F-barES-FEM-T4: a new finite element formulation of nearly incompressible solids with tetrahedral elements

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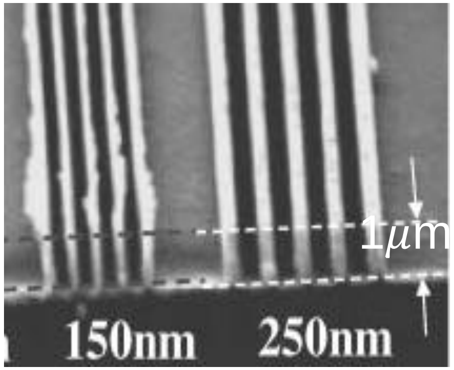
In this presentation,
there are no structural elements and no collapse...
But there are a few dynamic analyses of continuum elements.
I hope they interest you.

Motivation & Background

Motivation

We want to analyze **severe large deformation** of nearly incompressible solids ***accurately and stably***!

(Target: automobile tire, thermal nanoimprint, etc.)

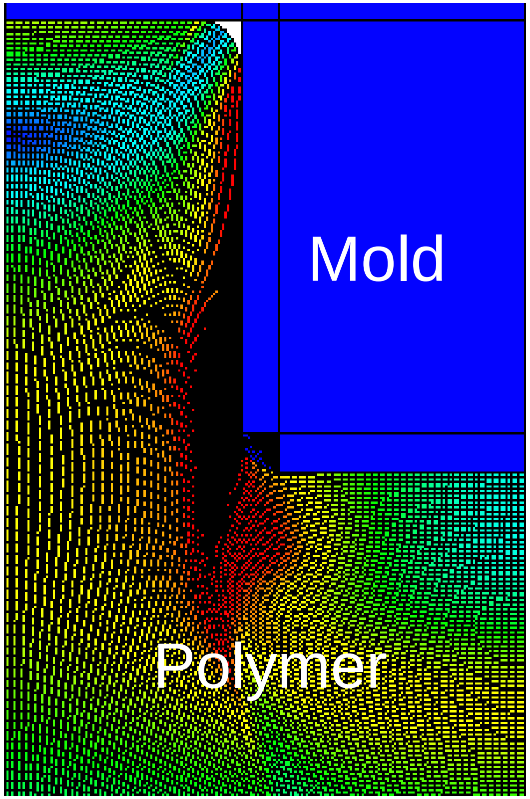


Background

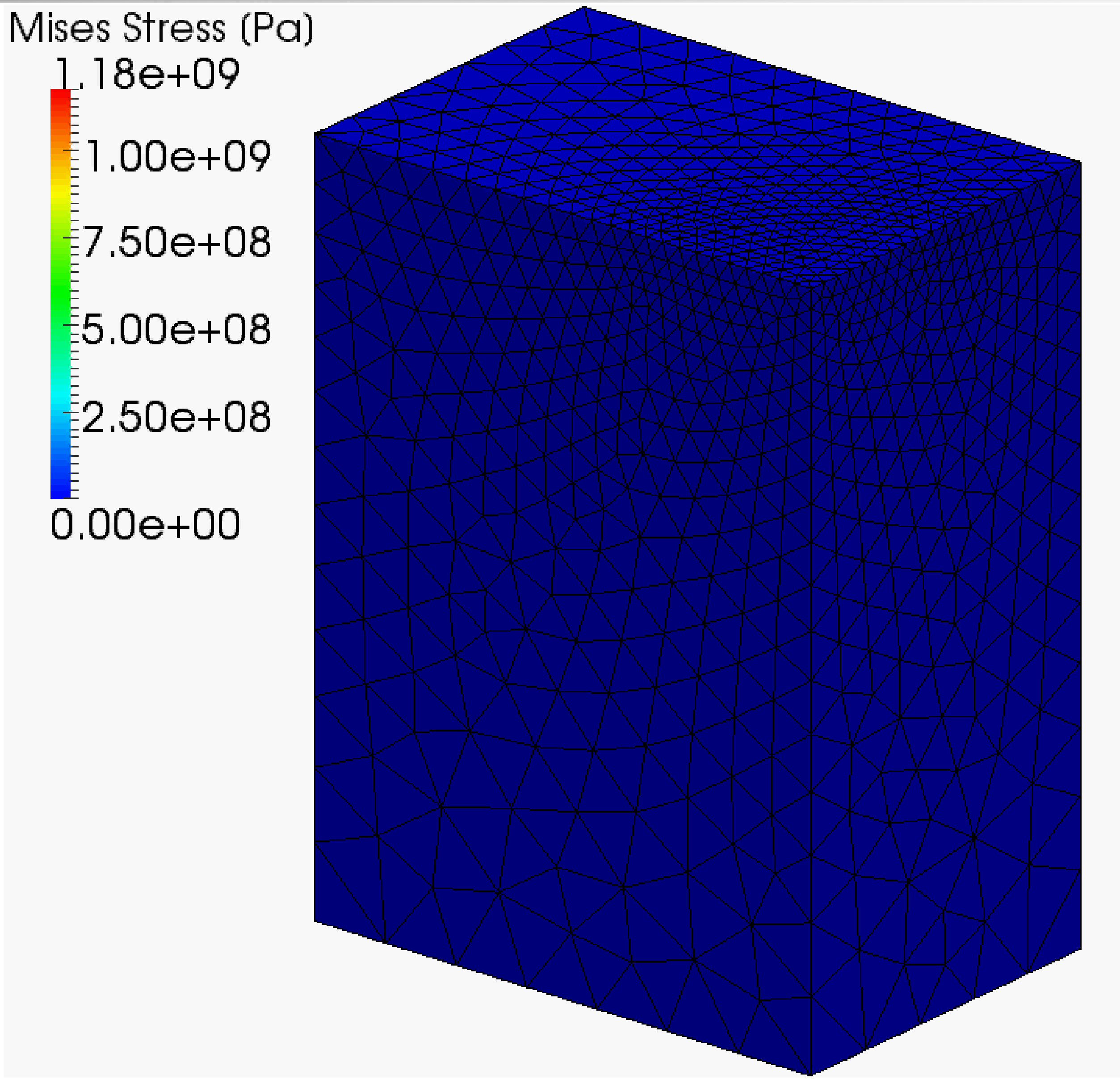
Finite elements are **distorted** in a short time, thereby resulting in convergence failure.

⇩

Mesh rezoning
(aka., remeshing)
is indispensable.



Our First Result in Advance



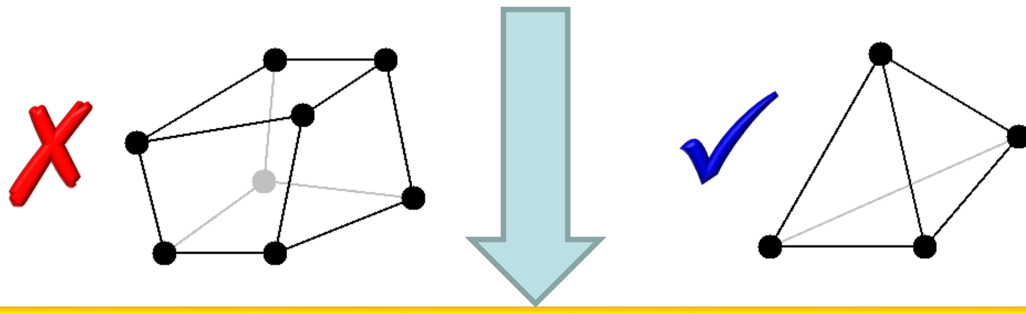
<u>What we want to do</u>:

- Large deformation
- Mesh rezoning
- Various kinds of analyses:
  - Static implicit
  - Dynamic explicit
  - Eigen Mode

Issues

The biggest issue
in large deformation mesh rezoning

It is impossible to remesh arbitrary deformed 3D shapes with hexahedral 8-node (H8) elements.



We have to use tetrahedral 4-node (T4) elements...

However, the *standard* (constant strain) T4 elements easily induce shear and volumetric locking, which leads to inaccurate results.

Conventional Methods

- Higher order elements:
  - ✗ Not volumetric locking free; Unstable in contact analysis; No good in large deformation due to intermediate nodes.
- EAS method:
  - ✗ Unstable due to spurious zero-energy modes.
- B-bar, F-bar and selective integration method:
  - ✗ Not applicable to T4 mesh directly.
- F-bar patch method:
  - ✗ Difficult to construct good patches. Not shear locking free.
- u/p hybrid elements based on the mixed variational principle:
  - ✗ No sufficient formulation for T4 mesh so far. (There are almost acceptable hybrid elements such as C3D4H of ABAQUS.)
- Smoothed finite element method (S-FEM):
  - ? Unknown potential (since 2007~). ***It's worth trying!!***

Various Types of S-FEMs

■ Basic type

- Node-based S-FEM (NS-FEM) — ✗ Spurious zero-energy
- Face-based S-FEM (FS-FEM)
- Edge-based S-FEM (ES-FEM)

(FS-FEM, ES-FEM) — ✗ Volumetric Locking

■ Selective type

- Selective FS/NS-FEM
- Selective ES/NS-FEM

✗ Restriction of constitutive model, Pressure oscillation, Corner locking (detailed later)

■ Bubble-enhanced or Hat-enhanced type

- bFS-FEM, hFS-FEM
- bES-FEM, hES-FEM

✗ Pressure oscillation, Short-lasting

■ F-bar type

NEW
- F-barES-FEM — ? Unknown potential

Objective

Develop a new S-FEM, F-barES-FEM-T4, by combining F-bar method and ES-FEM-T4 for large deformation problems of rubber-like materials

Table of Body Contents

- Methods: Formulation of F-barES-FEM-T4
- Results: Some verification analyses
- Summary

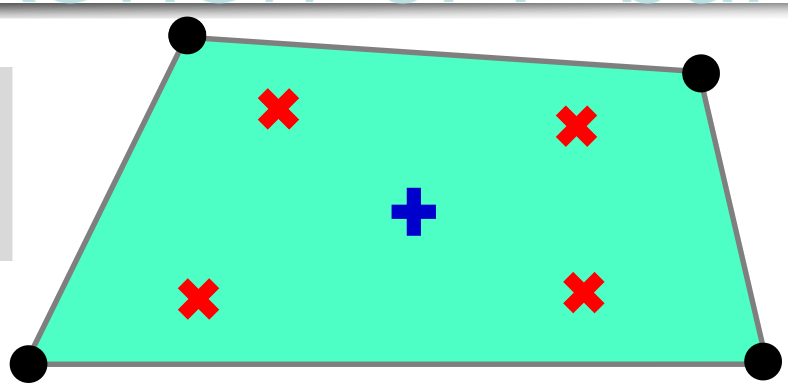
Methods

Formulations of F-barES-FEM-T4

(F-barES-FEM-T3 in 2D is explained for simplicity.)

Quick Review of F-bar Method

For quadrilateral (Q4) or hexahedral (H8) elements



Algorithm

1. Calculate deformation gradient $\boldsymbol{F}$ at the element center, and then make the relative volume change $\bar{J}$ $(= \det(\boldsymbol{F}))$.
2. Calculate deformation gradient $\boldsymbol{F}$ at each gauss point as usual, and then make $\boldsymbol{F}^{\mathrm{iso}}$ $(= \boldsymbol{F} \,/\, J^{1/3})$.
3. Modify $\boldsymbol{F}$ at each gauss point to obtain $\bar{\boldsymbol{F}}$ as

$$\bar{\boldsymbol{F}} = \bar{J}^{1/3}\, \boldsymbol{F}^{\mathrm{iso}}.$$

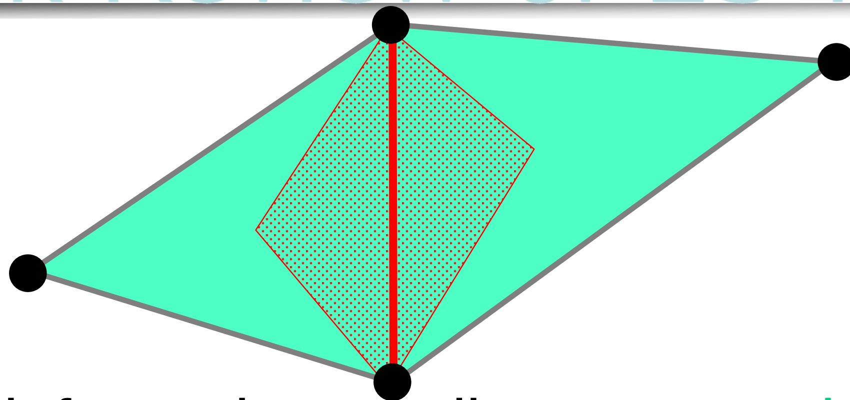
A kind of low-pass filter for J

4. Use $\bar{\boldsymbol{F}}$ to calculate the stress, nodal force and so on.

F-bar method is used to **avoid volumetric locking** in Q4 or H8 elements. Yet, it **cannot avoid shear locking**.

Quick Review of ES-FEM

For triangular (T3) or tetrahedral (T4) elements.



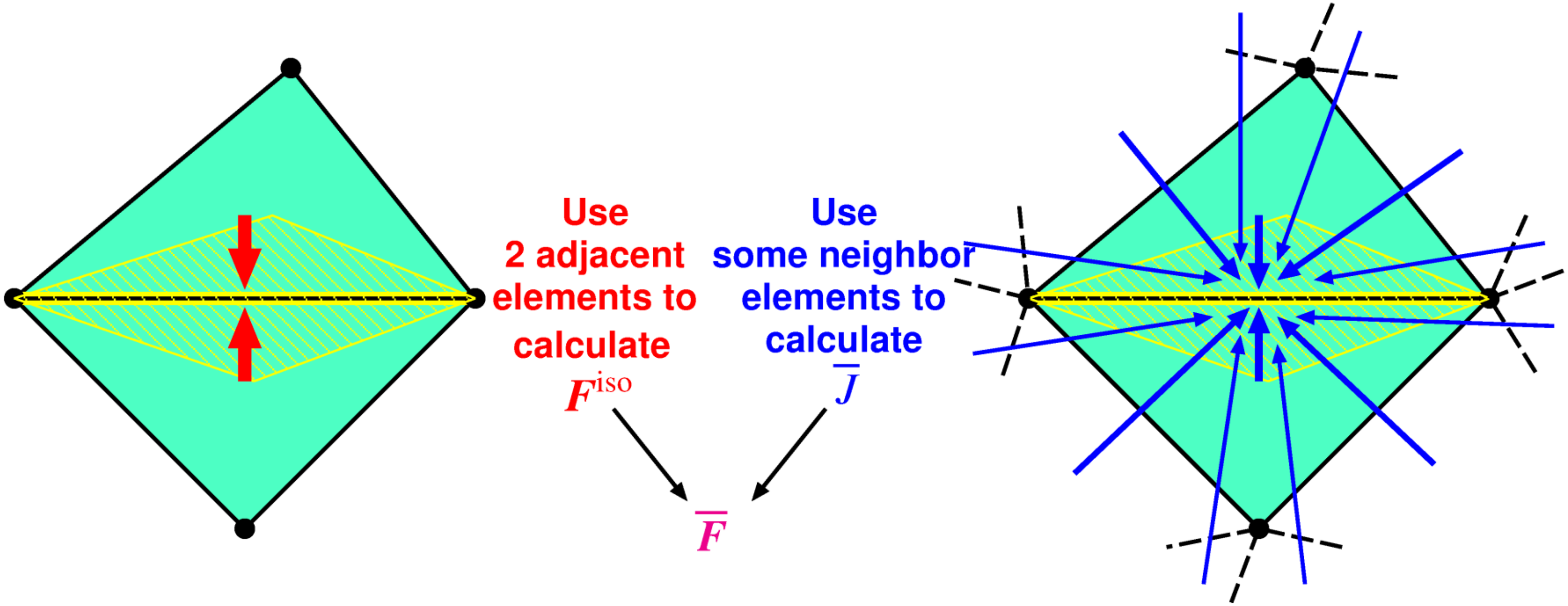
Algorithm:

1. Calculate the deformation gradient $\boldsymbol{F}$ at each element as usual.
2. Distribute the deformation gradient $\boldsymbol{F}$ to the connecting edges with area weights to make ${}^{\mathrm{Edge}}\boldsymbol{F}$ at each edge.
3. Use ${}^{\mathrm{Edge}}\boldsymbol{F}$ to calculate the stress, nodal force and so on.

ES-FEM is used to **avoid shear locking** in T3 or T4 elements. Yet, it **cannot avoid volumetric locking**.

Outline of F-barES-FEM

Concept Combination of F-bar method and ES-FEM



- ${}^{\mathrm{Edge}}\boldsymbol{F}^{\mathrm{iso}}$ is given by ES-FEM.
- ${}^{\mathrm{Edge}}\bar{J}$ is given by Cyclic Smoothing (detailed later).
- ${}^{\mathrm{Edge}}\bar{\boldsymbol{F}}$ is calculated in the manner of F-bar method:

$$ {}^{\mathrm{Edge}}\bar{\boldsymbol{F}} = {}^{\mathrm{Edge}}\bar{J}^{1/3}\ {}^{\mathrm{Edge}}\boldsymbol{F}^{\mathrm{iso}}. $$

Outline of F-barES-FEM

Brief Formulation

A kind of low-pass filter for J

1. Calculate ${}^{\mathrm{Elem}}J$ as usual.
2. Smooth ${}^{\mathrm{Elem}}J$ at nodes and get ${}^{\mathrm{Node}}\tilde{J}$.
3. Smooth ${}^{\mathrm{Node}}\tilde{J}$ at elements and get ${}^{\mathrm{Elem}}\tilde{J}$.
4. Repeat 2. and 3. as necessary (c times).

Cyclic Smoothing of J

⋮ (c layers of ~)

5. Smooth ${}^{\mathrm{Elem}}\tilde{\tilde{J}}$ at edges to make ${}^{\mathrm{Edge}}\bar{J}$.
6. Combine ${}^{\mathrm{Edge}}\bar{J}$ and ${}^{\mathrm{Edge}}\boldsymbol{F}^{\mathrm{iso}}$ of ES-FEM as

$$ {}^{\mathrm{Edge}}\bar{\boldsymbol{F}} = {}^{\mathrm{Edge}}\bar{J}^{1/3}\ {}^{\mathrm{Edge}}\boldsymbol{F}^{\mathrm{iso}}. $$

Hereafter, F-barES-FEM-T4 with c cycles of smoothing is called “F-barES-FEM-T4(c)”.

Equations to Satisfy

Implicit static

- Balance equation: $\{f^{\text{ext}}\} - \{f^{\text{int}}\} = \{0\}$.
- In Newton-Raphson loop, there is a need for solving the matrix equation:

$$[K]\{\delta u\} = \{f^{\text{ext}}\} - \{f^{\text{int}}\}.$$

Dynamic explicit

- Motion equation: $[M]\{\ddot{u}\} = \{f^{\text{ext}}\} - \{f^{\text{int}}\}$.
- Using the lumped mass matrix for $[M]$, there is no need for solving any matrix equations.
- Velocity Verlet method is applied for time integration.

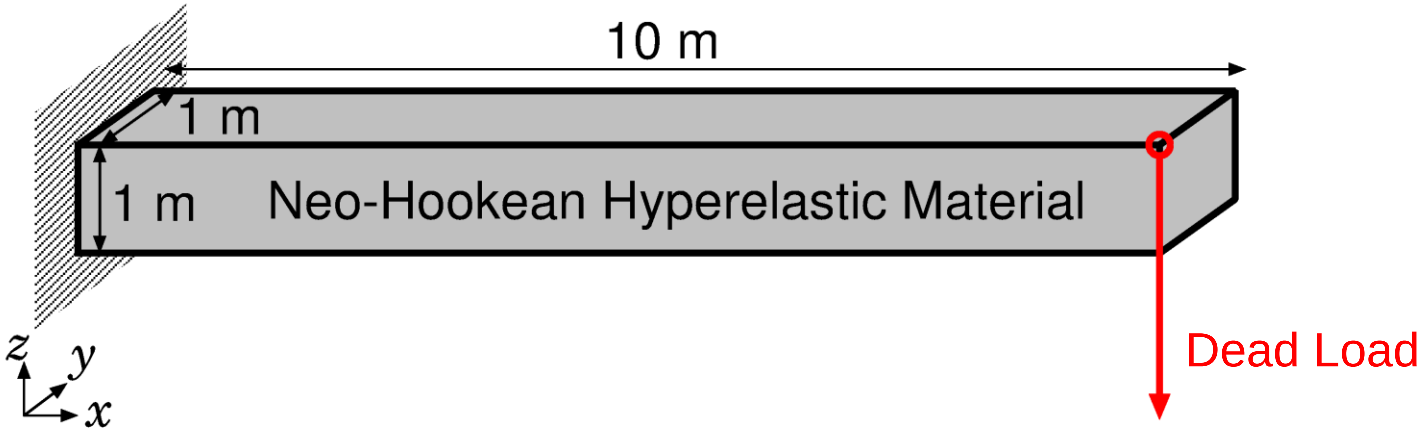
Results

Some verification analyses

(Analyses for hyperelastic materials
without mesh rezoning
are presented for pure verification.)

Bending of a Cantilever

Outline



- Neo-Hookean hyperelastic material
- Initial Poisson's ratio: 0.49 or 0.499.
- Two types of T4 meshes:
  a structured mesh and an unstructured mesh.
- Compared to ABAQUS C3D4H (1st-order hybrid tetrahedral element).

Bending of a Cantilever

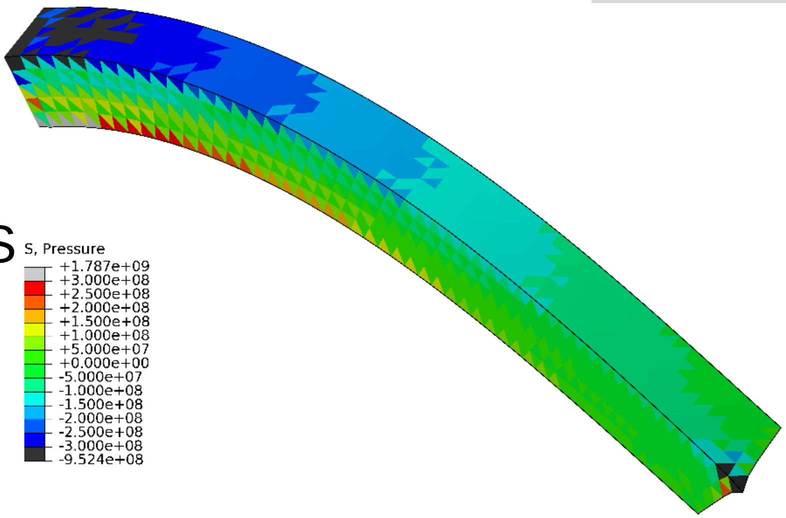
Pressure Distributions

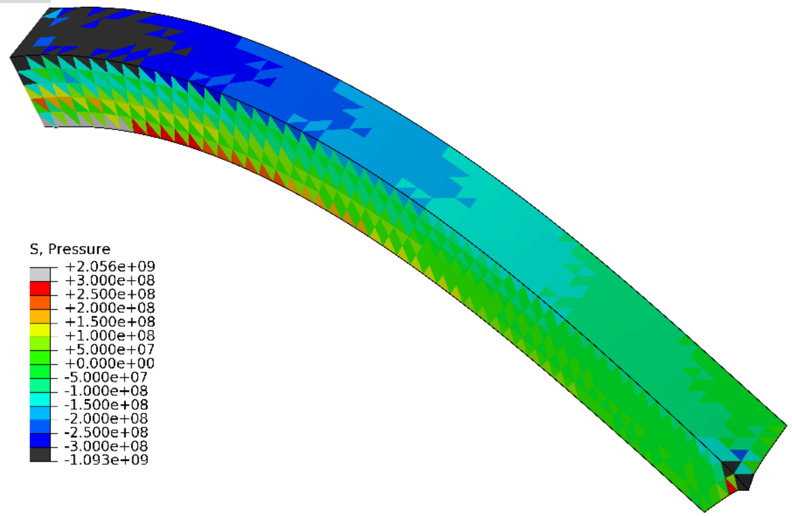
$\nu^{\mathrm{ini}} = 0.49$

Structured Mesh

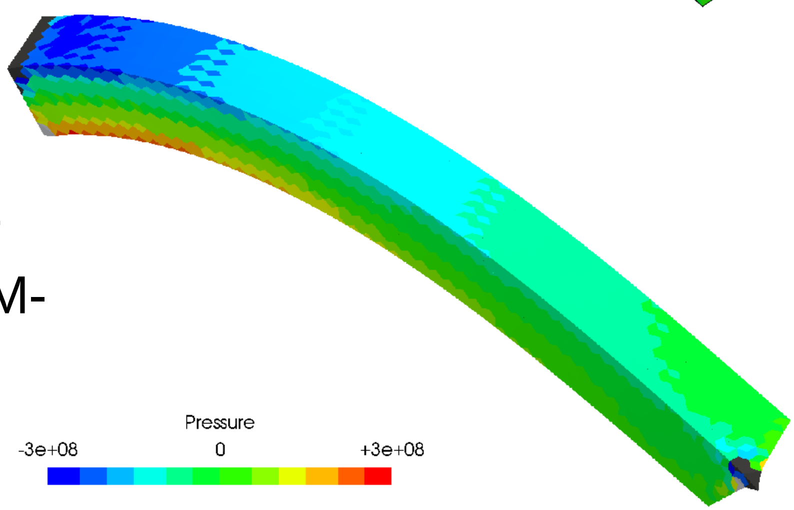
$\nu^{\mathrm{ini}} = 0.499$

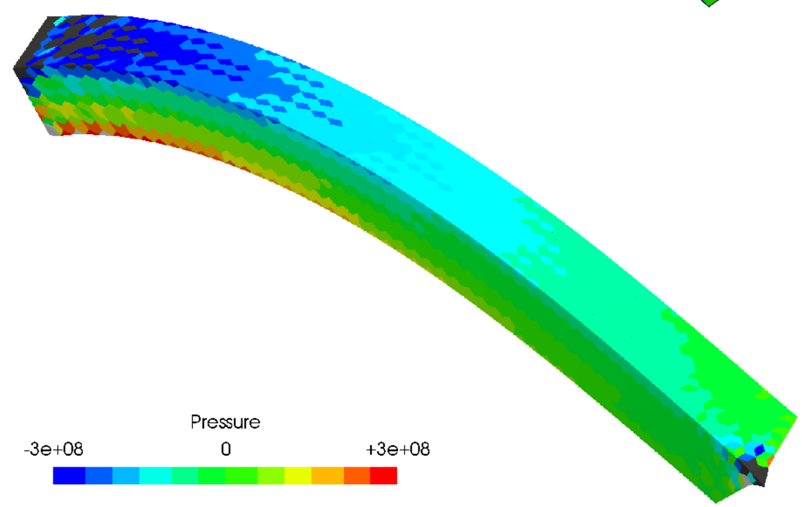
ABAQUS C3D4H





F-bar ES-FEM-T4(1)





Static Implicit

Bending of a Cantilever

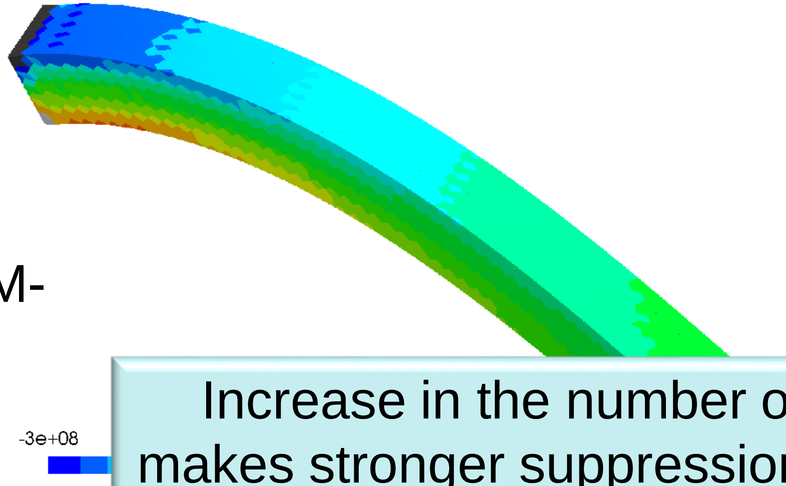
Pressure Distributions

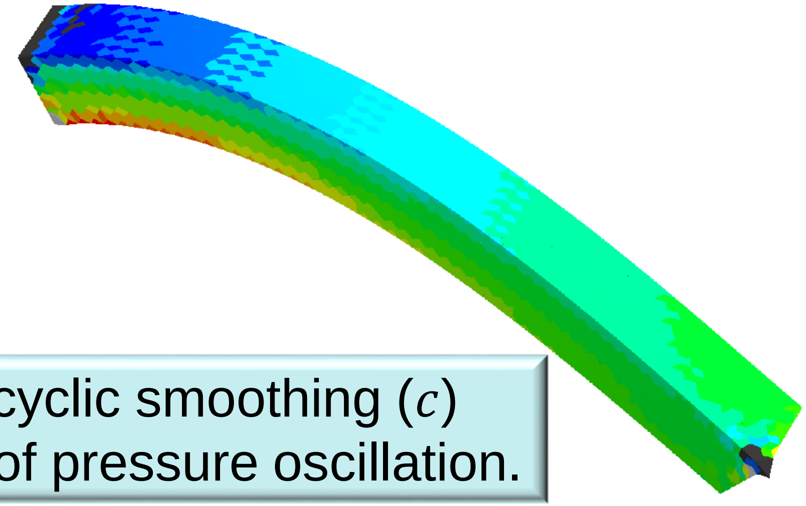
$\nu^{\mathrm{ini}} = 0.49$

Structured Mesh

$\nu^{\mathrm{ini}} = 0.499$

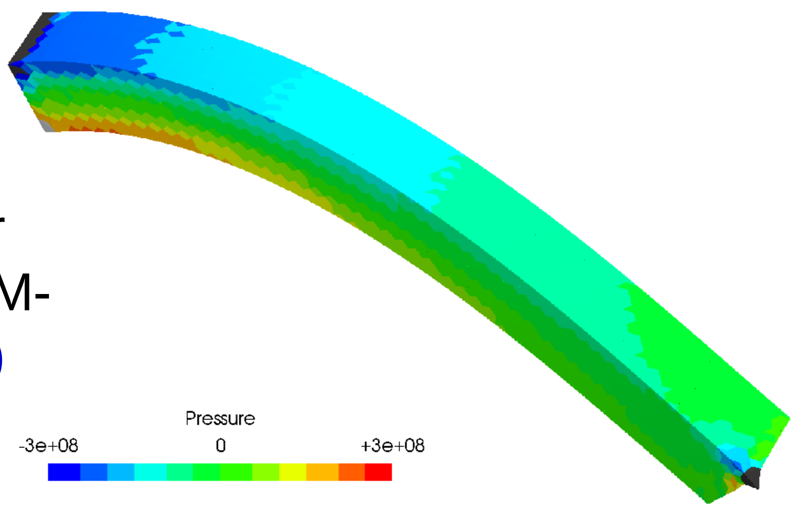
F-bar ES-FEM-T4(2)

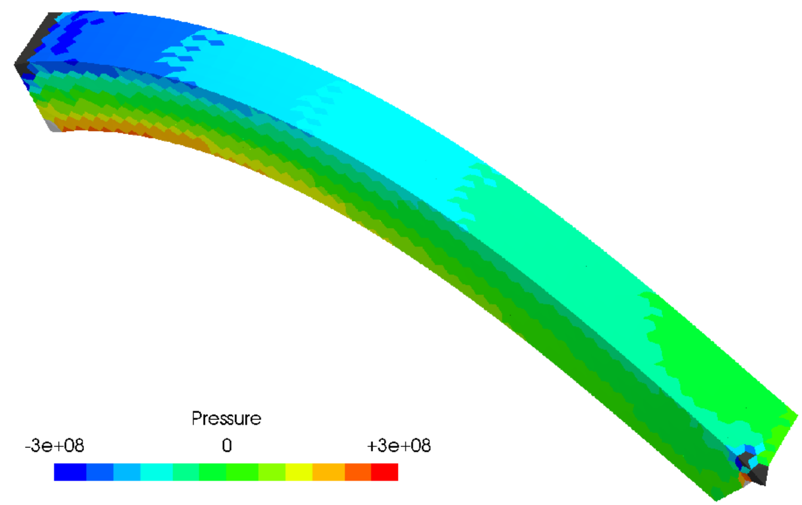




Increase in the number of cyclic smoothing (c) makes stronger suppression of pressure oscillation.

F-bar ES-FEM-T4(3)





Static Implicit

Bending of a Cantilever

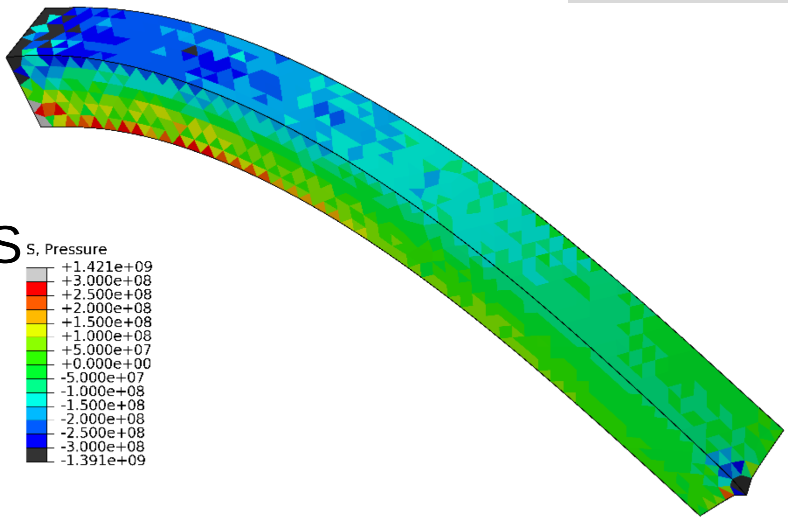
Pressure Distributions

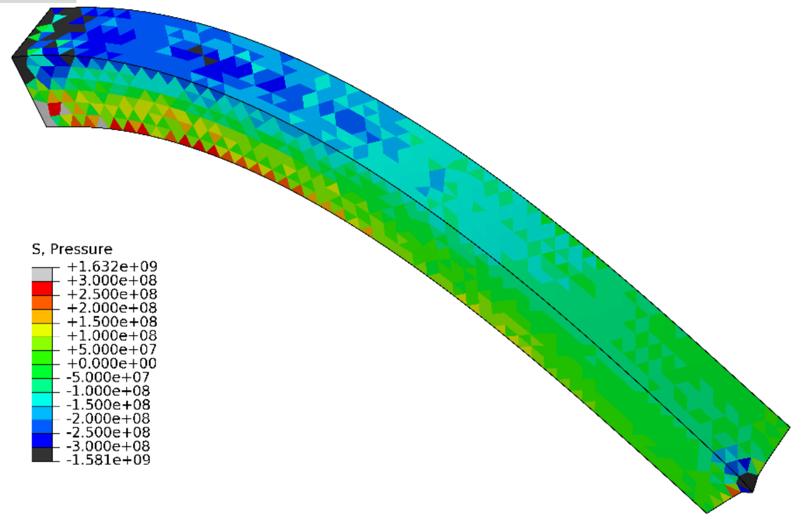
$\nu^{\text{ini}} = 0.49$

Unstructured Mesh

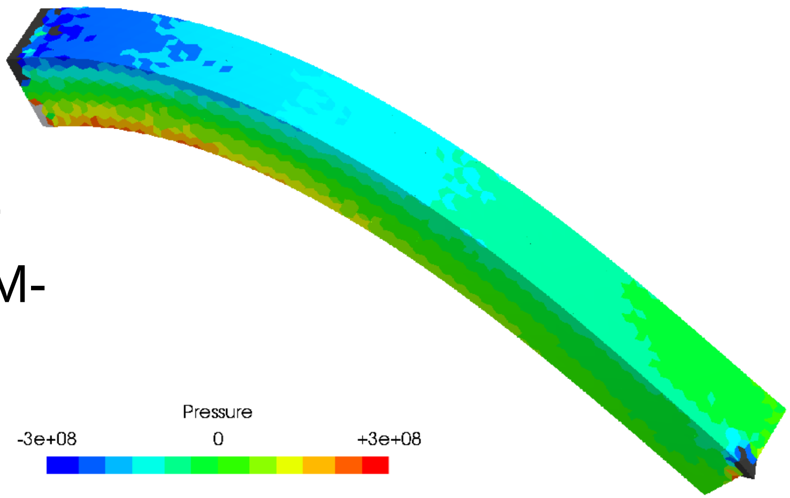
$\nu^{\text{ini}} = 0.499$

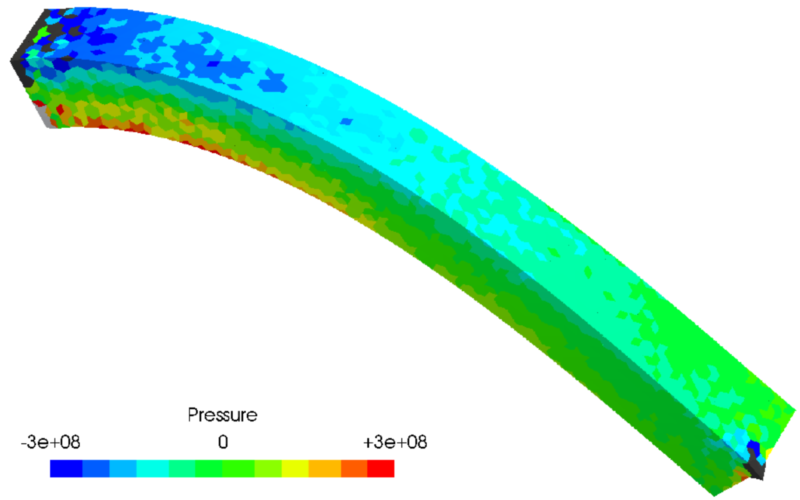
ABAQUS C3D4H





F-bar ES-FEM-T4(1)





Static Implicit

Bending of a Cantilever

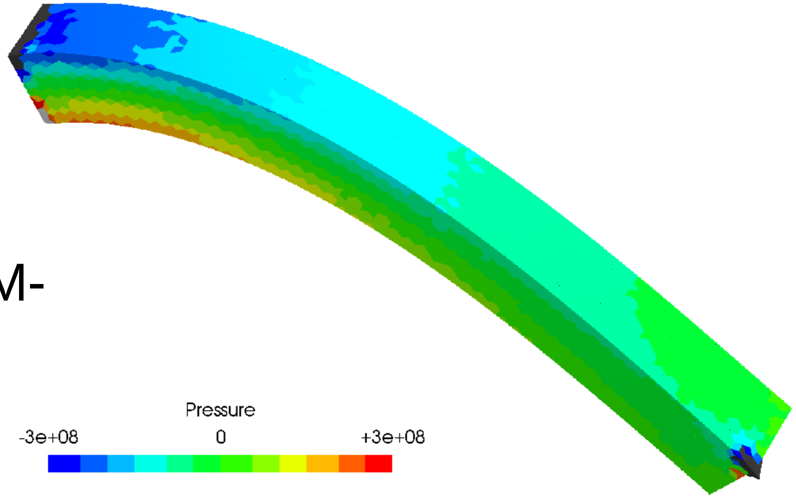
Pressure Distributions

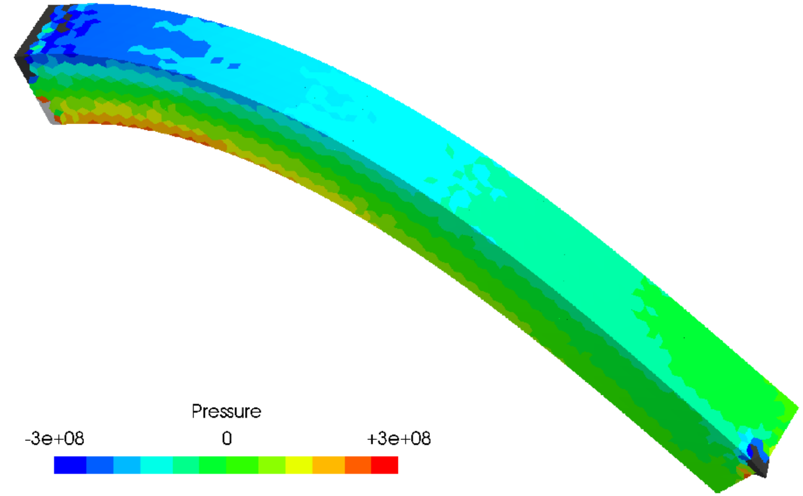
$\nu^{\text{ini}} = 0.49$

Unstructured Mesh

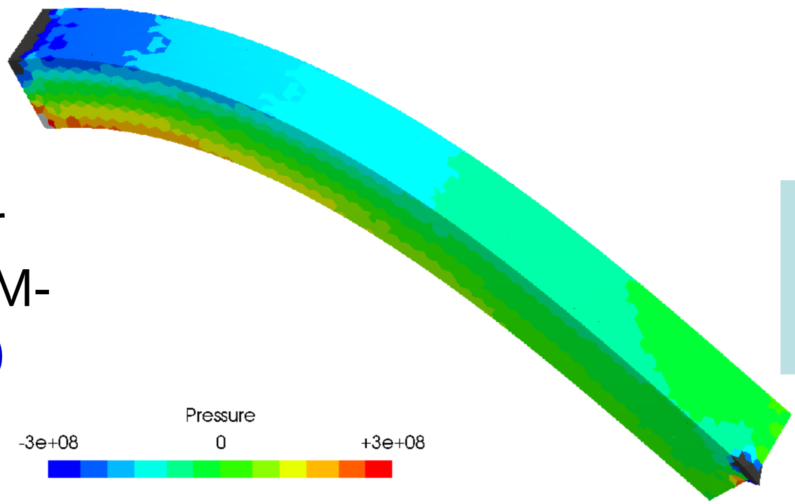
$\nu^{\text{ini}} = 0.499$

F-bar ES-FEM-T4(2)

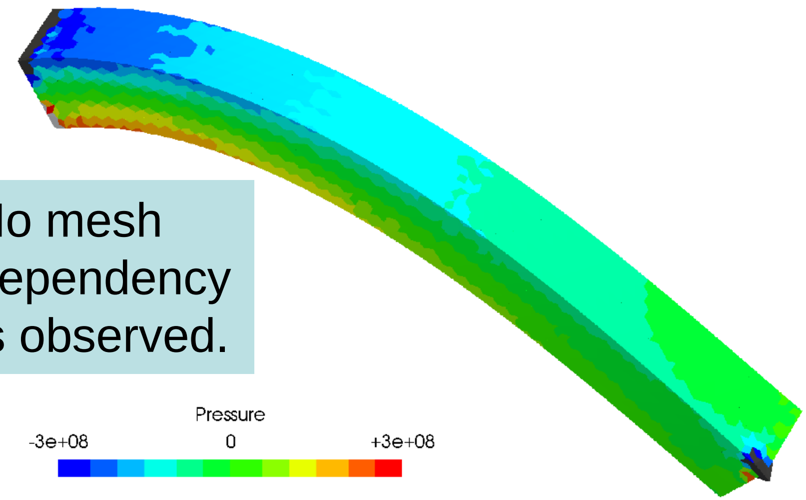




F-bar ES-FEM-T4(3)

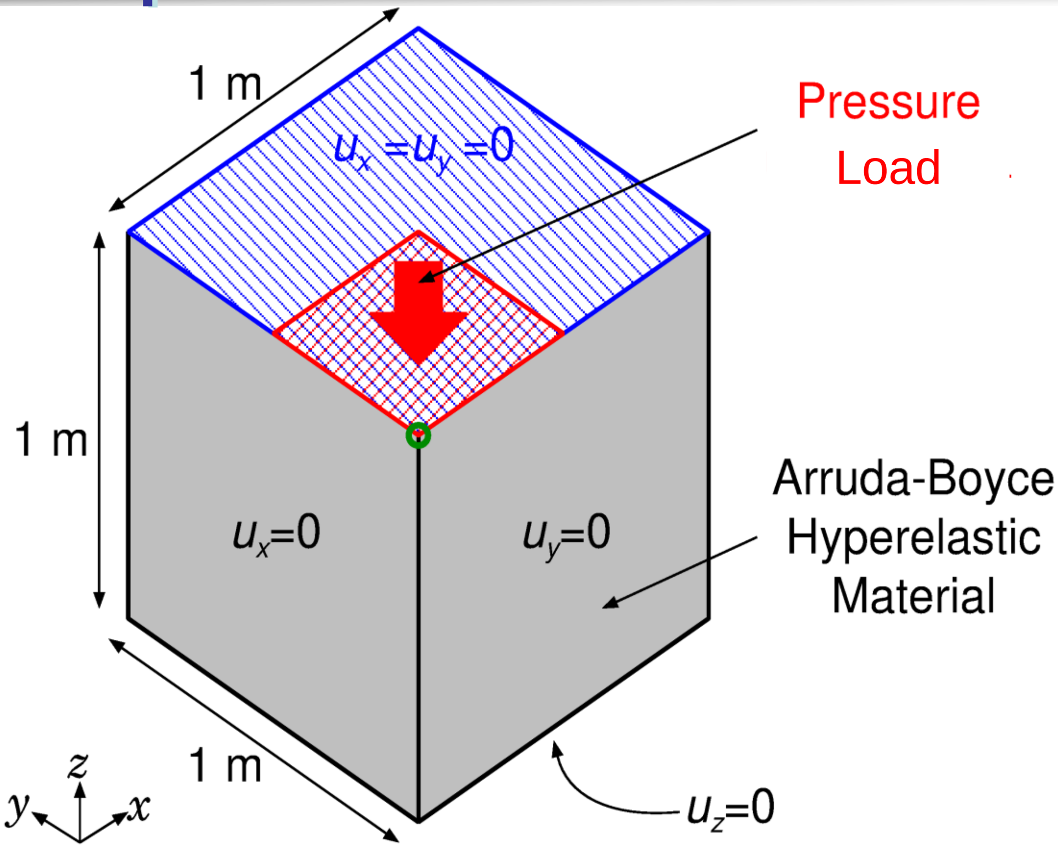


No mesh dependency is observed.



Compression of a Block

Outline



- Arruda-Boyce hyperelastic material ($\nu_{\text{ini}} = \mathbf{0.499}$).
- Applying pressure on ¼ of the top face.
- Compared to ABAQUS C3D4H with the same unstructured T4 mesh.

Static Implicit

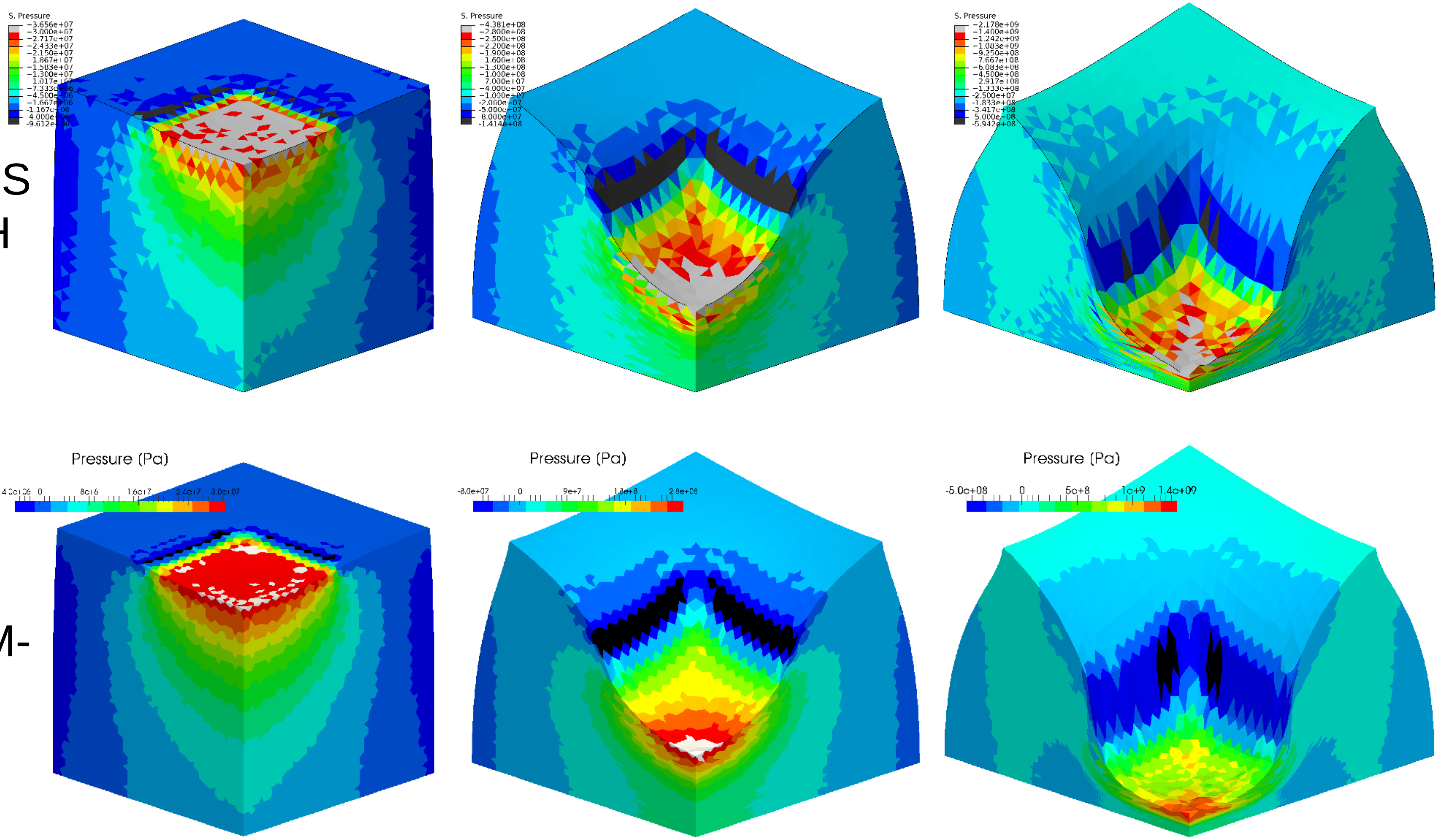
Compression of a Block

Pressure Distribution

Early stage | Middle stage | Later stage

ABAQUS C3D4H

F-bar ES-FEM-T4(2)



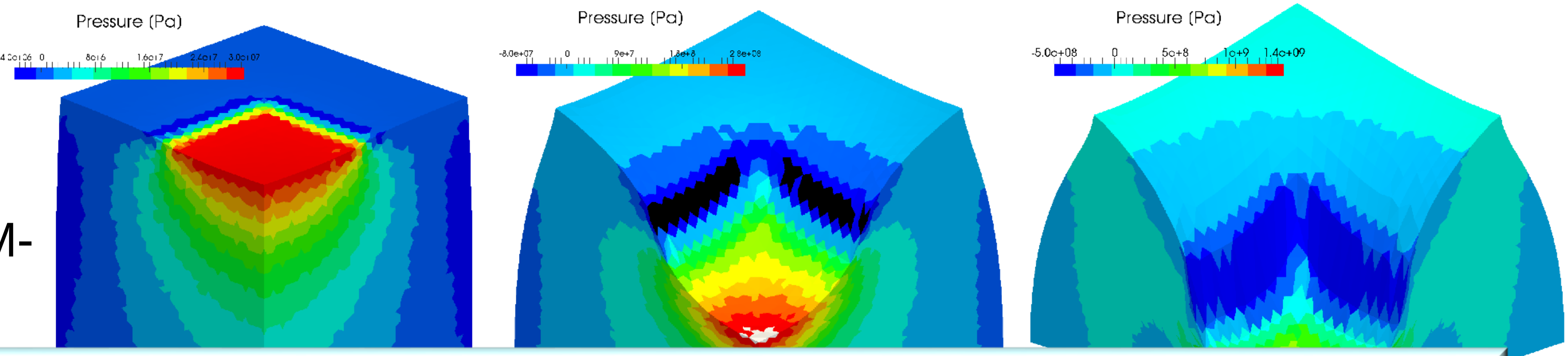
Static Implicit

Compression of a Block

Pressure Distribution

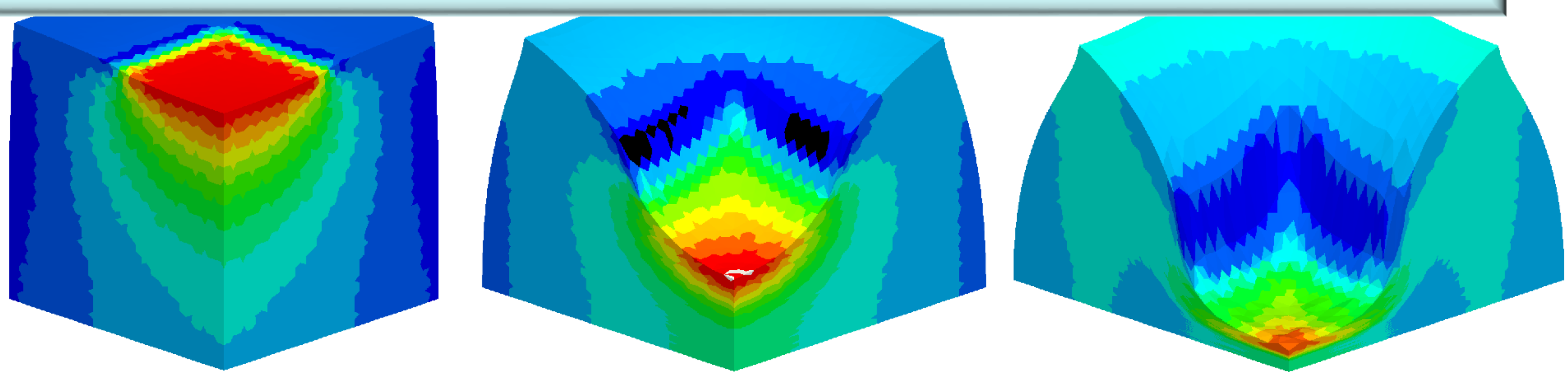
Early stage | Middle stage | Later stage

F-bar ES-FEM-T4(3)



In case the Poisson's ratio is 0.499,
F-barES-FEM-T4(2) or later resolves the pressure oscillation issue.

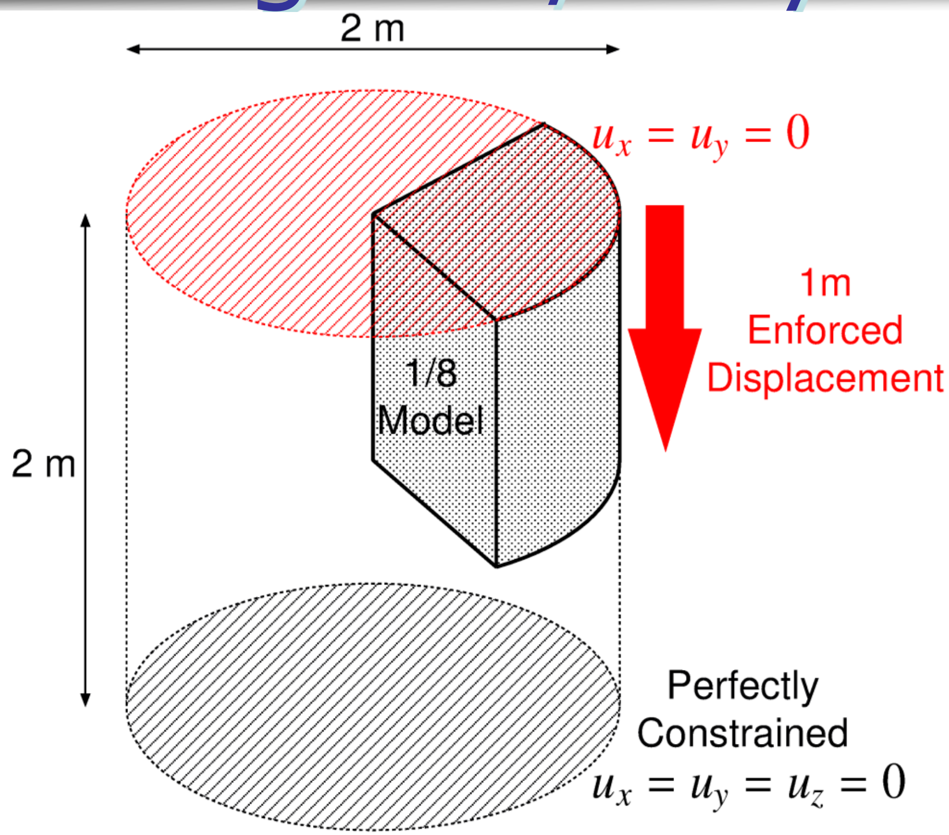
F-bar ES-FEM-T4(4)



Static Implicit

Barreling of 1/8 Cylinder

Outline



- Neo-Hookean hyperelastic material ($\nu_{\mathrm{ini}} = \mathbf{0.499}$).
- Enforced displacement is applied to the top surface.
- Compared to ABAQUS C3D4H with the same unstructured T4 mesh.

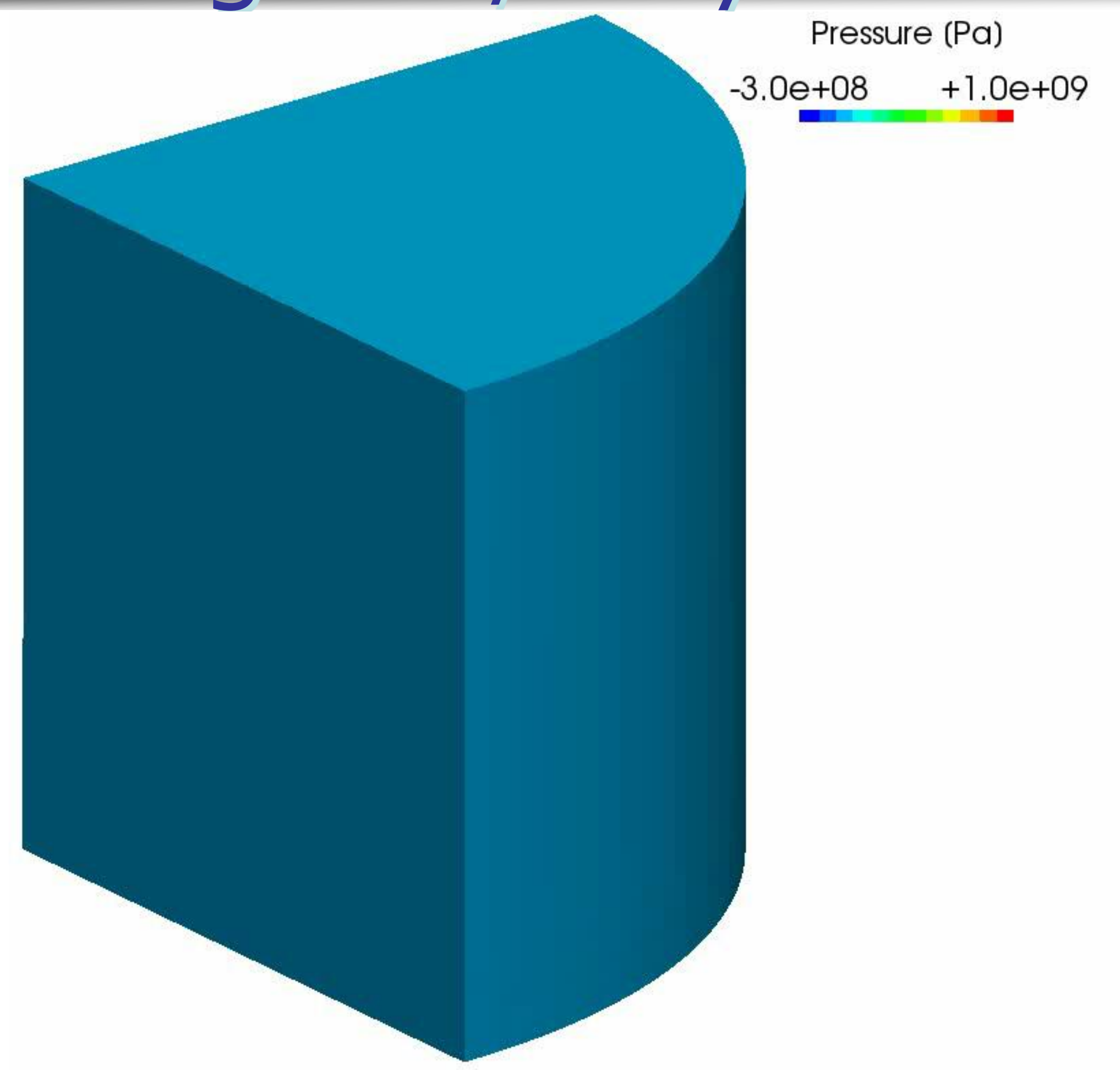
Static Implicit

Barreling of 1/8 Cylinder

Result of F-bar ES-FEM(2) (Pressure)

50% nominal compression

Almost smooth pressure distribution is obtained except just around the rim.



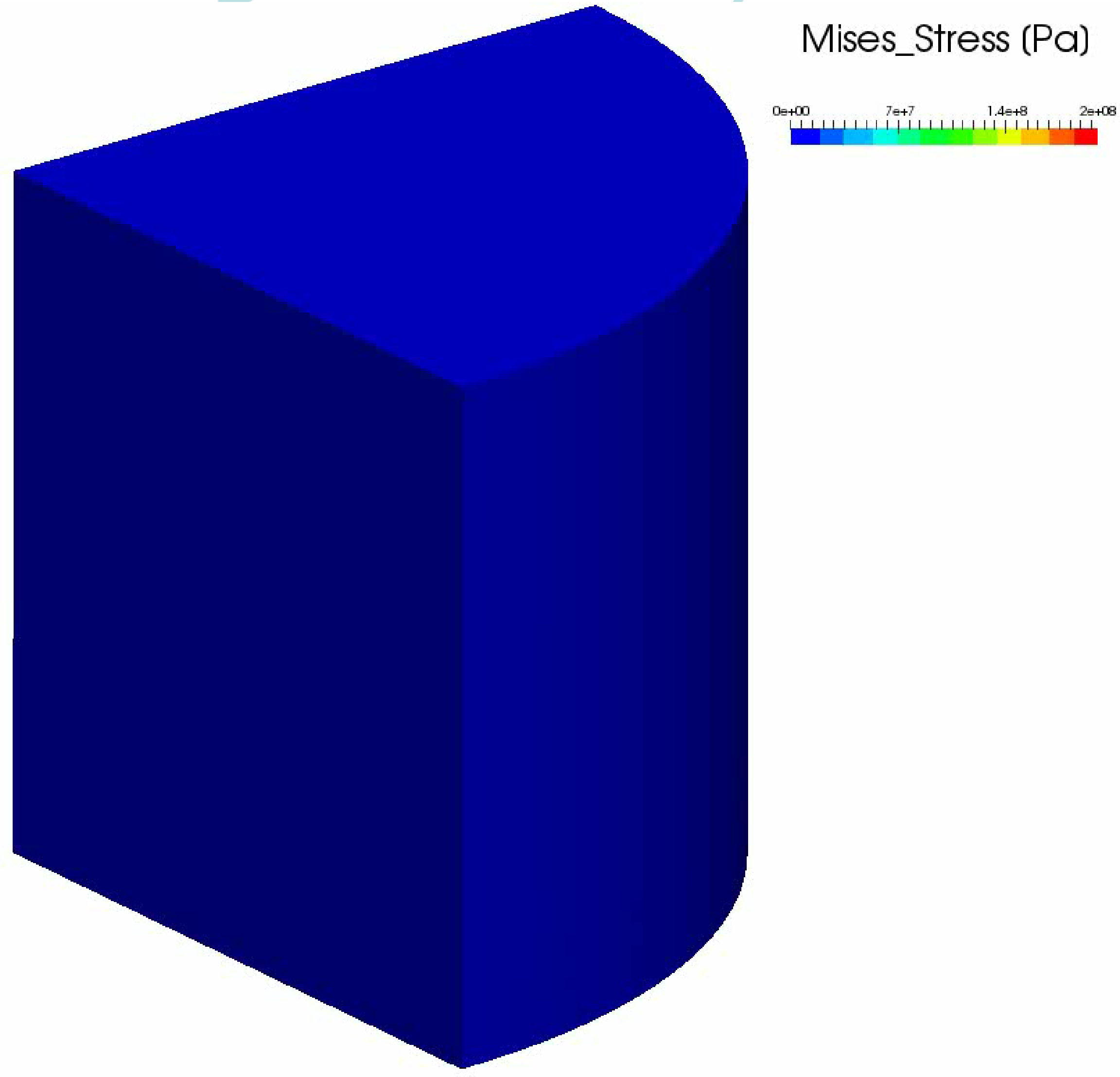
Static Implicit

Barreling of 1/8 Cylinder

Result of F-bar ES-FEM(2) (Mises Stress)

50% nominal compression

Smooth Mises stress distribution is obtained except just around the rim.



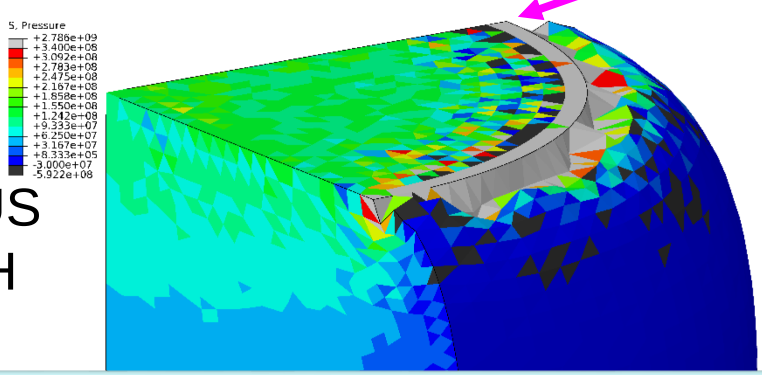
Static Implicit

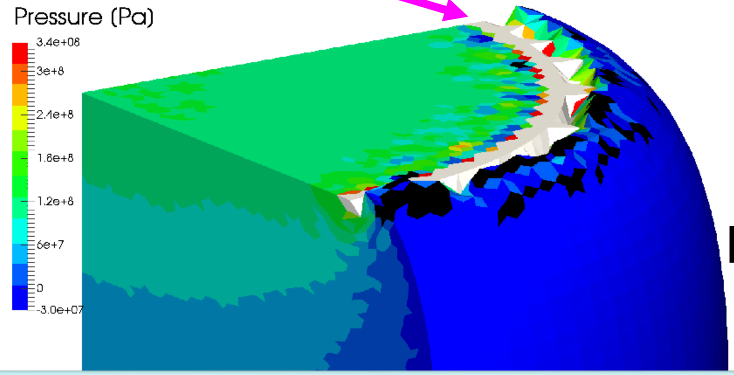
Barreling of 1/8 Cylinder

Pressure Distribution

Strange deformation (corner locking) around the rim

ABAQUS C3D4H

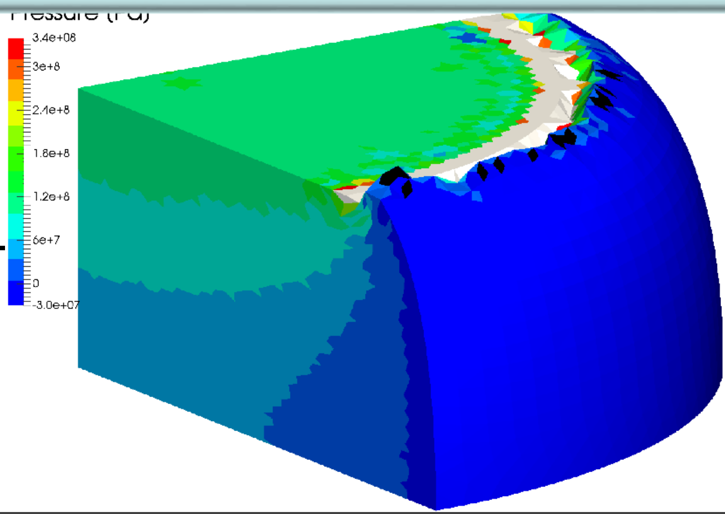


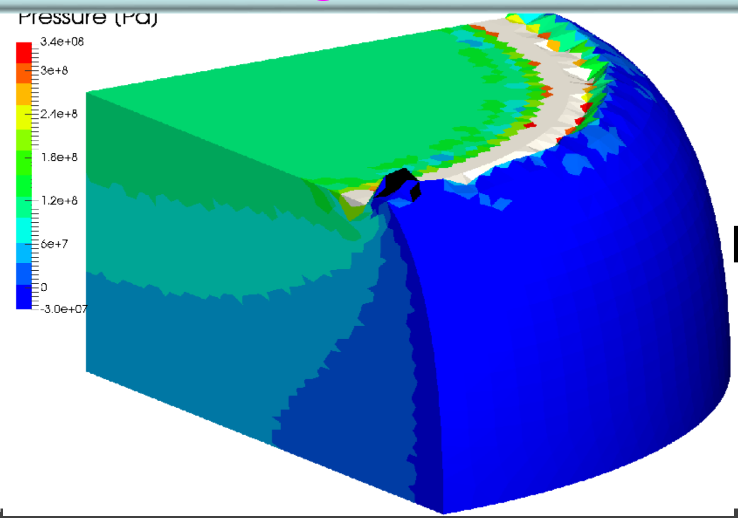


F-bar ES-FEM-T4(2)

F-barES-FEM-T4 with a sufficient cyclic smoothing can resolve the corner locking issue.

F-bar ES-FEM-T4(3)

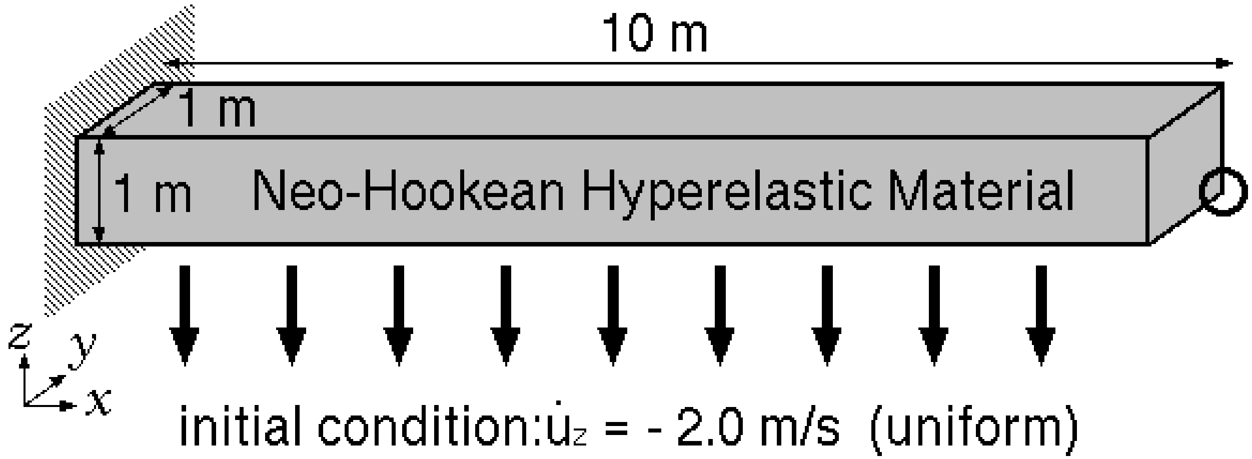




F-bar ES-FEM-T4(4)

Bending of a Cantilever

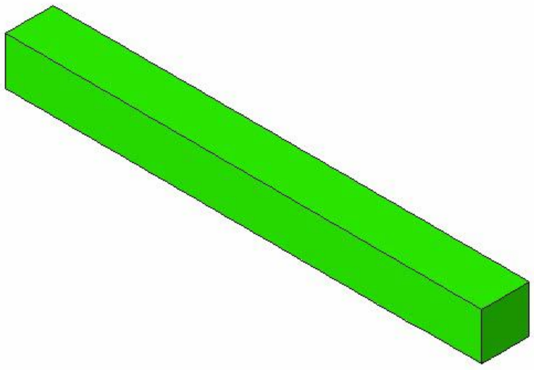
Outline



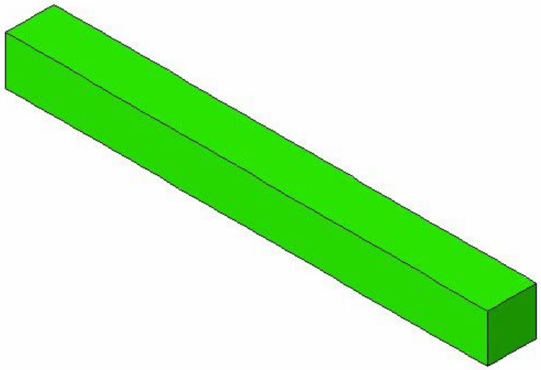
- Neo-Hookean hyperelastic material: $E_{\mathrm{ini}} = 6\ \mathrm{MPa},\ \nu_{\mathrm{ini}} = \mathbf{0.499},\ \rho = 1000\ \mathrm{kg/m^3}$.
- Uniform initial velocity: $\dot{u}_z = -2\ \mathrm{m/s}$.
- Compared to ABAQUS/Explicit C3D4 (NOT C3D4H) & C3D8 (hexahedral selective reduced integration).

Bending of a Cantilever

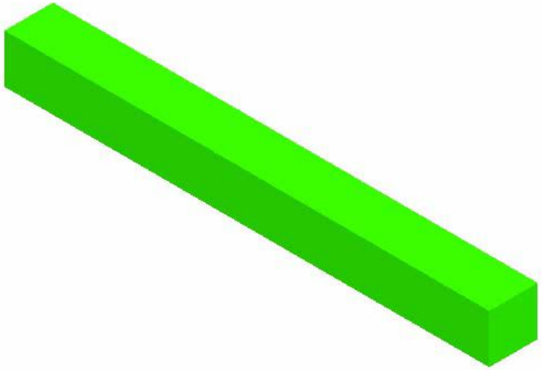
Pressure sign distributions



ABAQUS/Explicit C3D4
(✗ Locking &
Pressure oscillation)



ABAQUS/Explicit C3D8



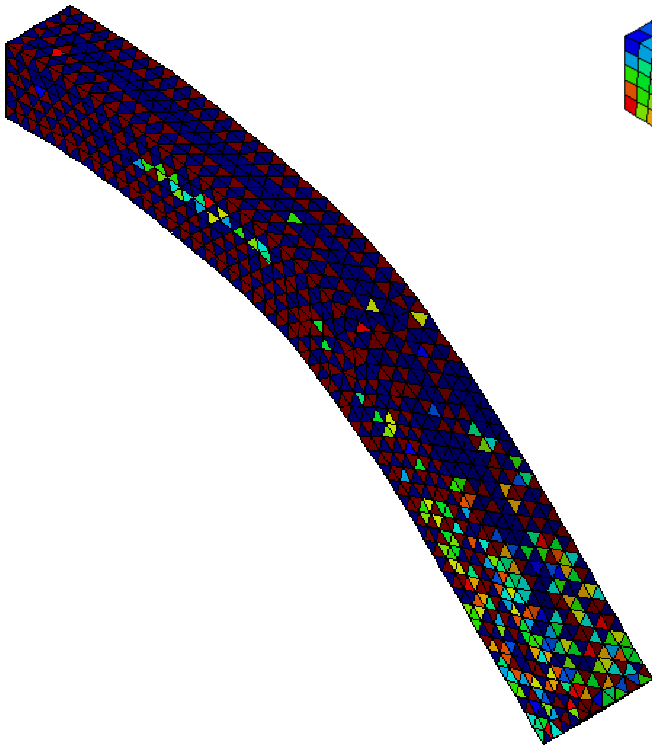
F-barES-FEM-T4(2)

F-barES-FEM has no locking & less pressure oscillation in dynamic explicit analysis.

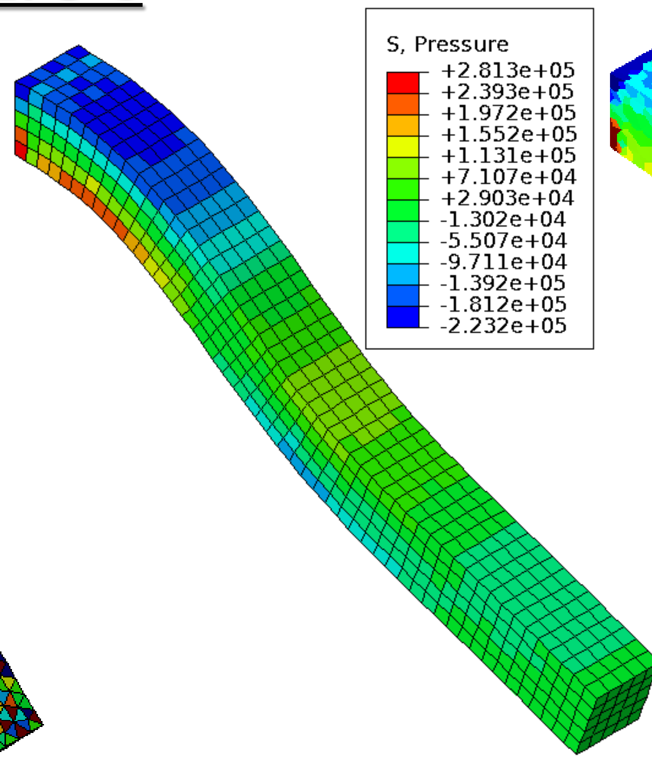
Dynamic
Explicit

Bending of a Cantilever

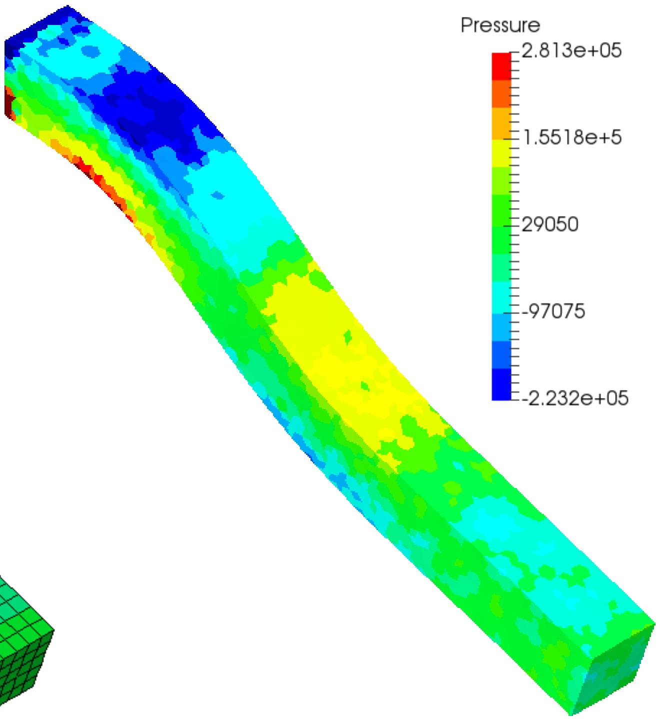
Pressure at $t = 1.5$ s



ABAQUS/Explicit C3D4



ABAQUS/Explicit C3D8



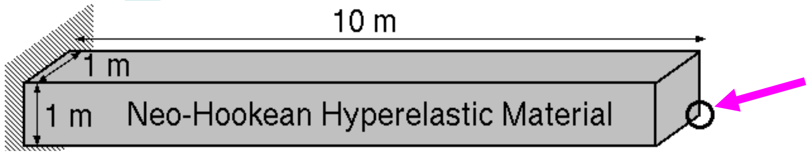
F-barES-FEM-T4(2)

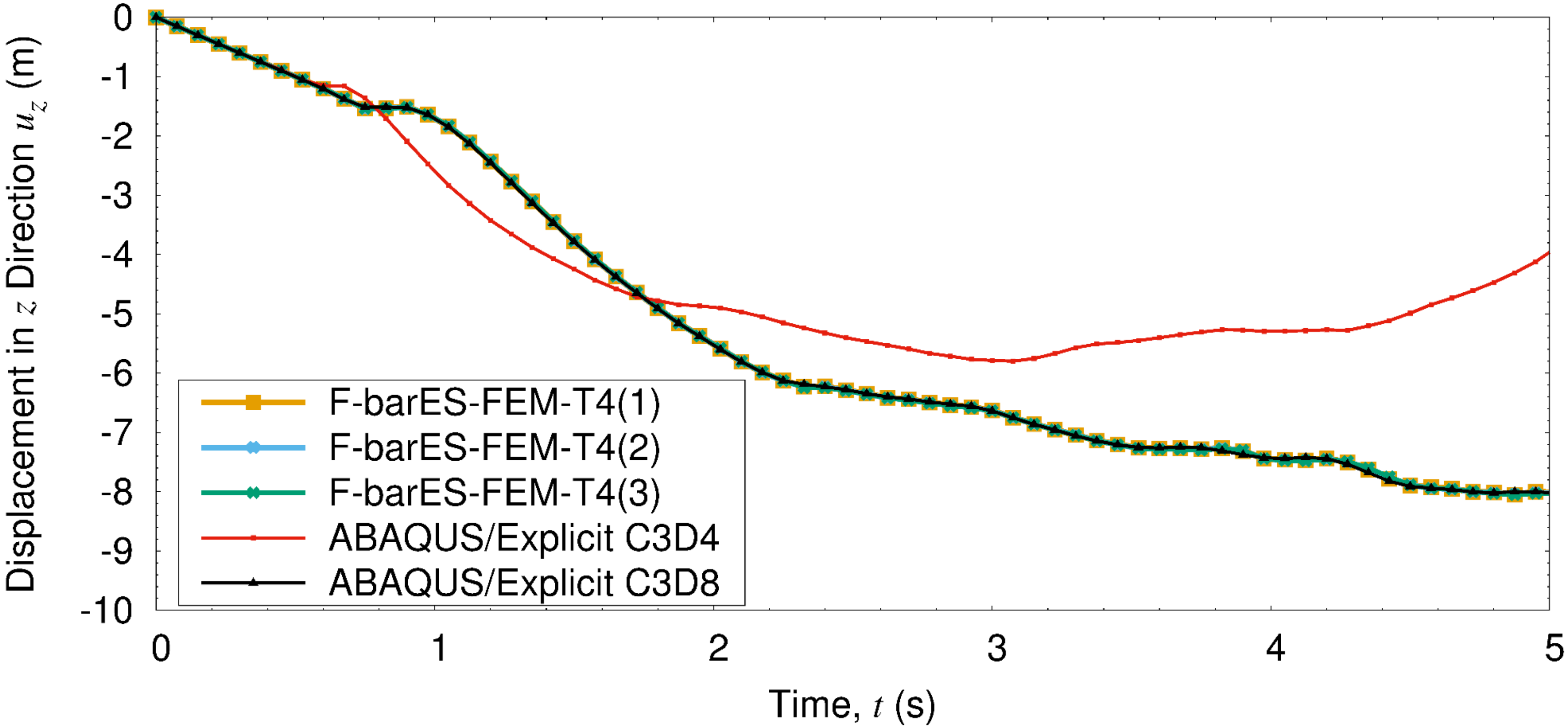
F-barES-FEM-T4 has good accuracy in pressure.

Dynamic Explicit

Bending of a Cantilever

Deflection

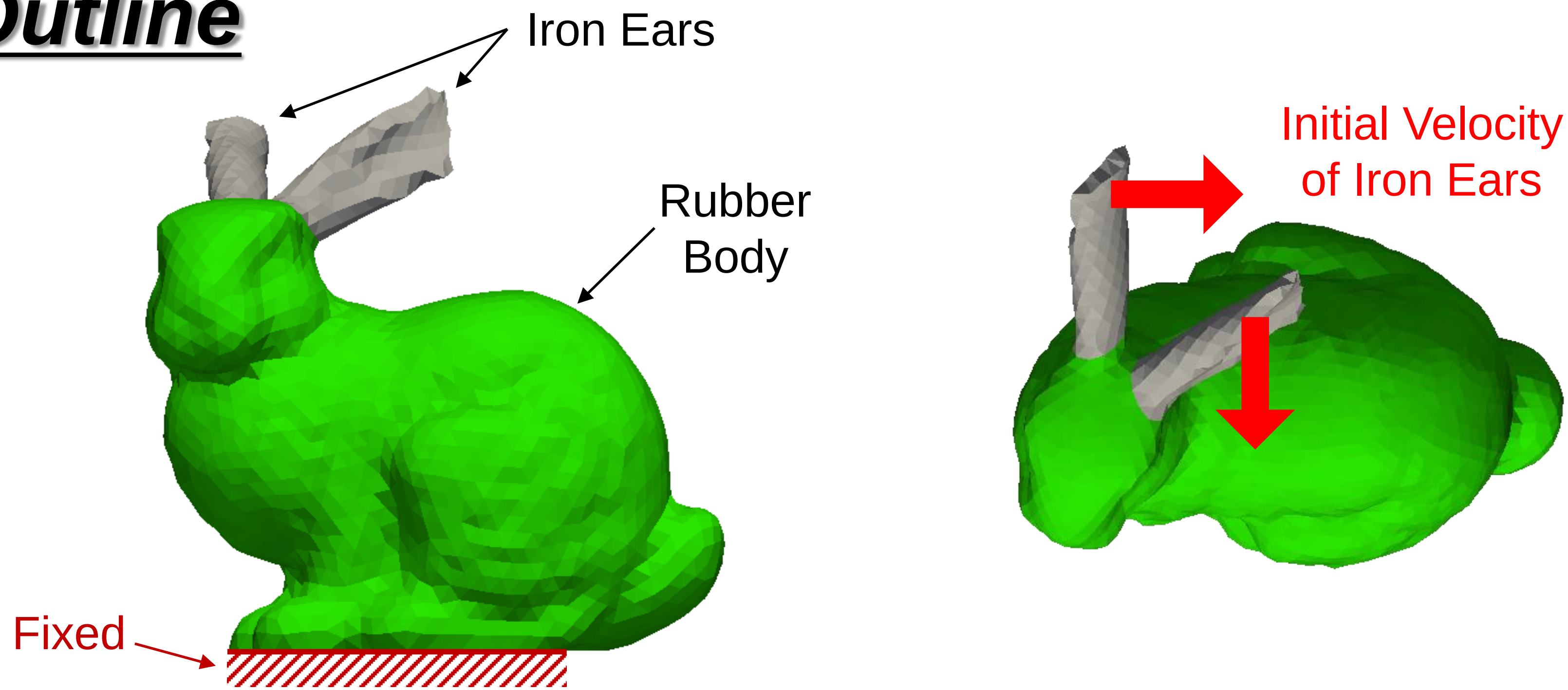




Displacement accuracy of F-barES-FEM is independent of the number of cyclic smoothing.

Swinging of Bunny Ears

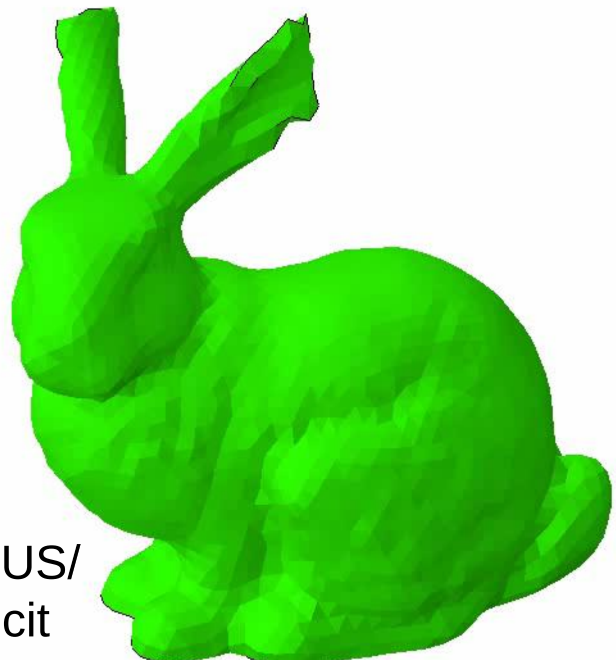
Outline



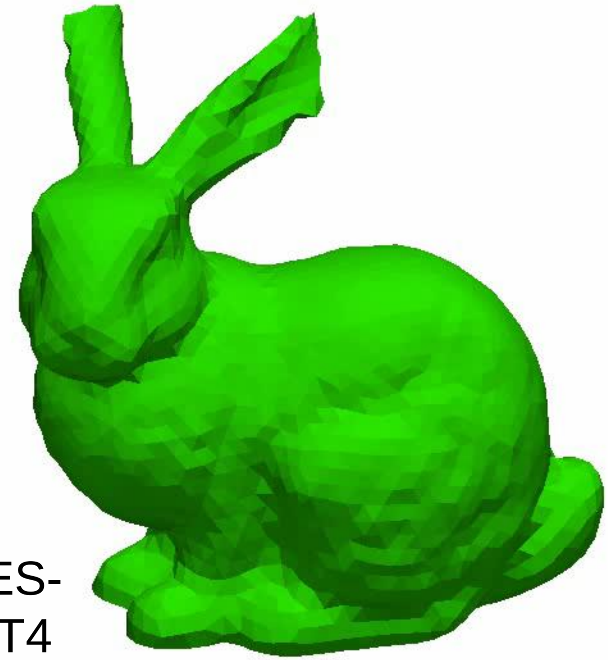
- Iron ears: $E_{\mathrm{ini}} = 200\ \mathrm{GPa},\ \nu_{ini} = \mathbf{0.3},\ \rho = 7800\ \mathrm{kg/m^3}$, Neo-Hookean, **No cyclic smoothing.**
- Rubber body: $E_{\mathrm{ini}} = 6\ \mathrm{MPa},\ \nu_{ini} = \mathbf{0.49},\ \rho = 920\ \mathrm{kg/m^3}$, Neo-Hookean, **1 cycle of smoothing.**
- Compared to ABAQUS/Explicit C3D4.

Swinging of Bunny Ears

Pressure sign distributions



ABAQUS/
Explicit
C3D4

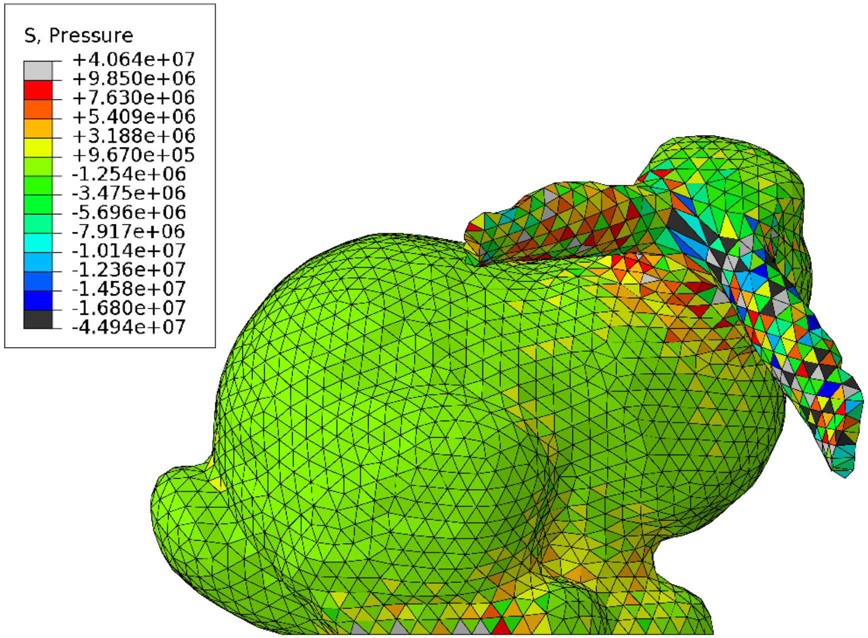


F-barES-
FEM-T4

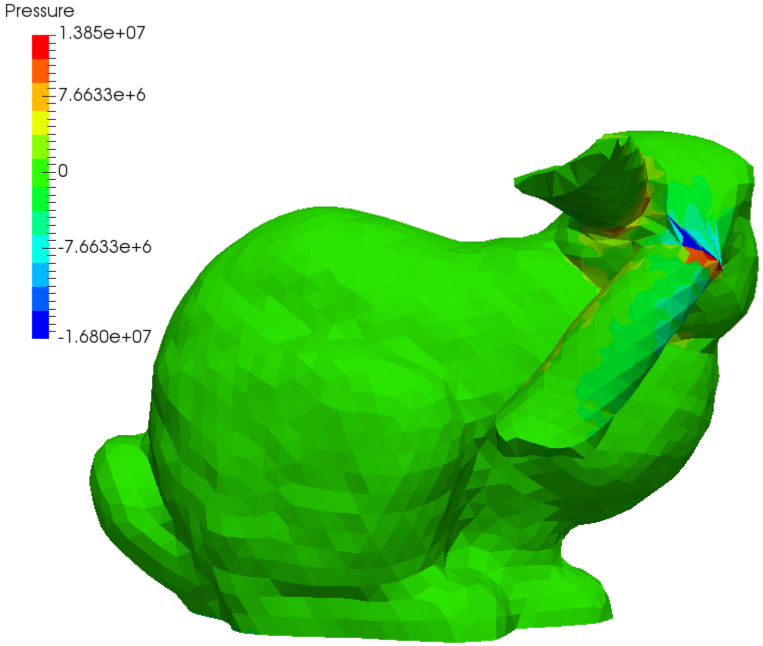
- F-barES-FEM is apparently softer than C3D4.
- F-barES-FEM shows patchy pressure patterns due to complex reflections of pressure waves.
- C3D4 shows typical checkerboard patterns.

Swinging of Bunny Ears

Pressure distributions (back shot)



ABAQUS/Explicit C3D4



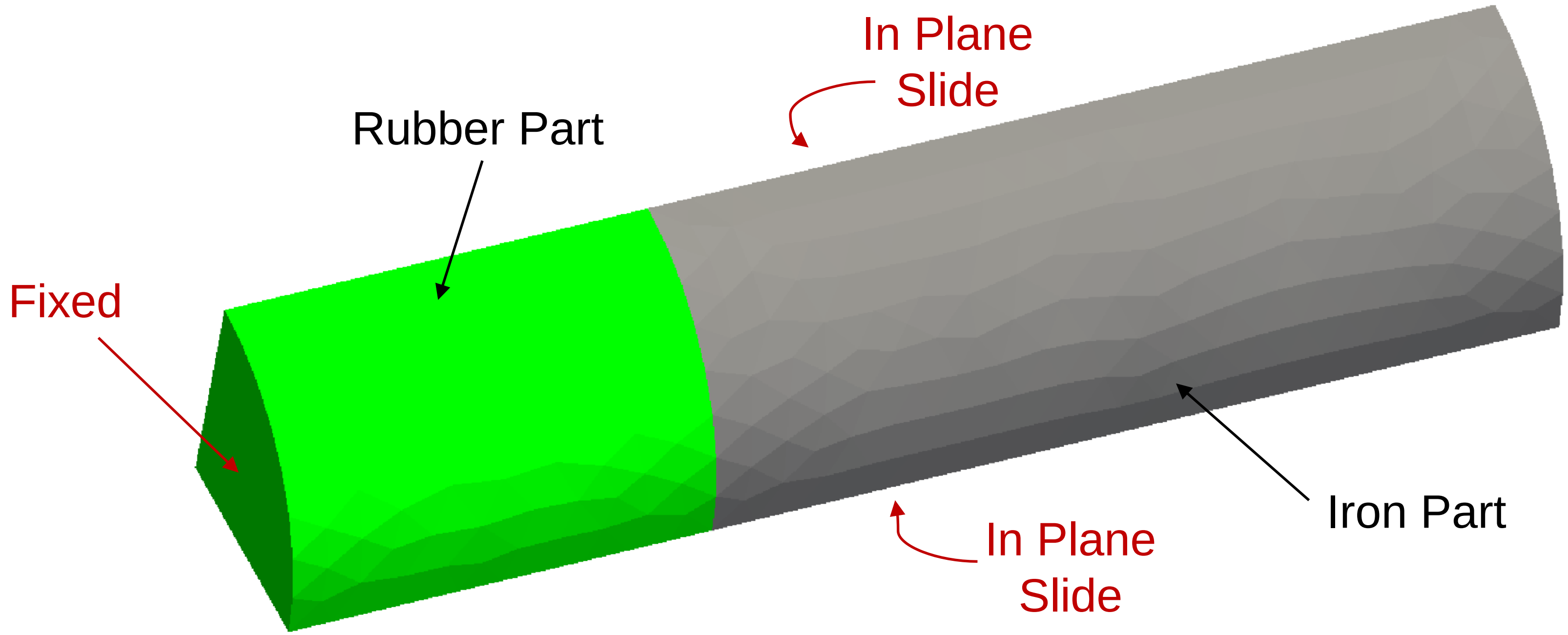
F-barES-FEM-T4

- C3D4 shows checkerboard patterns even on the iron ears. (Because of the negative influence from rubber body???)
- F-barES-FEM-T4 shows smooth pressure distribution except the connection interfaces (corners).

Eigen Mode

Natural Modes of ¼ Cylinder

Outline

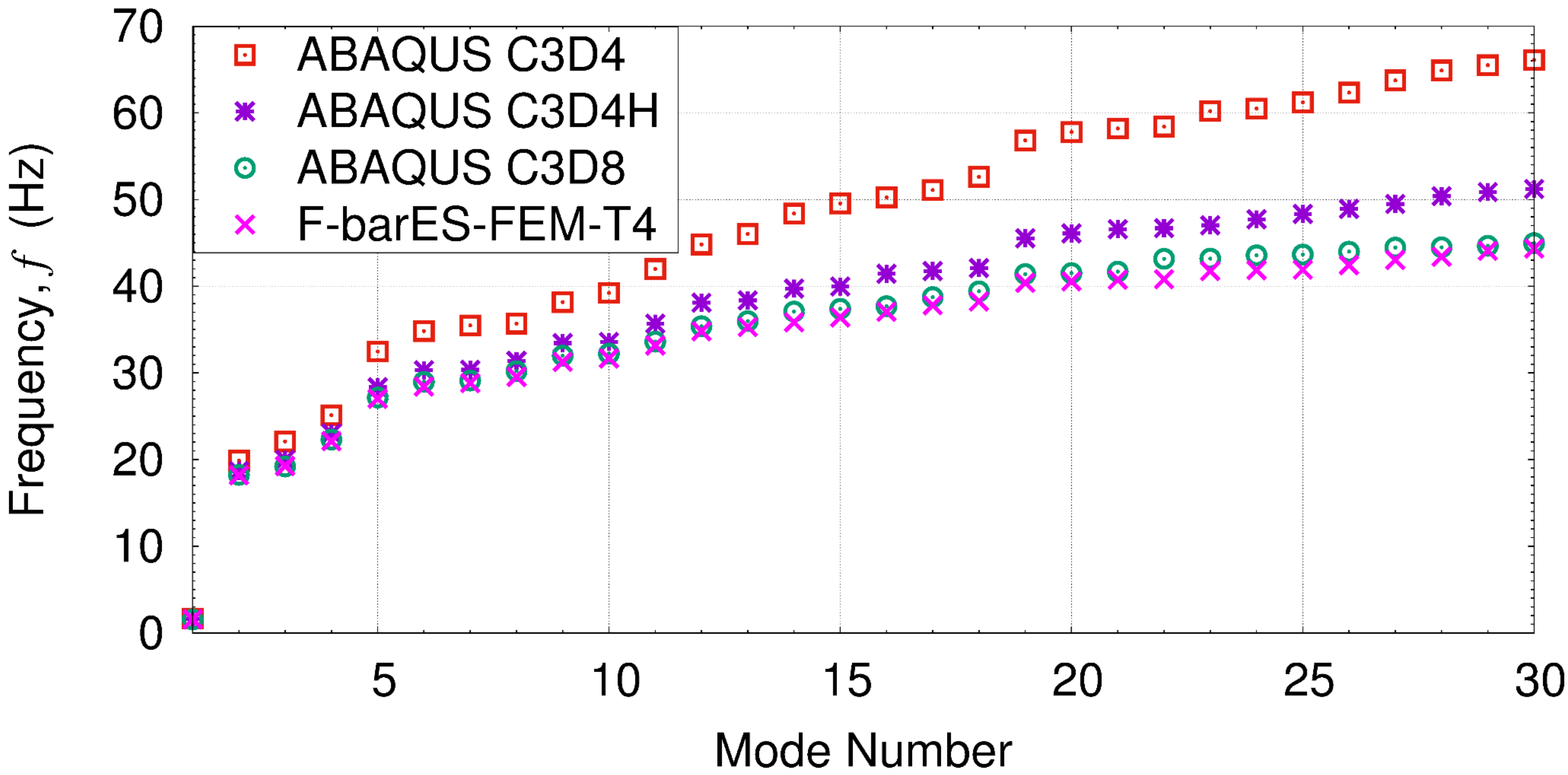


- Iron part: $E_{\mathrm{ini}} = 200\ \mathrm{GPa},\ \nu_{ini} = \mathbf{0.3},\ \rho = 7800\ \mathrm{kg/m^3}$, Elastic, **No cyclic smoothing.**
- Rubber part: $E_{\mathrm{ini}} = 6\ \mathrm{MPa},\ \nu_{ini} = \mathbf{0.499},\ \rho = 920\ \mathrm{kg/m^3}$, Elastic, **2 cycles of smoothing.**
- Compared to ABAQUS C3D4, C3D4H, and C3D8.

Eigen Mode

Natural Modes of ¼ Cylinder

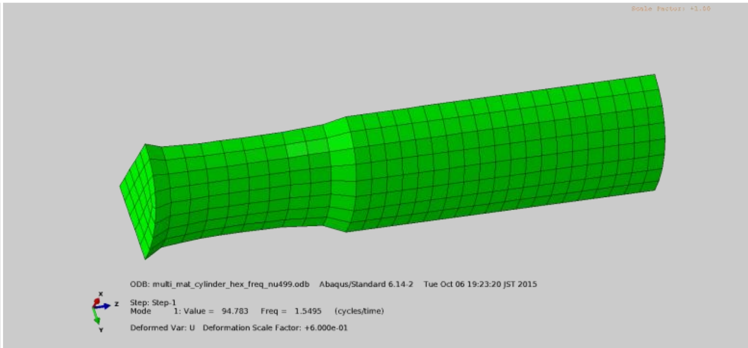
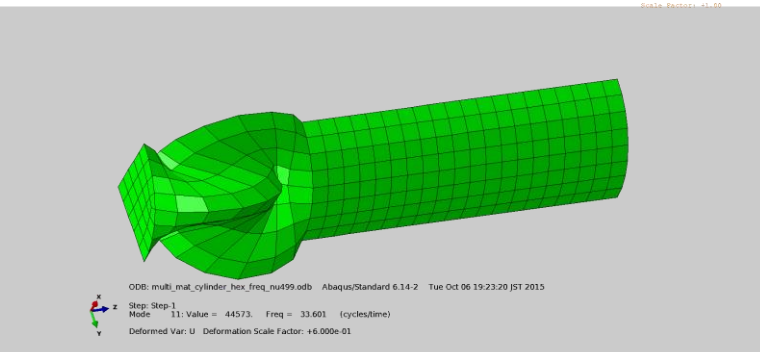
Eigen frequencies



- C3D4 and C3D4H show higher frequencies (stiffer results).
- F-barES-FEM-T4 and C3D8 are in good agreement.

Natural Modes of ¼ Cylinder

Eigen modes

<table>
<tr><th></th><th>1st Mode</th><th>11th Mode</th></tr>
<tr><td>ABAQUS C3D8</td><td>
</td><td>
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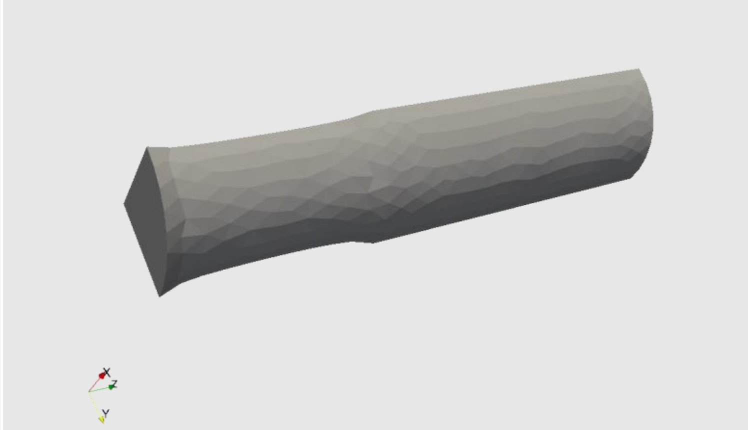
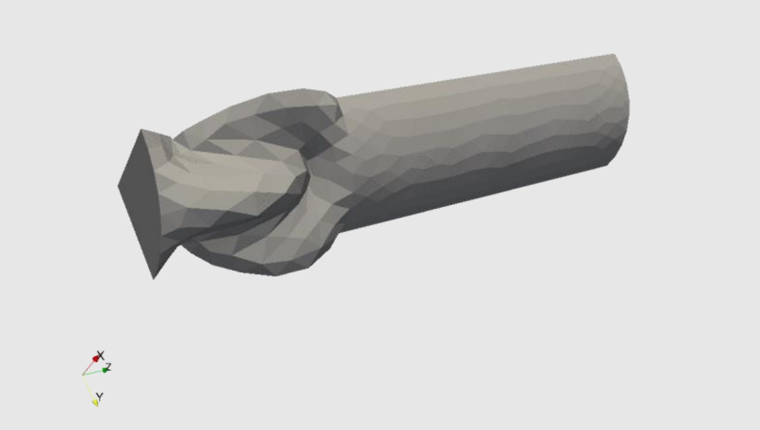
<tr><td>F-barES-FEM-T4</td><td>
</td><td>
</td></tr>
</table>

F-barES-FEM-T4 has good accuracy in eigen mode analysis.

Summary

Benefits and Drawbacks of F-barES-FEM-T4

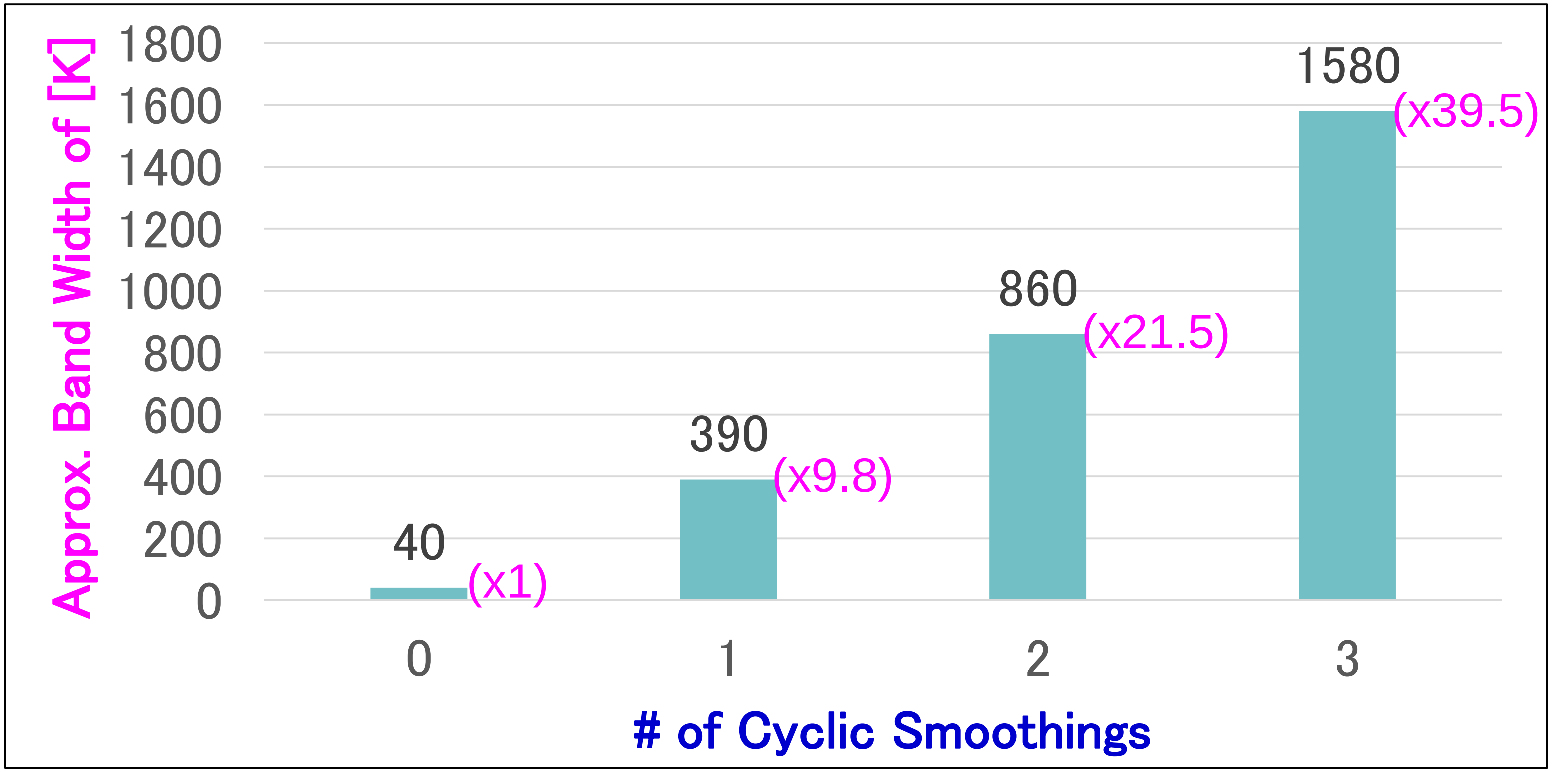
Benefits

- ✓ Locking-free with 1st -order tetra meshes.
  No difficulty in severe strain or contact analysis.
- ✓ No increase in DOF.
  No need of static condensation.
- ✓ No restriction of material constitutive model.
- ✓ Less pressure oscillation in rubber-like materials.
- ✓ Less corner locking.
- ✓ Applicable to static/dynamic, implicit/explicit, and modal analyses.

Benefits and Drawbacks of F-barES-FEM-T4

Drawbacks

✗ Slow speed of calculation.

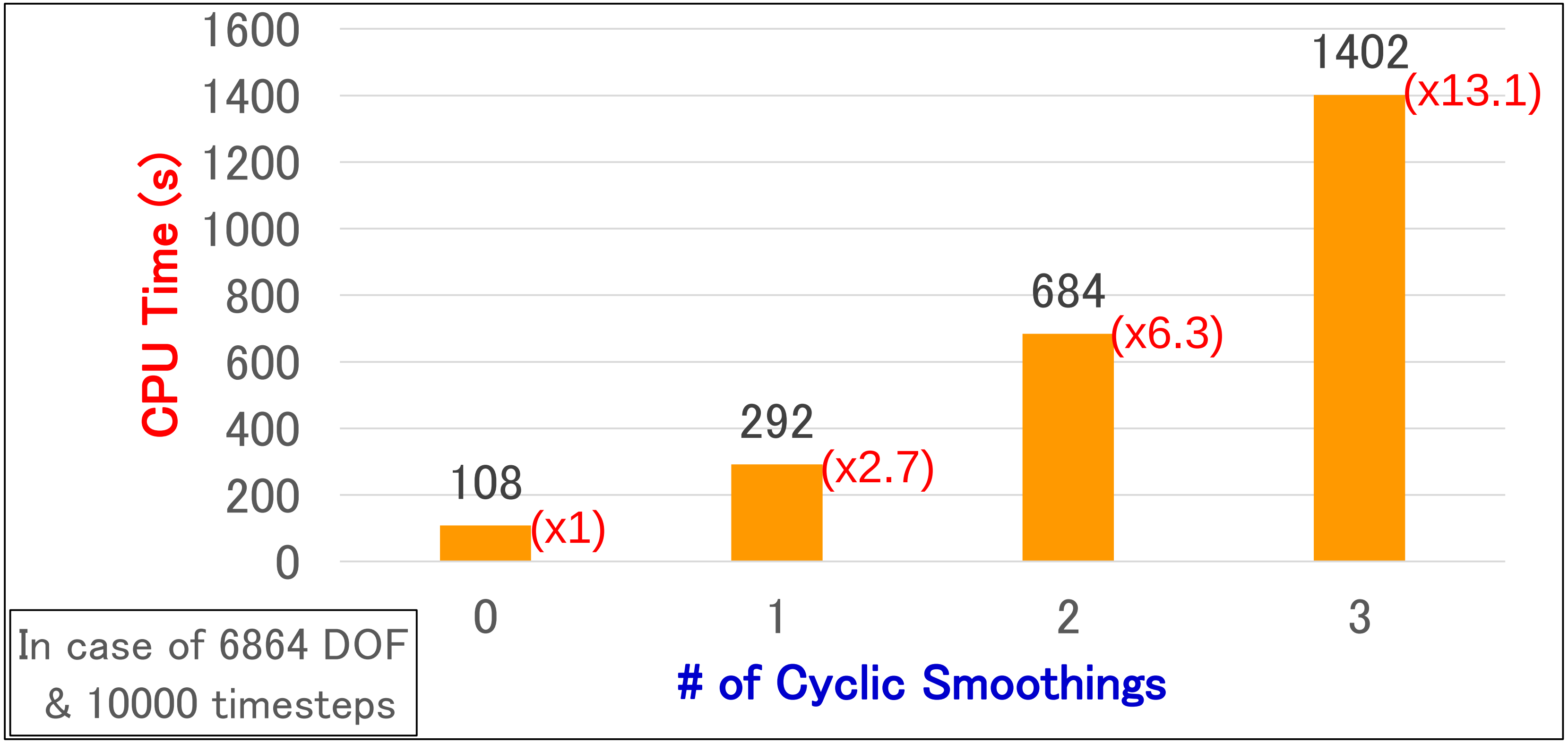


In ***implicit*** analyses, CPU Time is roughly linear with respect to the band width of [K].

Benefits and Drawbacks of F-barES-FEM-T4

Drawbacks

✗ Slow speed of calculation.



In ***explicit*** analyses, [K] is unnecessary; yet, CPU Time increases gradually with the # of cyclic smoothings.

Conclusion

F-barES-FEM-T4 has excellent accuracy, but needs some effort for speed-up.

Thank you for your kind attention!