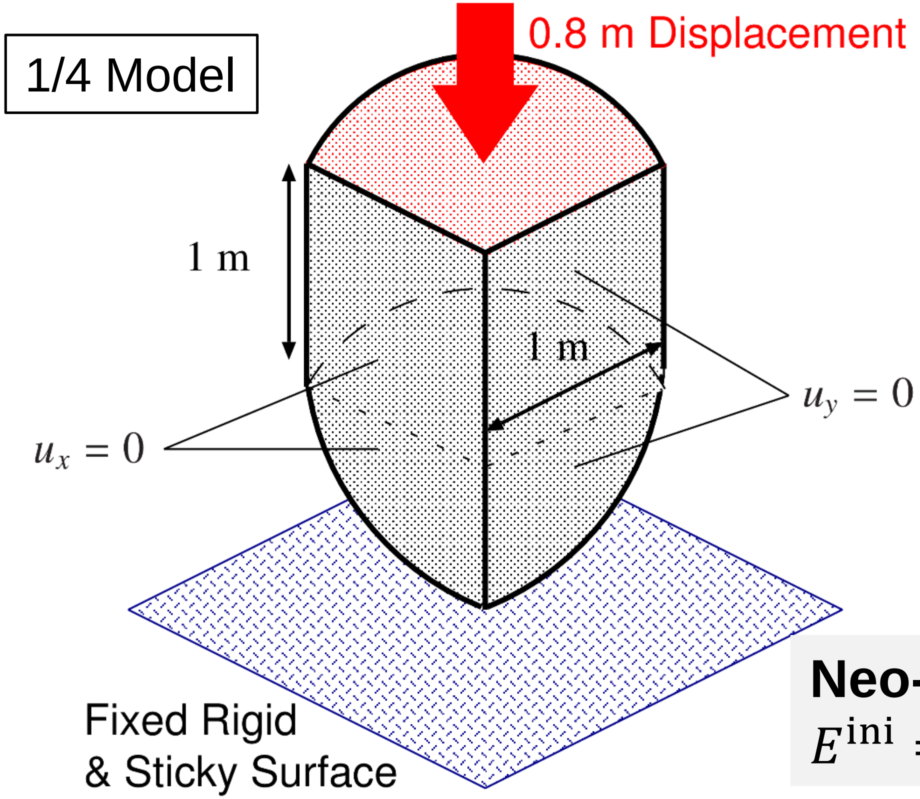


F-barES
-FEM-T4

T4 elements including F-barES-FEM-T4 present no contact force oscillation.

Contact Press of Bullet

Outline



1484 elements &
373 nodes (T4)
or
2446 nodes (T10)

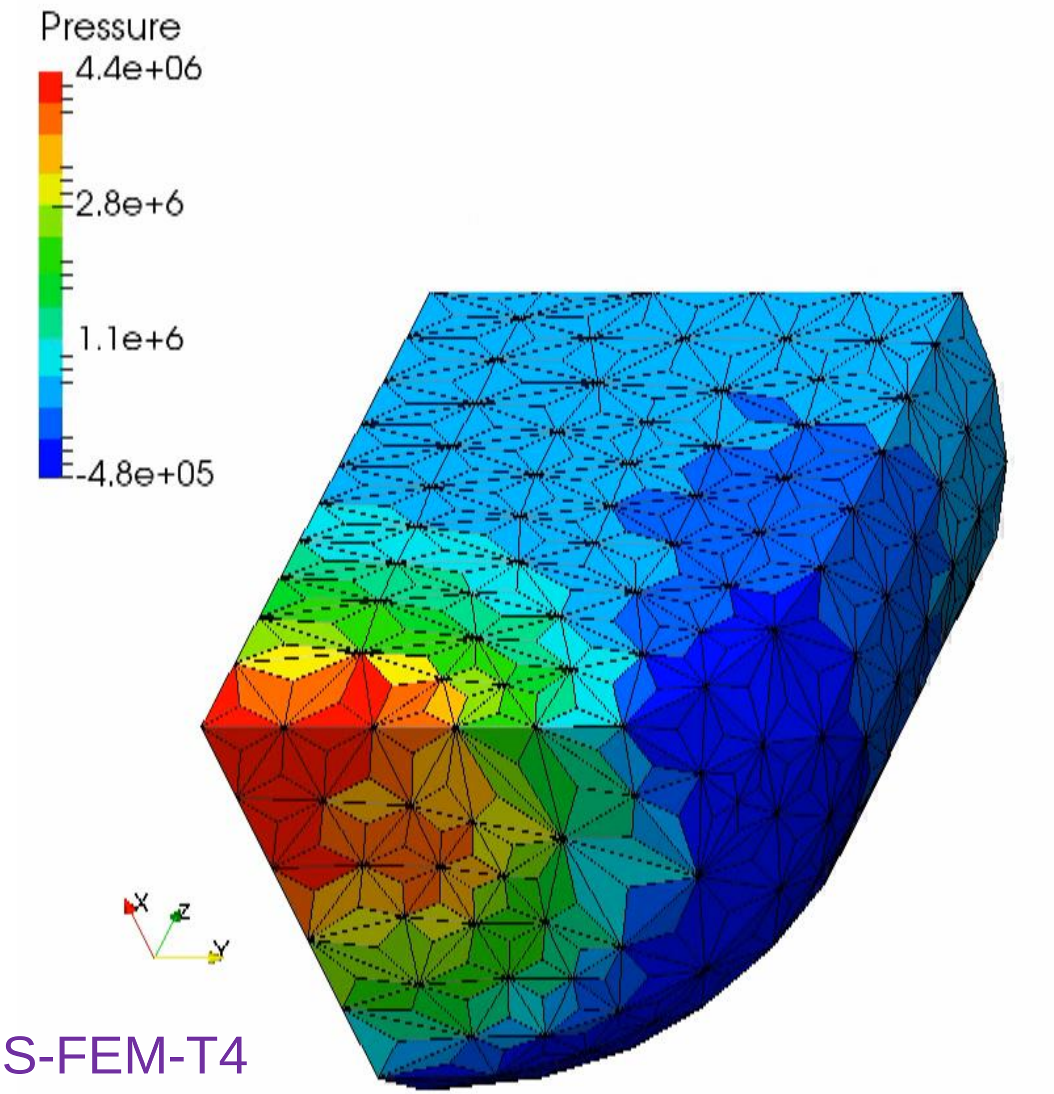
Neo-Hookean hyperelastic:
$E^{\mathrm{ini}} = 6\ \mathrm{MPa}, \nu^{\mathrm{ini}} = 0.49$

- Sticky contact between a rigid surface & a bullet.
- Enforced displacement on the top face of the bullet.
- Compared to ABAQUS C3D10H, C3D10HS, C3D10MH and C3D4H with the same Tet mesh.

Static
Implicit

Contact Press of Bullet

Pressure dist.



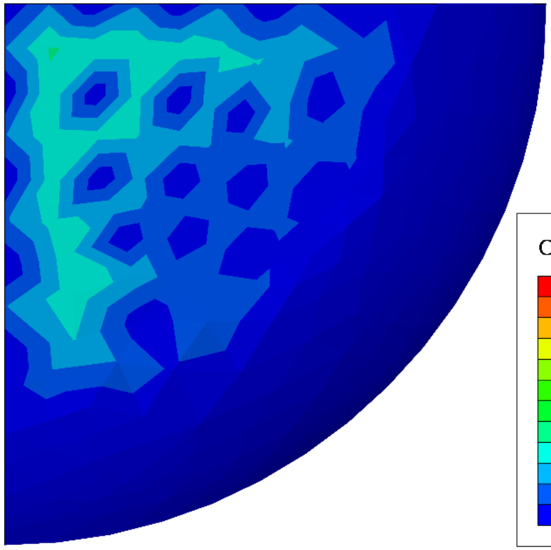
F-barES-FEM-T4

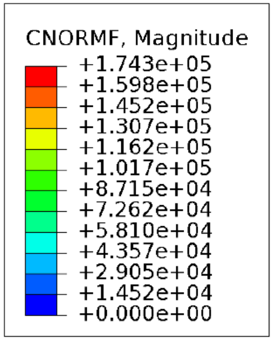
Static
Implicit

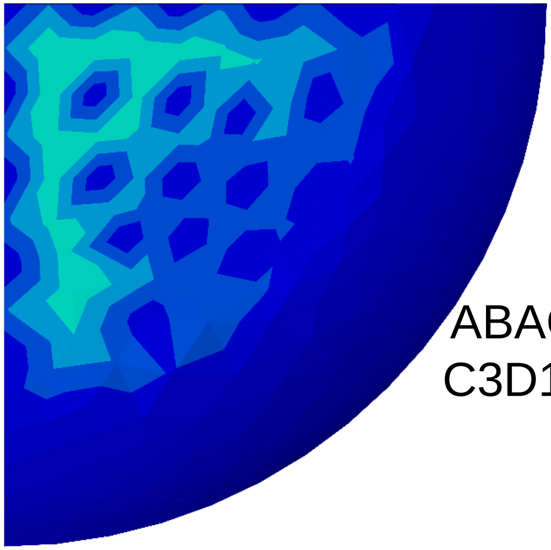
Contact Press of Bullet

Nodal normal contact force dist.

ABAQUS
C3D10H







ABAQUS
C3D10HS

ABAQUS
C3D10MH

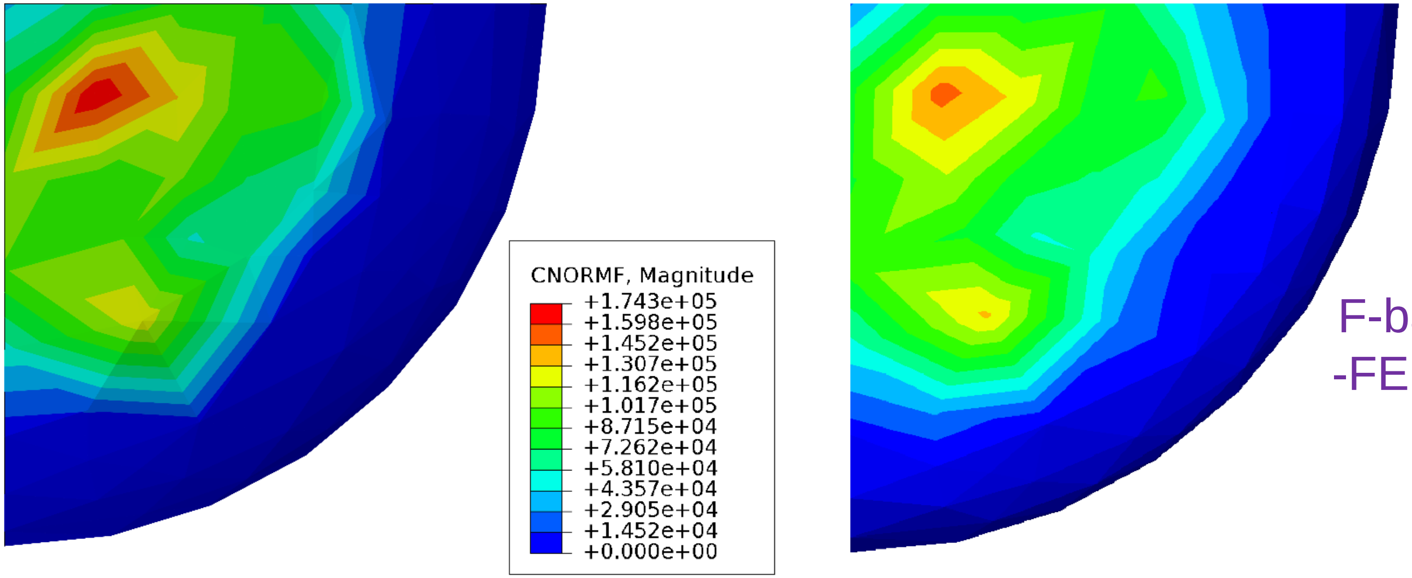
Same as last,
T10 elements
present contact
force oscillation
except for
C3D10MH.

Static Implicit

Contact Press of Bullet

Nodal normal contact force dist.

ABAQUS C3D4H



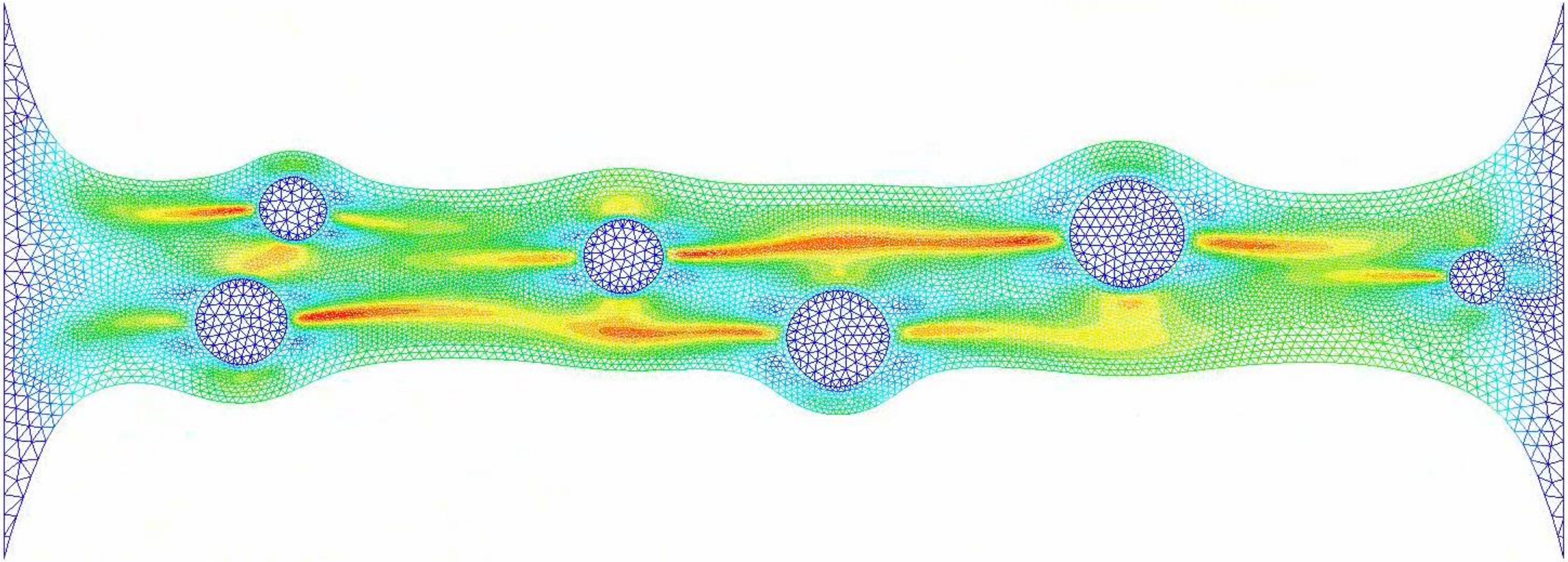
F-barES -FEM-T4

Same as last,
T4 elements including F-barES-FEM-T4
present no contact force oscillation.

Static Implicit

Stretch of Filler-containing Rubber with Remesing

- Several hard circular fillers are distributed in a square soft matrix rubber (neo-Hookean hyperelastic with $\nu_{\mathrm{ini}} = 0.49$).
- E_{ini} of the filler is 100 times larger than E_{ini} of the matrix.
- Left side is constrained and right side is displaced.



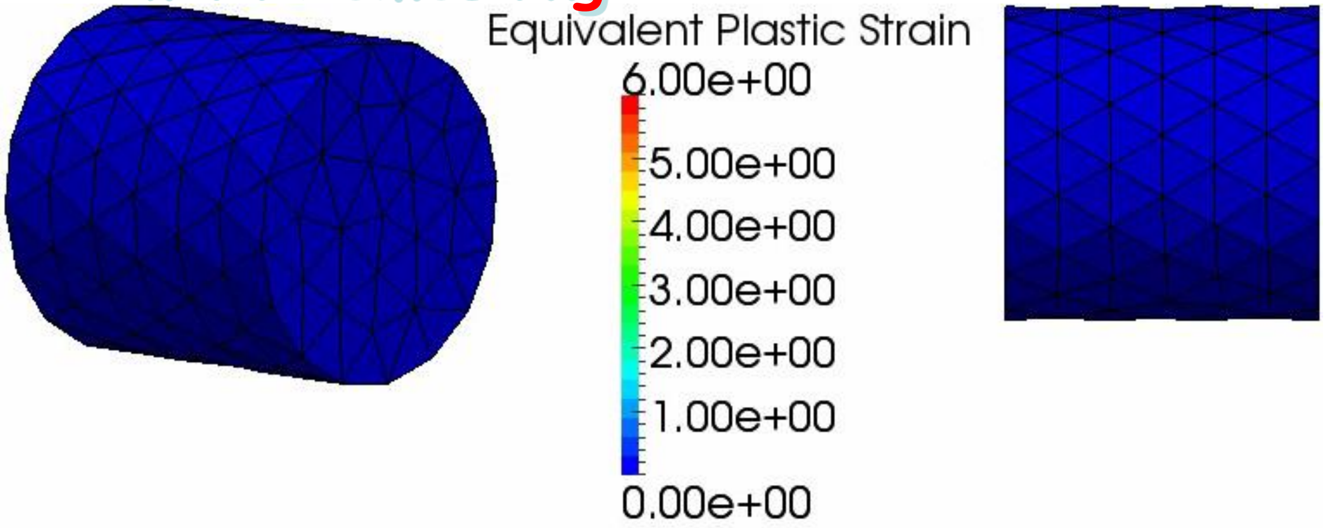
Valid Mises stress dist. is obtained after many time remeshings.

Static
Implicit

Shear-tensioning of Elasto-plastic cylinder with Remeshing

- Aluminium cylinder subjected to enforced disp.
- Pure shear at the initial stage, but stretch dominates at the later stage.
- Necking occurs in the end.

Valid plastic strain dist. is obtained after many time remeshings.



Final stretch at the neck is more than 7000%.

Summary

Benefits and Drawbacks of F-barES-FEM-T4

Benefits

✓ Locking-free with 1st order tetra meshes.

No difficulty in severe strain or contact analysis.

✓ No increase in DOF.

Purely displacement-based formulation.

✓ No restriction of material constitutive model.

Pressure dependent models are acceptable.

✓ Less corner locking & pressure checkerboarding.

More accurate than Selective ES/NS-FEM!

Drawbacks

✗ The more cyclic smoothing necessitates the more CPU time due to the wider bandwidth.

Slower than Selective ES/NS-FEM…

Take-Home Messages

F-barES-FEM-T4 is the current best T4 FE formulation especially for the large deformation of

- Rubber-like materials,
- Viscoelastic materials, and
- Elastoplastic materials.

Thank you for your kind attention!