Modal and dynamic explicit analyses with the latest tetrahedral smoothed finite element method

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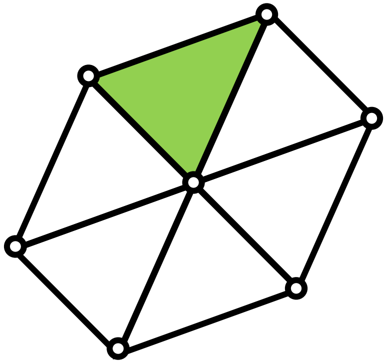
What is S-FEM?

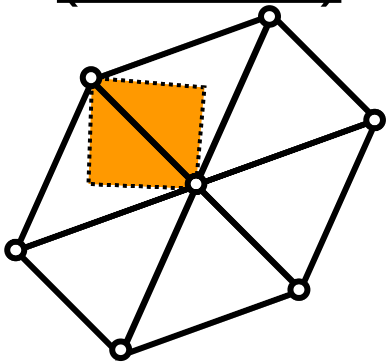
- ***Smoothed*** finite element method (**S-FEM**) is a relatively new FE formulation proposed by Prof. G. R. Liu in 2006.
- S-FEM is one of the **strain smoothing** techniques.
- There are several types of classical S-FEMs depending on the **domains of strain smoothing**.

For example in a 2D triangular mesh:

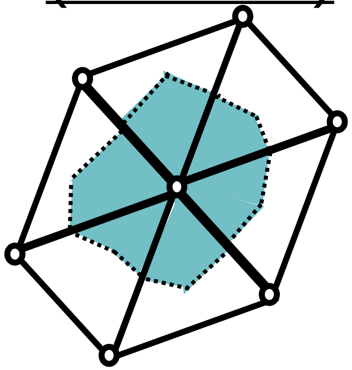
Standard FEM



Edge-based S-FEM (ES-FEM)



Node-based S-FEM (NS-FEM)



What are the major benefits of S-FEM?

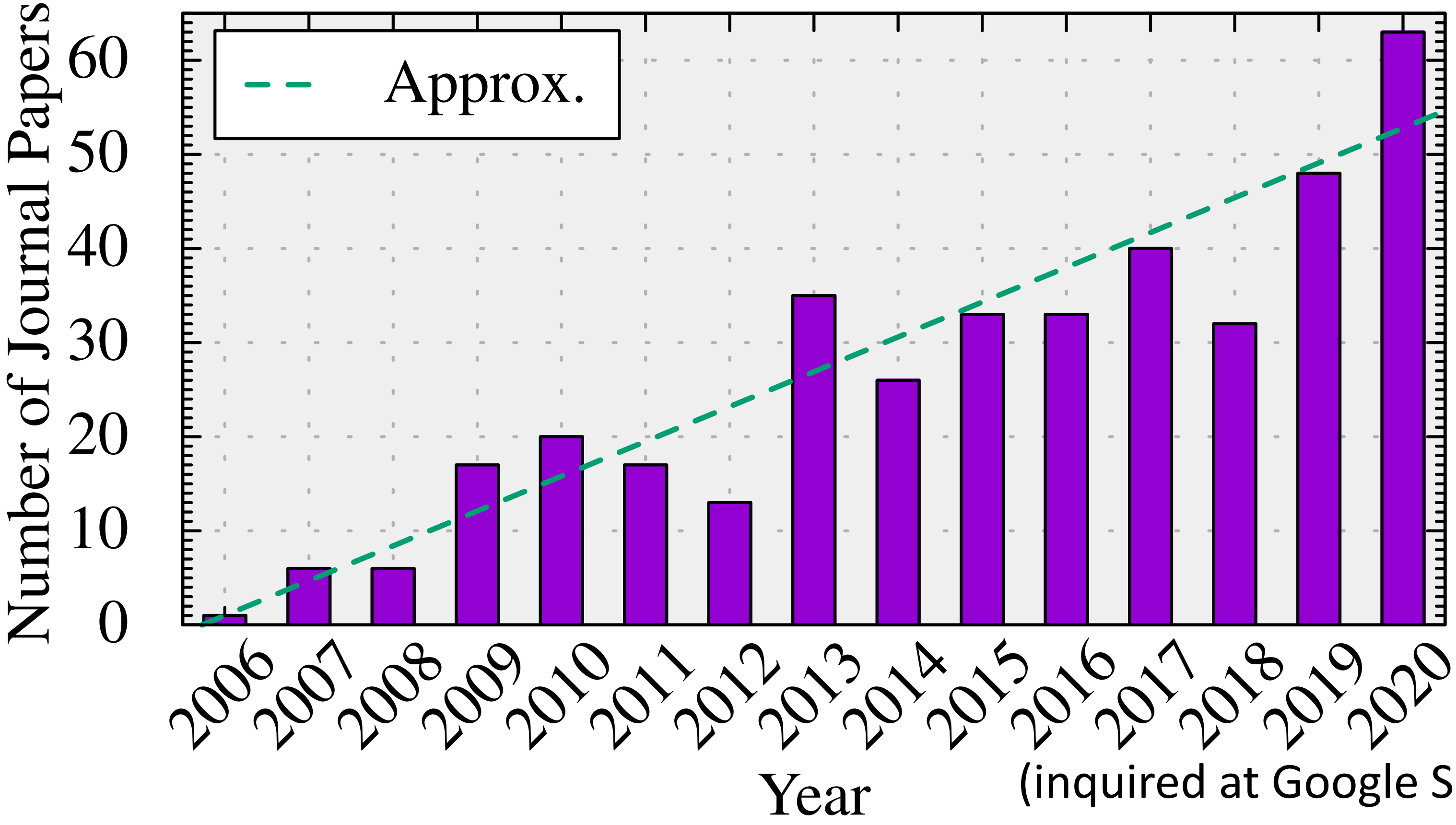
1. **Super-linear mesh convergence rate.**
   (Almost same rate as 2nd-order elements with T4 mesh.)
2. **Shear locking free with ES-FEM.**
   (Excellent accuracy with T4 mesh.)
3. **Little accuracy loss with skewed meshes.**
   (No problem with complex geometry.)

T4: 4-node Tetrahedra

S-FEM is a powerful method
suitable for practical industrial applications.

How popular is S-FEM?

Number of journal papers whose **title** contains "**smoothed finite element**":



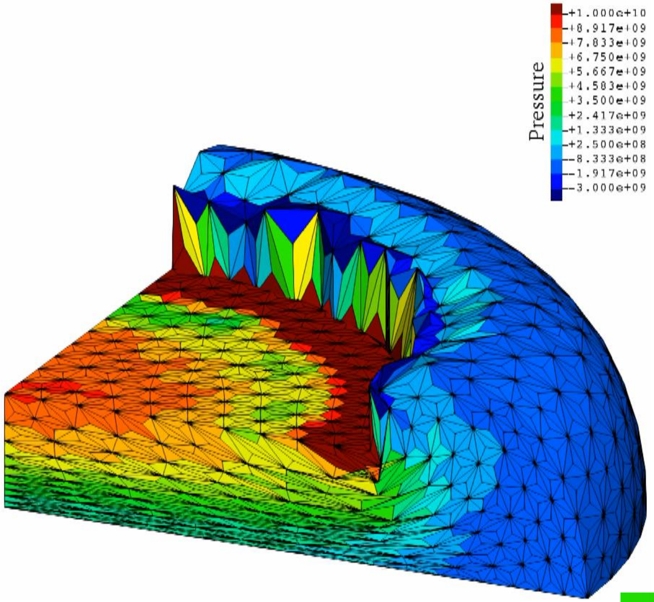
(inquired at Google Scholar)

The attraction of S-FEM is expanding continuously.

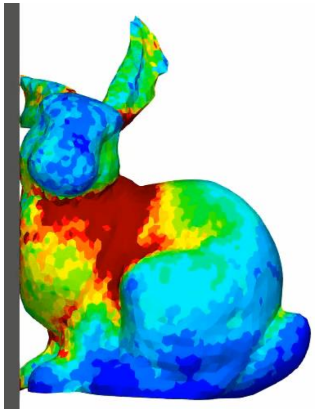
Applications of S-FEMs in Our Lab

- **Solid mechanics (still in academic)**

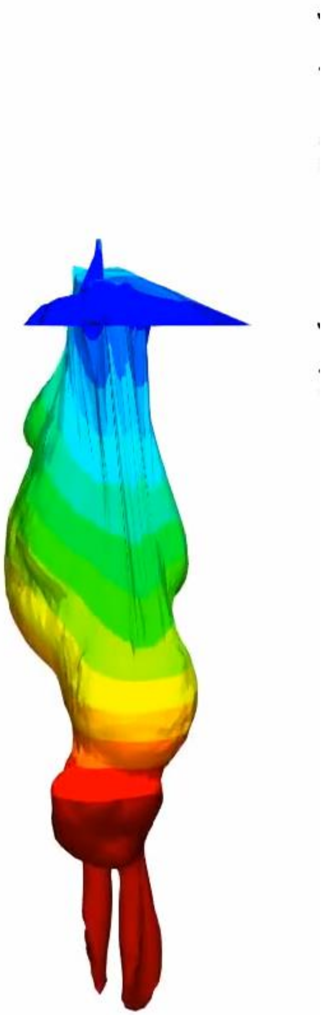
Static Implicit



Dynamic Explicit

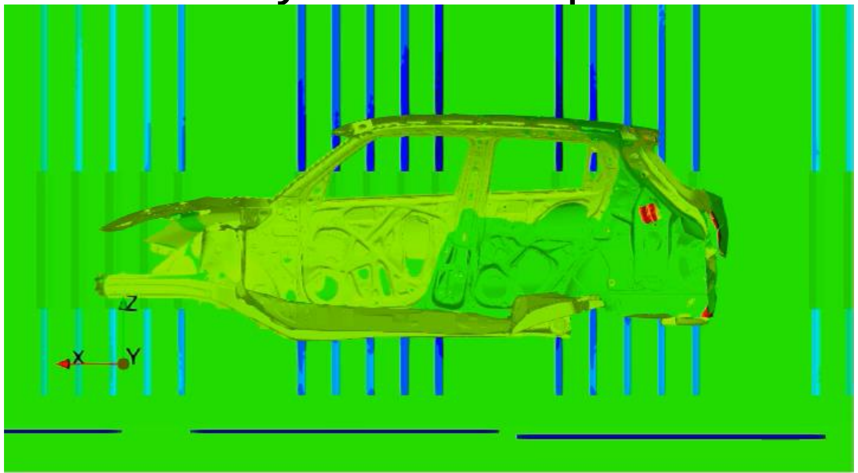


Viscous Implicit



- **Electrostatic**
  (already in practice)

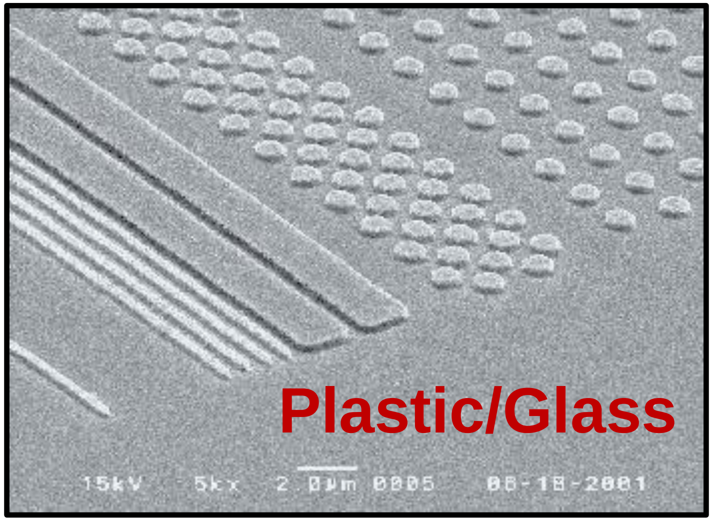
Carbody Electro Deposition



Motivation

What we want to do:

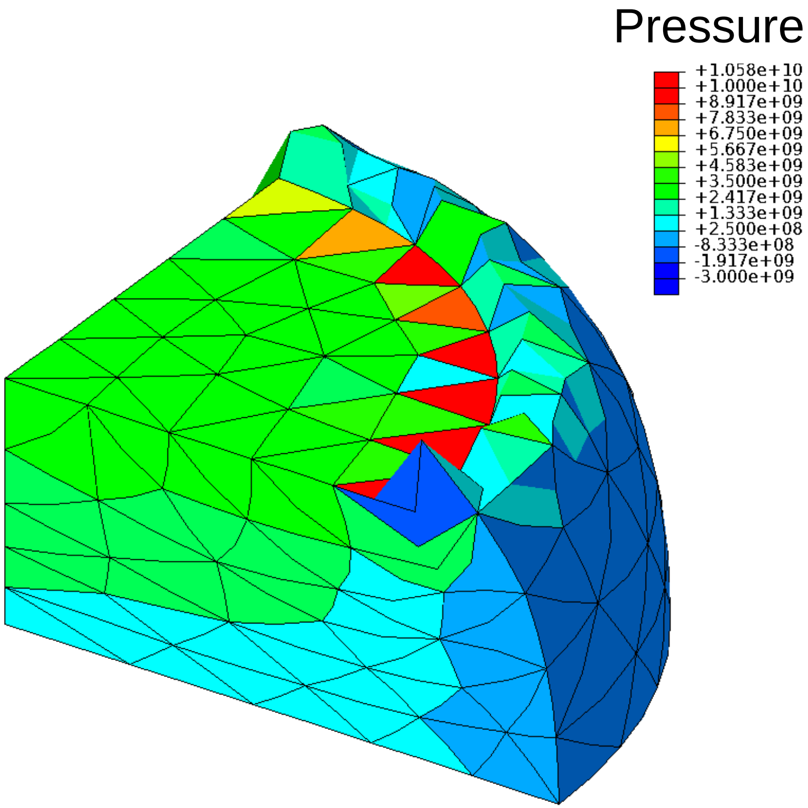
- Solve **severe large deformation** analyses accurately and robustly.
- Treat complex geometries with **tetrahedral meshes**.
- Consider **nearly incompressible materials** $(\nu \simeq 0.5)$.
- Support **contact** problems.
- Handle **auto re-meshing**.



Issues (e.g., barreling analysis of rubber cylinder)

Neo-Hookean ***hyperelastic*** body with $\nu_{\text{ini}} = 0.49$

With the best tetrahedral element in ABAQUS



T10:
10-node Tetrahedra

2$^{\text{nd}}$ order modified hybrid T10 (ABAQUS C3D10MH)

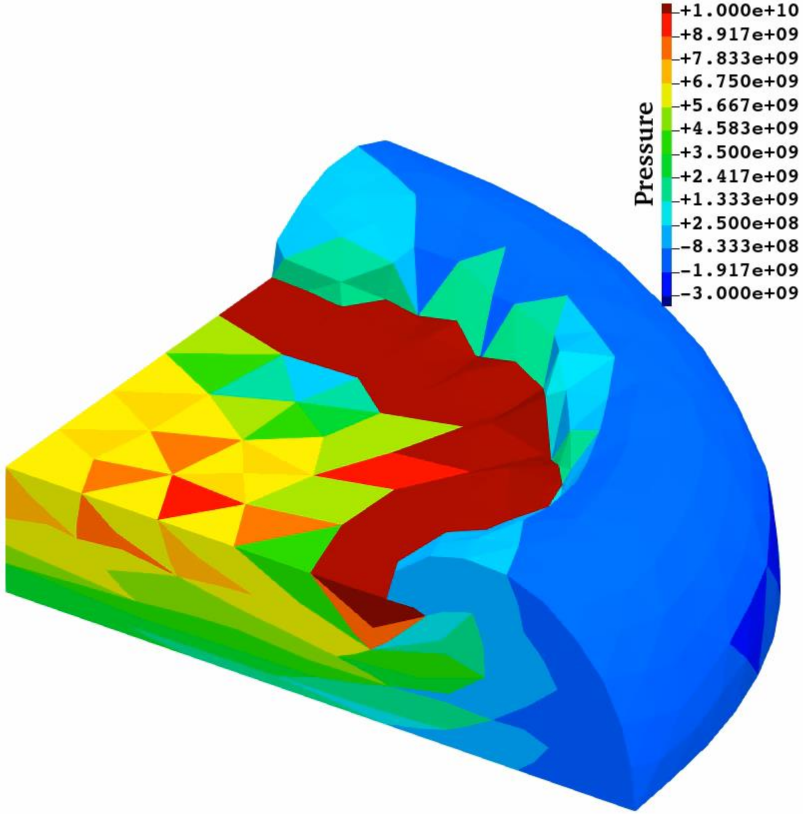
- ✓ No shear/volumetric locking
- ✗ Short lasting (weak to severe deformation)
- ✗ Low interpolation accuracy

Our Approach (e.g., barreling analysis of rubber cylinder)

Neo-Hookean ***hyperelastic*** body with $\nu_{\mathrm{ini}} = 0.49$

With the latest S-FEM tetrahedral element

Same mesh & contour range as C3D10MH case.



Selective CS-FEM-T10 is much better than conventional tetrahedral elements in static analyses.

Y. Onishi, IJCM, (2021).

Further evaluation is necessary in dynamic analyses.

Latest S-FEM T10 (SelectiveCS-FEM-T10)

- ✓ No shear/volumetric locking
- ✓ Less pressure checkerboarding
- ✓ Long lasting (robust to severe deformation)
- ✓ Same CPU time as T10 elements.

Objective

1. Development of a dynamic version of SelectiveCS-FEM-T10
2. Evaluation of its accuracy and robustness in severe large deformation dynamic analyses.

Table of Body Contents

Methods:
Formulation of
SelectiveCS-FEM-T10

Concepts of SelectiveCS-FEM-T10

- Using T10 element and subdivide it into T4 sub-elements.

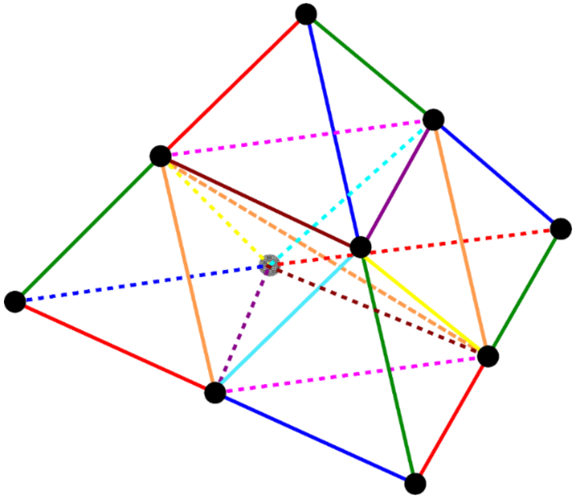
  ⟹ Overcomes the drawbacks of intermediate nodes.

- Adopting intra-element ES-FEM (a kind of CS-FEM) having no strain smoothing across multiple elements.

  ⟹ Becomes an independent element of existing FE codes.

- Applying selective reduced integration (SRI).

  ⟹ Overcomes volumetric locking.

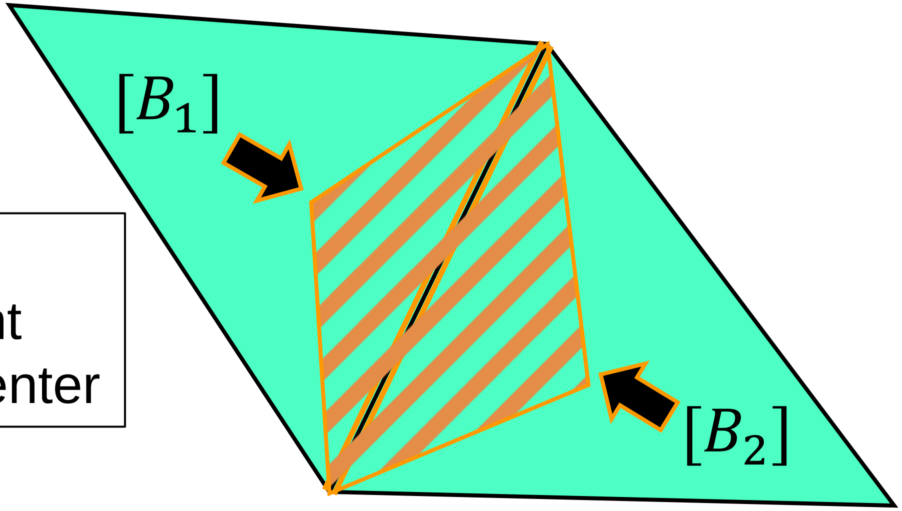


Brief Formulation of ES-FEM

Let us consider two 3-node triangular elements in 2D for simplicity.

- Calculate $[B](= \mathrm{d}\boldsymbol{N}/\mathrm{d}\boldsymbol{x})$ at each element as usual.
- Distribute each $[B]$ to the connecting edge with an area weight and build $[{}^{\mathrm{Edge}}B]$.
- Calculate deformation gradient $(\boldsymbol{F})$, Cauchy stress $(\boldsymbol{\sigma})$ and nodal internal force $\{f^{\mathrm{int}}\}$ in each edge smoothing domain.

As if putting
a Gauss point
on each edge center

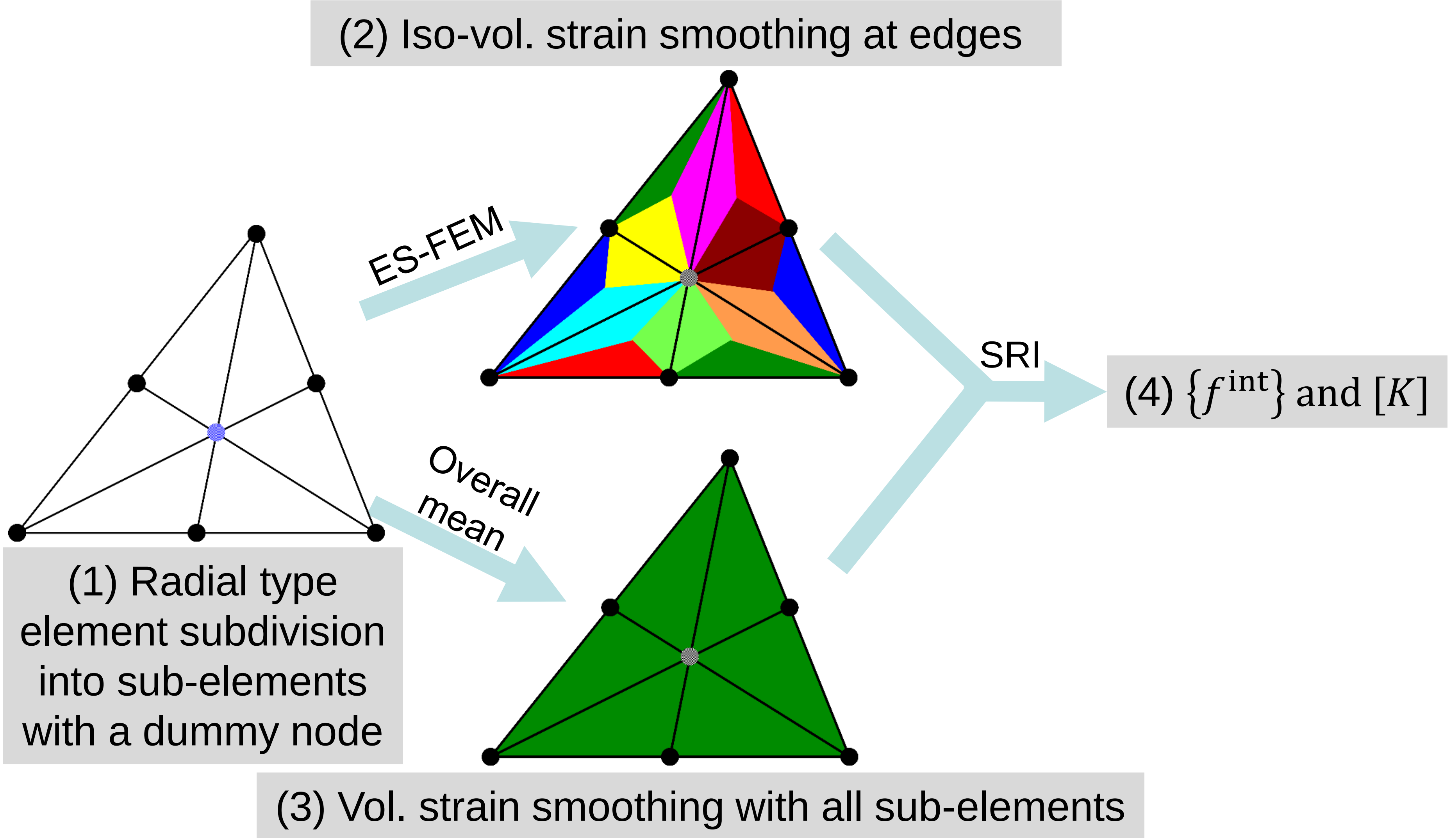


$[{}^{\mathrm{Edge}}B]$

↳ ${}^{\mathrm{Edge}}\boldsymbol{F}$, ${}^{\mathrm{Edge}}\boldsymbol{\sigma}$, $\{{}^{\mathrm{Edge}}f^{\mathrm{int}}\}$ etc.

Flowchart of SelectiveCS-FEM-T10

Explanation in 2D (6-node triangular element) for simplicity



(2) Iso-vol. strain smoothing at edges

ES-FEM

SRI

(4) $\{f^{\mathrm{int}}\}$ and $[K]$

Overall mean

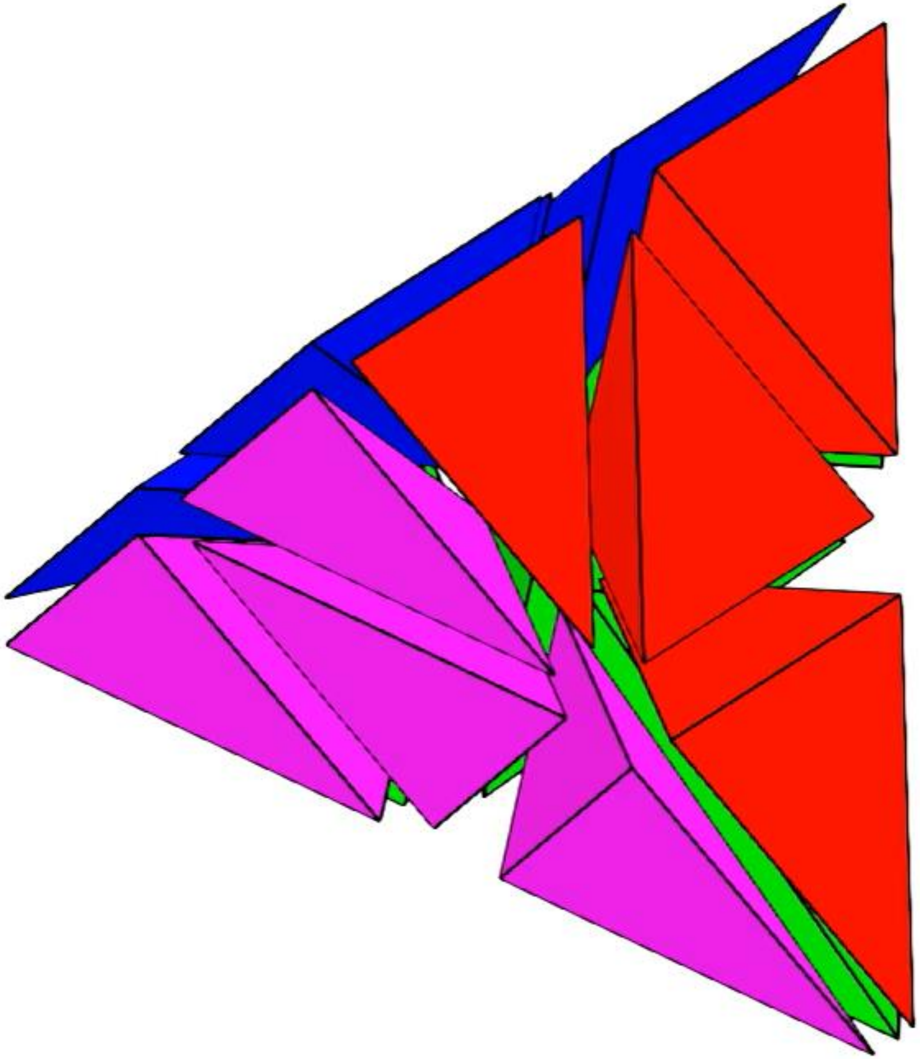
(1) Radial type element subdivision into sub-elements with a dummy node

(3) Vol. strain smoothing with all sub-elements

T10 Element Subdivision in 3D

Radial subdivision (30% shrunk mesh)

There are 16 T4 sub-elements in total.



Sub-elements have a little larger skewness.

but skewness is not a big issue for ES-FEM.

Strain on all 34 edges are smoothed by ES-FEM.

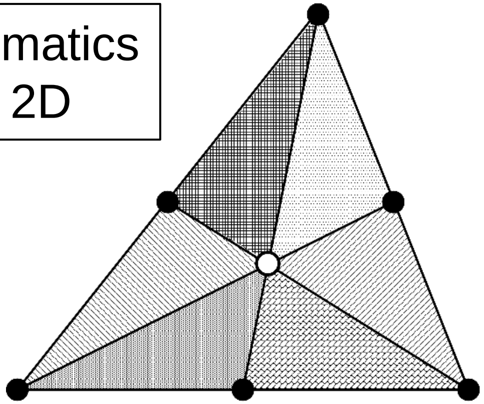
Building Lumped Mass Matrix

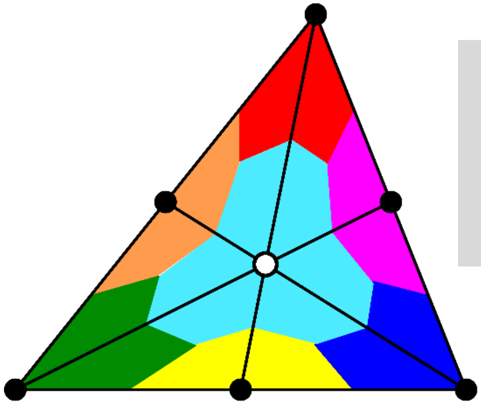
1. Calculate the mass of each sub-element.

2. Distribute it to composing 4 nodes.
   (3 nodes in 2D.)

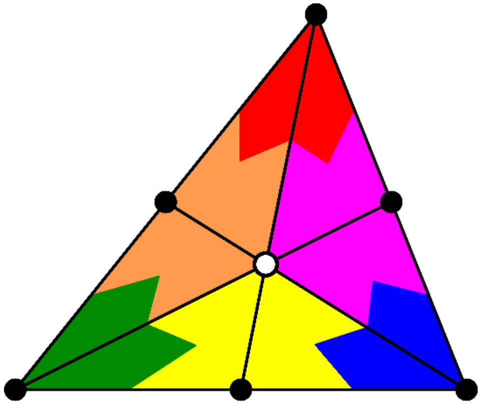
3. The mass of the dummy node is distributed to the connecting 6 mid-nodes.
   (3 mid-nodes in 2D.)

Schematics in 2D





Each color denotes the corresponding area for mass.

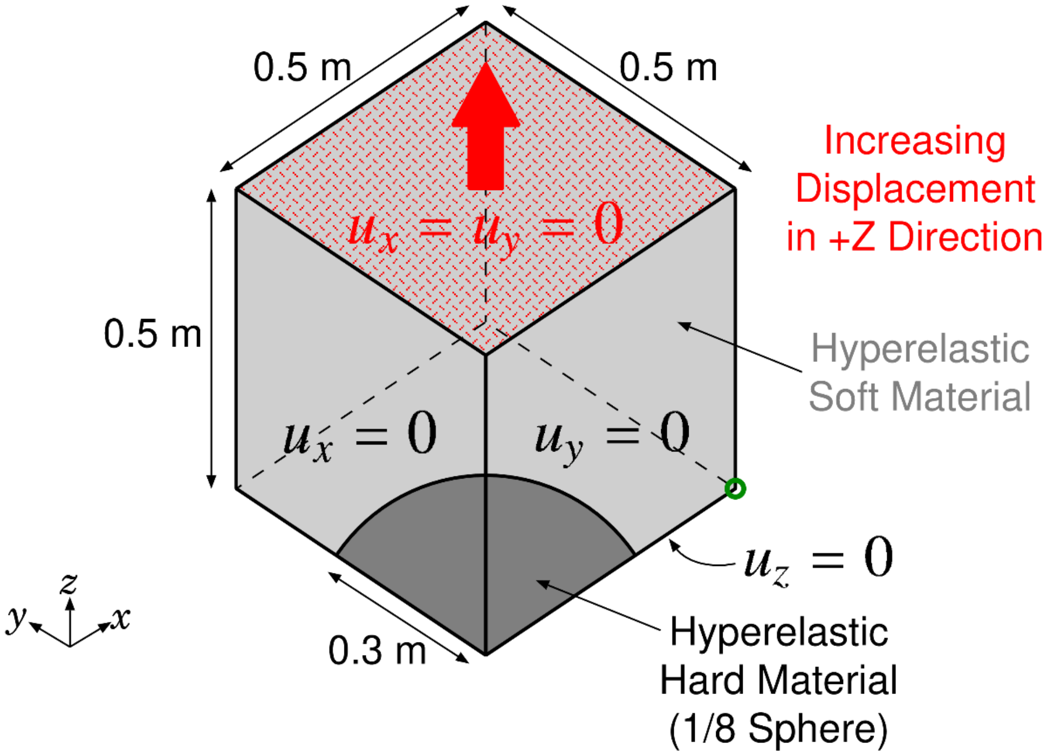


Results:
Demonstration of
SelectiveCS-FEM-T10

Static Implicit

Tensioning of Rubber-Filler Composite

Outline

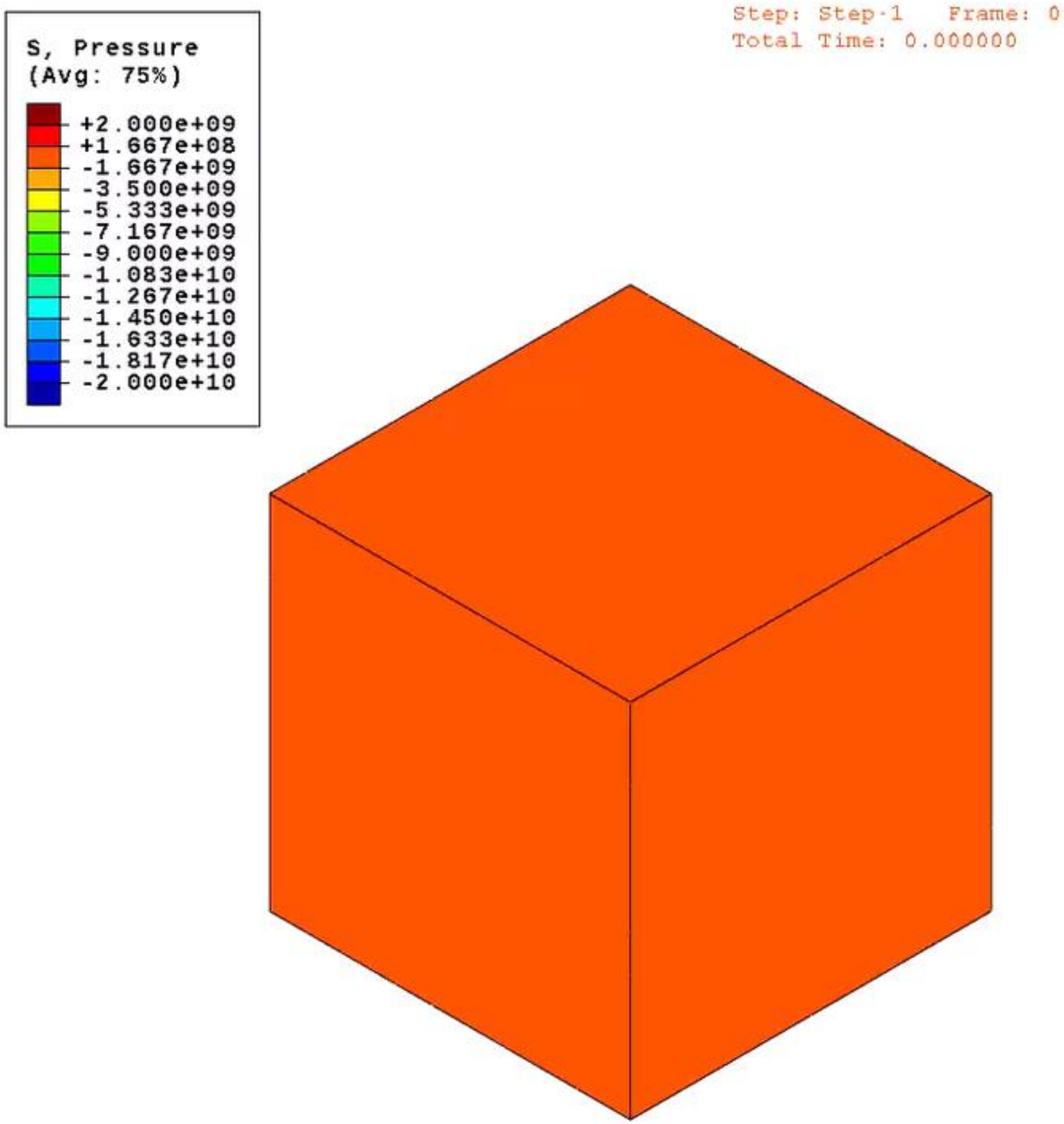


- Soft material: Neo-hookean, $E_{\mathrm{ini}} = 6\ \mathrm{GPa}$, $\boldsymbol{\nu}_{\mathbf{ini}} = \mathbf{0.49}$.
- Hard material: Neo-hookean, $E_{\mathrm{ini}} = 260\ \mathrm{GPa}$, $\boldsymbol{\nu}_{\mathbf{ini}} = \mathbf{0.3}$.
- Discretized into T10 mesh. (about 11,000 nodes and 7,000 elements)
- Compared to ABAQUS C3D10MH, the best T10 element of ABAQUS, with the same mesh.

Static
Implicit

Tensioning of Rubber-Filler Composite

Result of ABAQUS C3D10MH with pressure contour



Convergence failure at 69% nominal stretch (short lasting)

Static Implicit

Tensioning of Rubber-Filler Composite

Result of

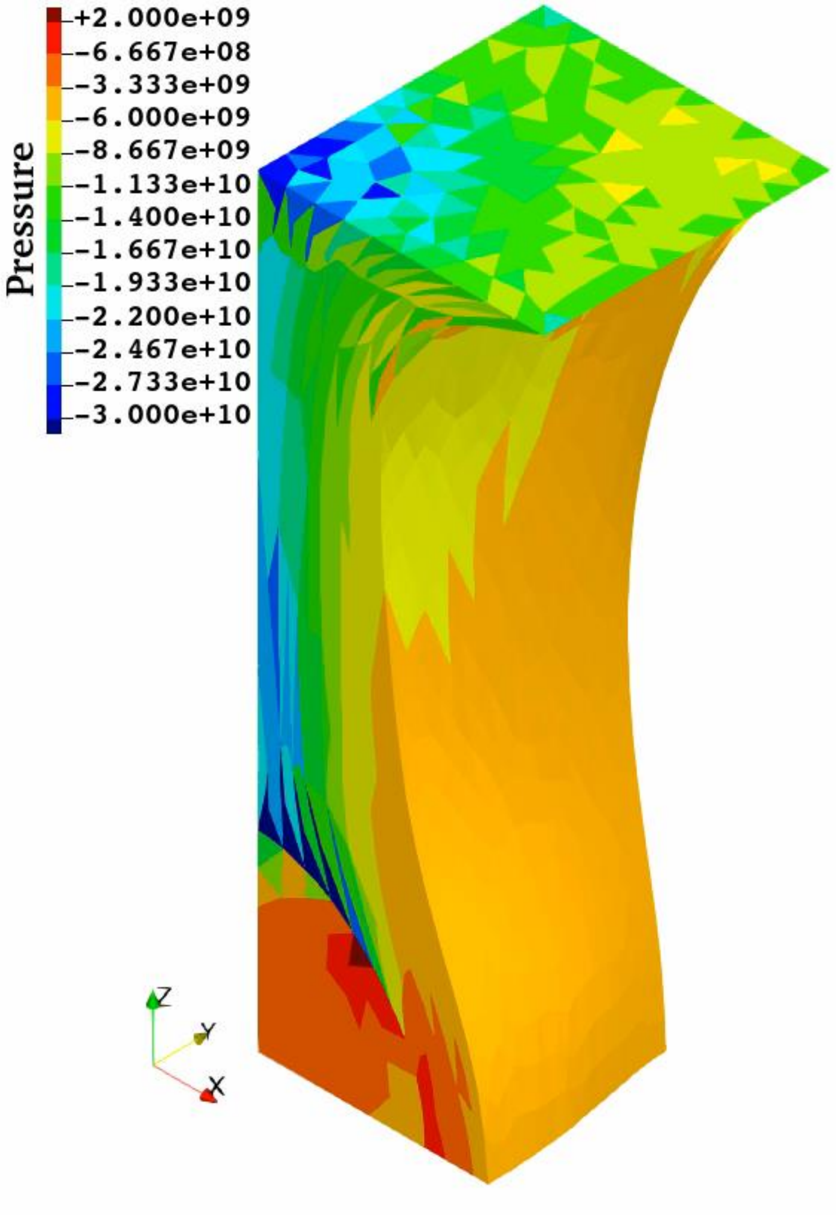
Selective

CS-FEM-T10

with

pressure

contour

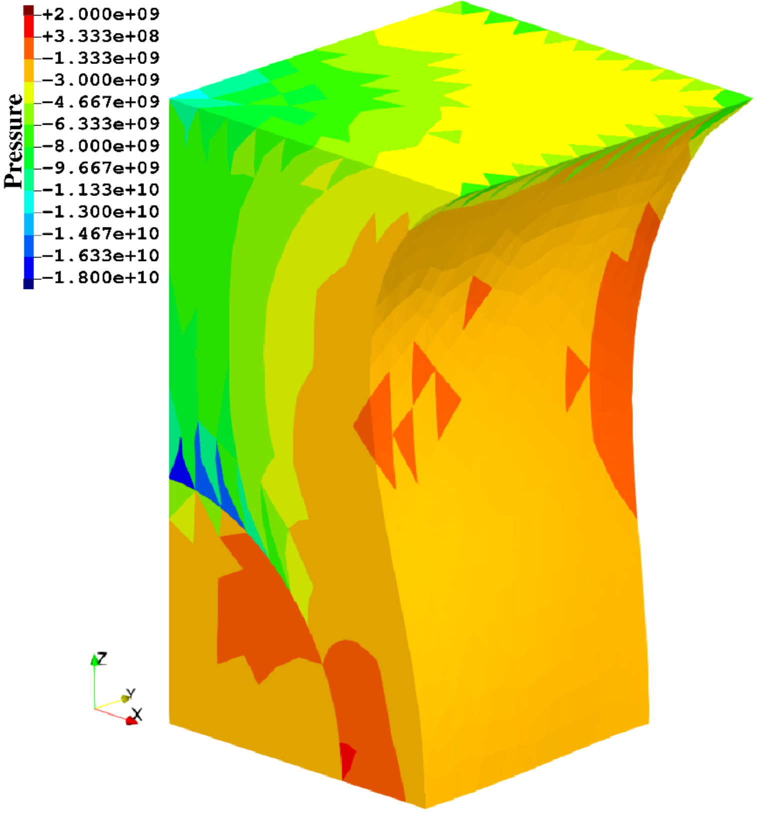


Convergence failure at 166% nominal stretch (long lasting)

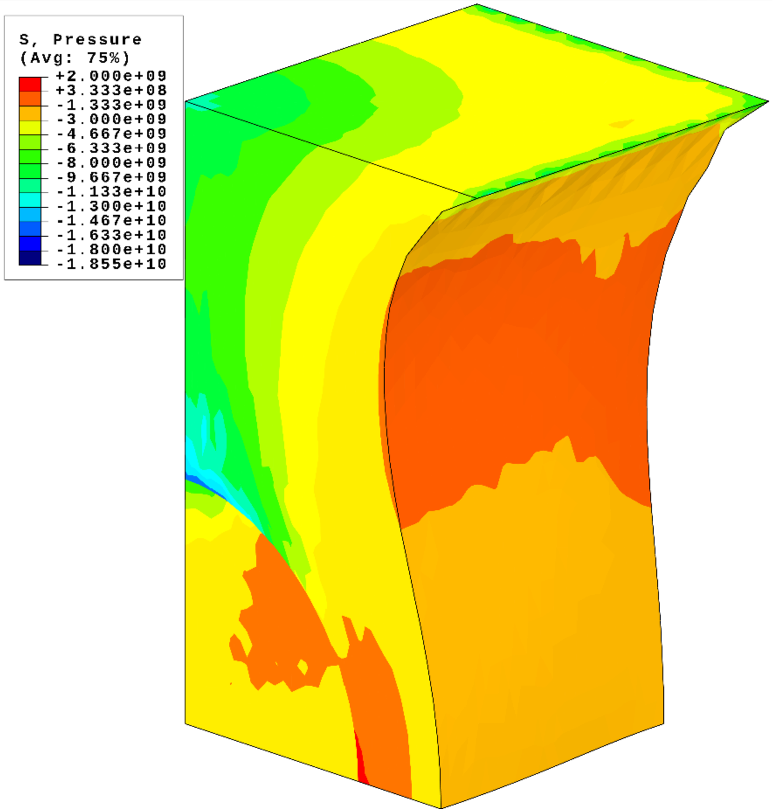
Static
Implicit

Tensioning of Rubber-Filler Composite

Comparison of pressure dist. at 60% nominal stretch



SelectiveCS-FEM-T10



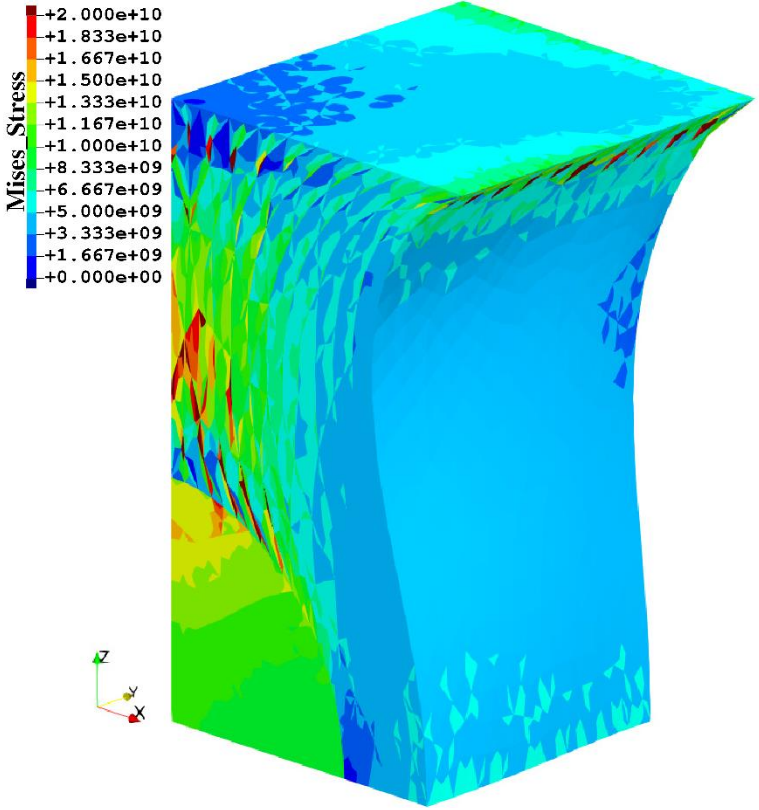
ABAQUS C3D10MH

SelectiveCS-FEM-T10 has good pressure accuracy.

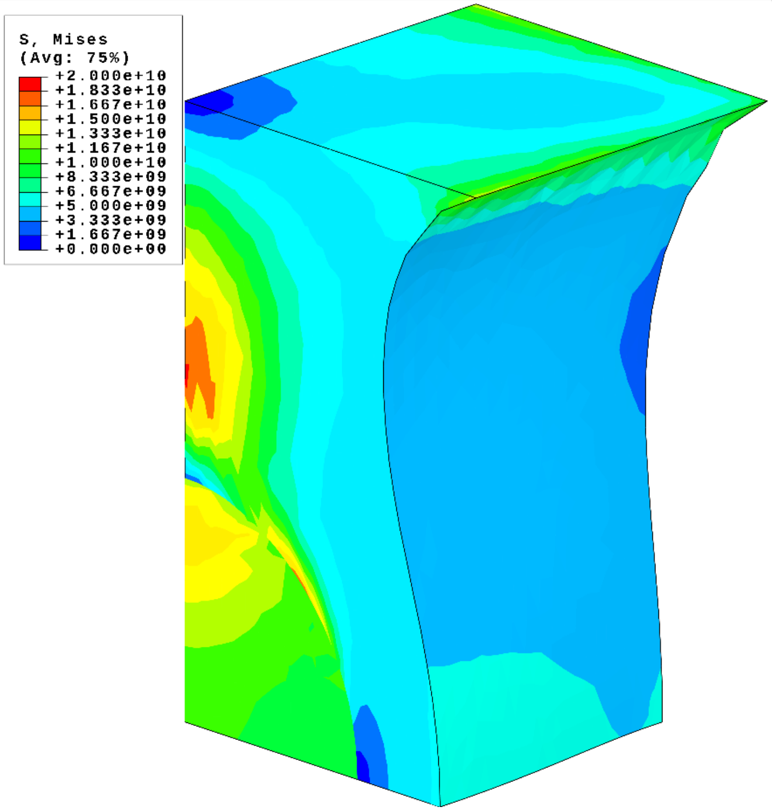
Static Implicit

Tensioning of Rubber-Filler Composite

Comparison of Mises stress dist. at 60% nominal stretch



SelectiveCS-FEM-T10



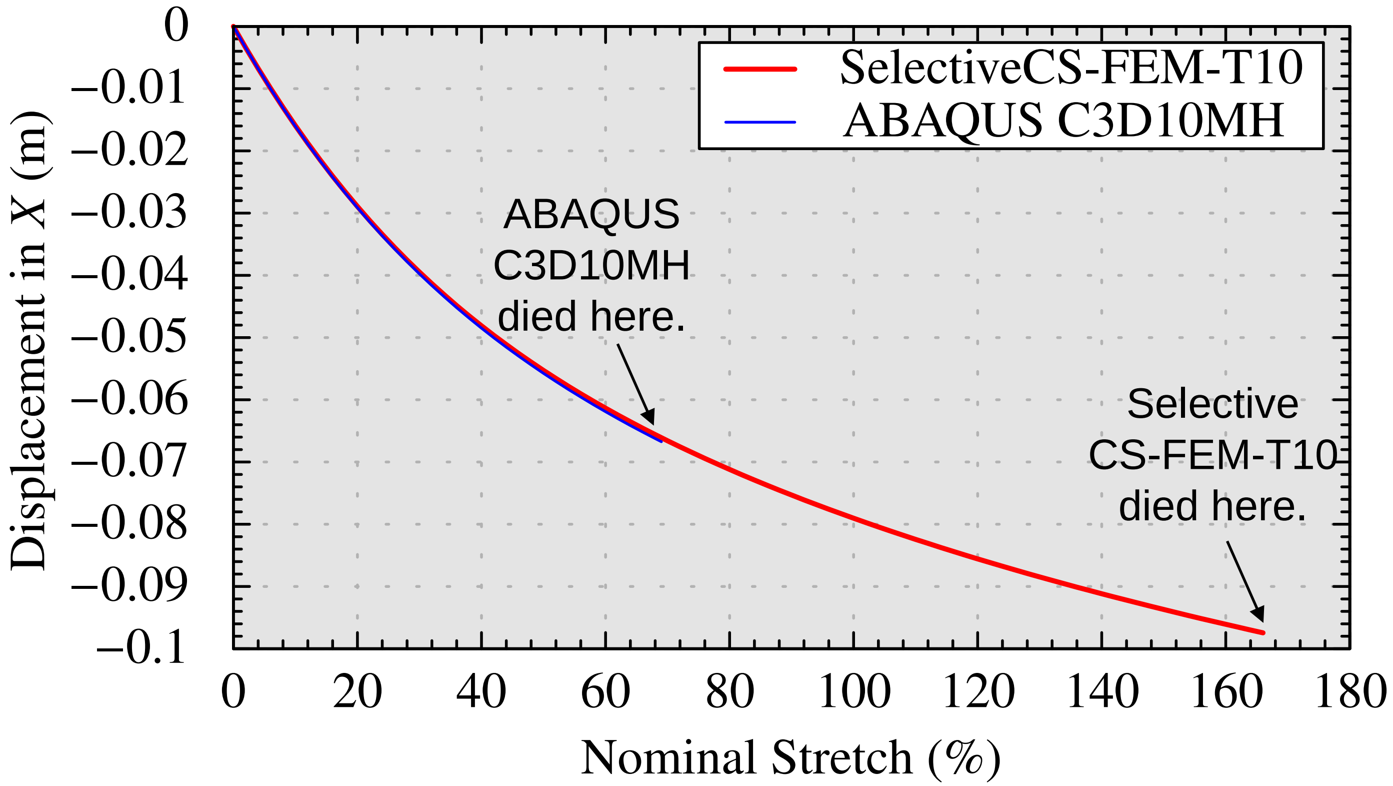
ABAQUS C3D10MH

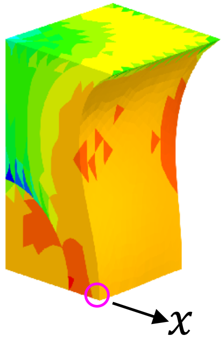
SelectiveCS-FEM-T10 has an issue of Mises stress oscillation, which should be resolved in the future.

Static
Implicit

Tensioning of Rubber-Filler Composite

Comparison of history of u_x at the bottom corner





SelectiveCS-FEM-T10 has enough accuracy in displacement (and force, also) in addition to large deformation robustness.

Deformation Modes of Armadillo

Outline

- Rubber body.
  (Young’s modulus: 5MPa,
  Poisson’s ratio: 0.49)
- Discretized in T10 mesh.
  (about 80,000 nodes
  and 52,000 elements)
- Both soles of the feet are perfectly constrained.
- Modal analysis up to 40 eigen modes.
  (This is not a large deformation analysis.)
- Compared to ABAQUS C3D10MH with the same mesh.

Eigen Mode

Deformation Modes of Armadillo

Eigen modes up to Mode 40 with SelectiveCS-FEM-T10

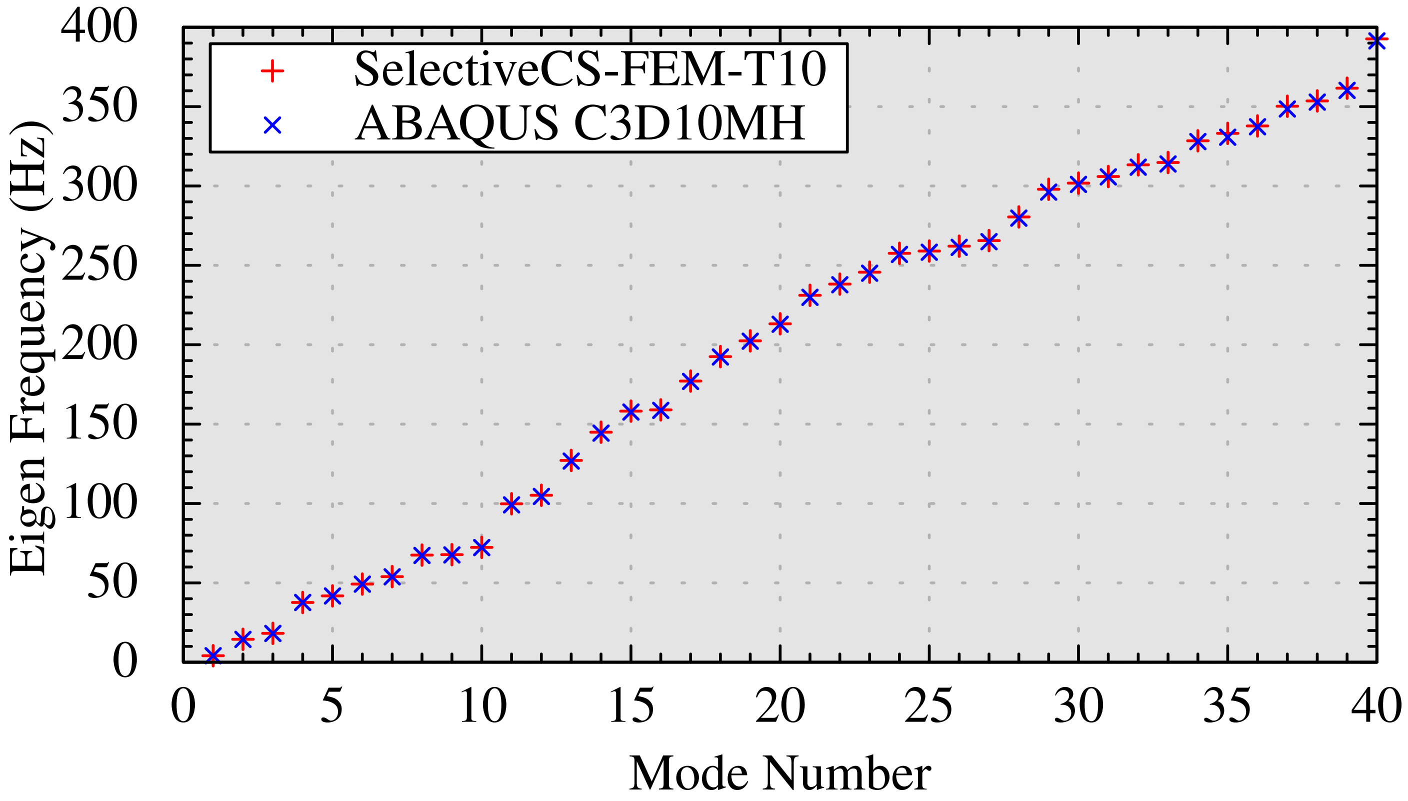
There are no unnatural modes.

SelectiveCS-FEM-T10 has no spurious low-energy modes like hour-glass modes.

Eigen Mode

Deformation Modes of Armadillo

Comparison of eigen frequencies

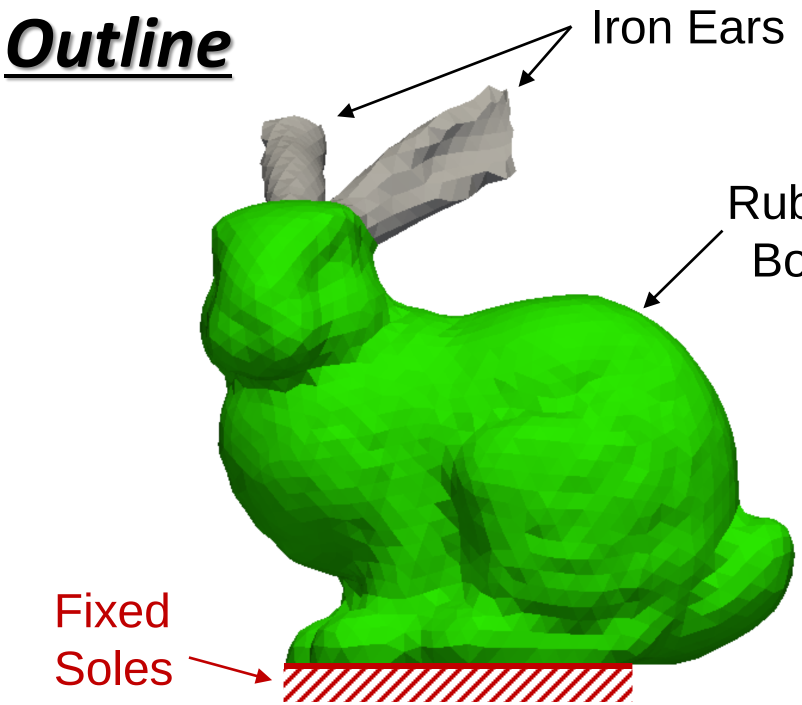


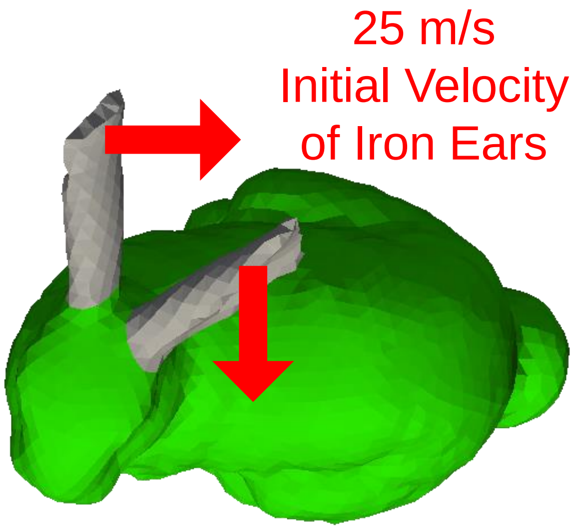
SelectiveCS-FEM-T10 has practical accuracy in modal analyses as ABAQUS C3D10MH; therefore, SelectiveCS-FEM-T10 would be stable in dynamic analyses.

Dynamic
Explicit

Swing of Bunny Ears

Outline





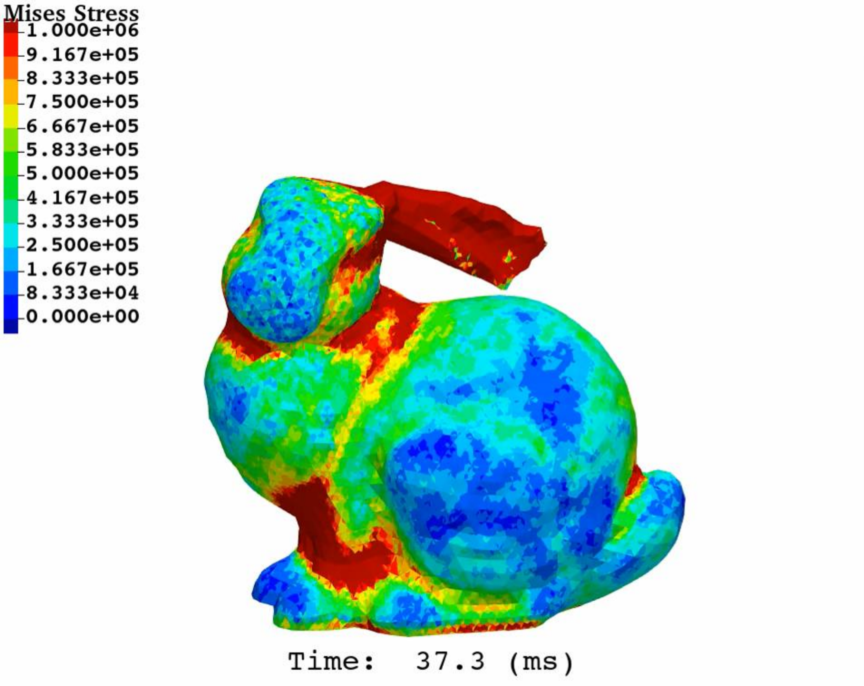
$\Delta t = 0.05\ \mu\text{s}$, which is recommended Δt for C3D10M

- Iron ears: Neo-Hookean, $E_{\text{ini}} = 200\ \text{GPa}$, $\boldsymbol{\nu}_{\textbf{ini}} = \mathbf{0.3}$, $\rho = 7800\ \text{kg/m}^3$.
- Rubber body: Neo-Hookean, $E_{\text{ini}} = 6\ \text{MPa}$, $\boldsymbol{\nu}_{\textbf{ini}} = \mathbf{0.49}$, $\rho = 920\ \text{kg/m}^3$.
- Discretized into T10 mesh. (about 61,000 nodes and 41,000 elements)
- Compared to ABAQUS/Explicit C3D10M (NOT C3D10MH) with the same mesh and Δt.
- Contact is not considered.

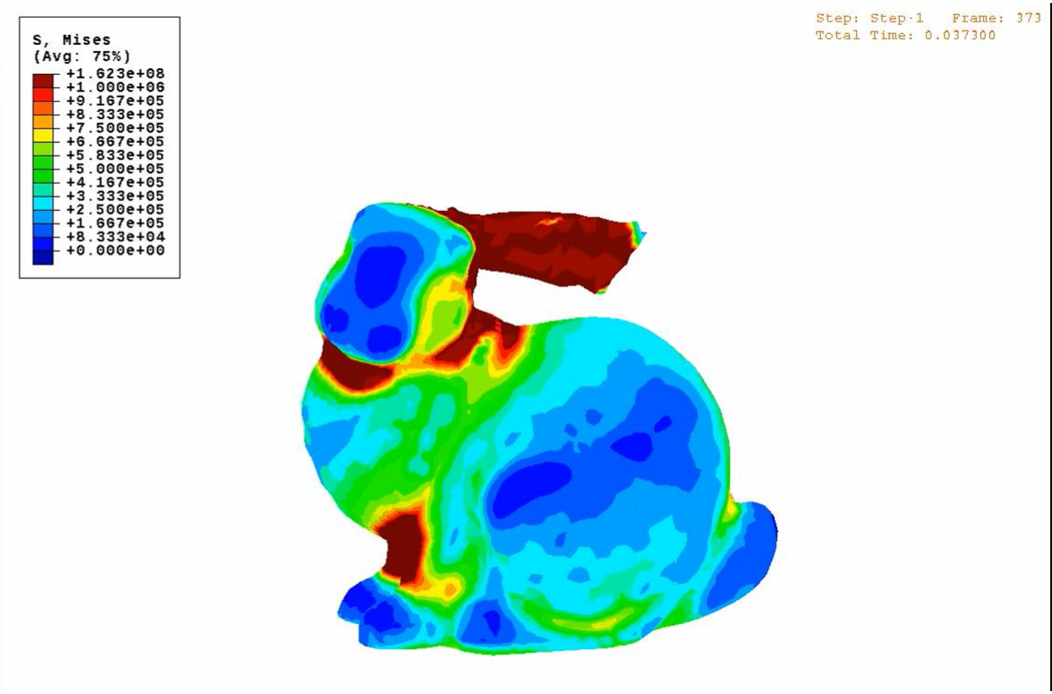
Dynamic
Explicit

Swing of Bunny Ears

Comparison of Mises stress animation



SelectiveCS-FEM-T10



ABAQUS C3D10M

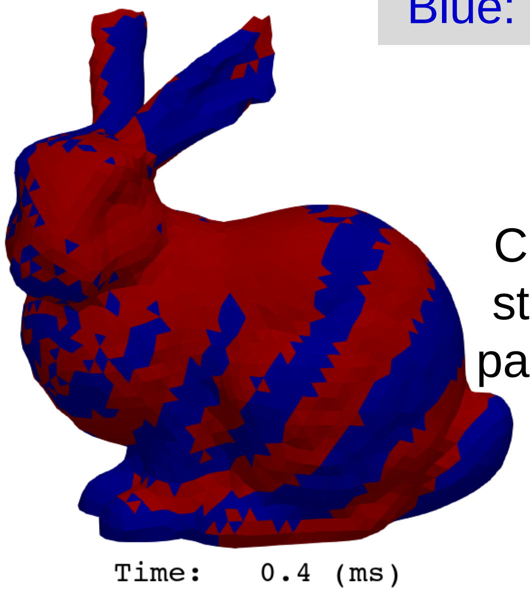
SelectiveCS-FEM-T10 has similar accuracy
in displacement and Mises stress
to ABAQUS C3D10M.

Dynamic
Explicit

Swing of Bunny Ears

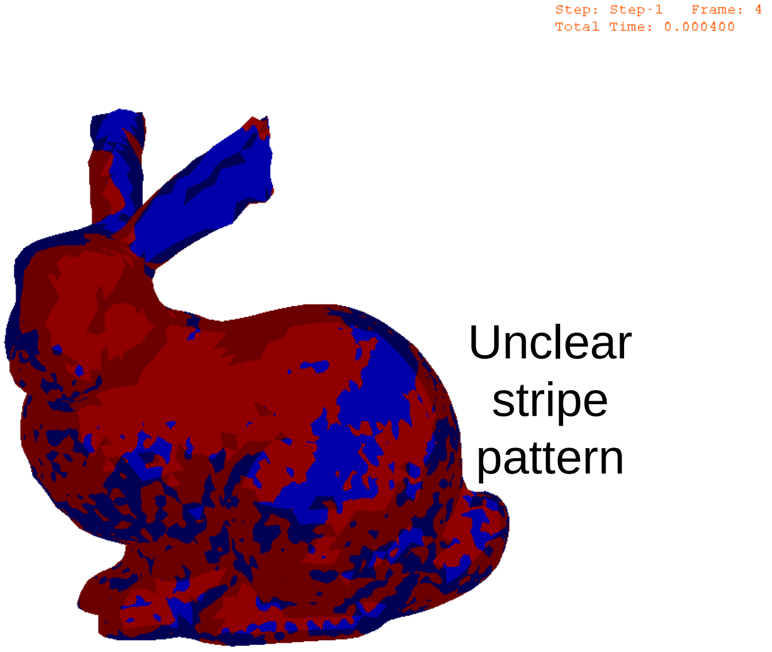
Comparison of pressure sign at *t* =0.4 ms (right after the stat)

Red: positive pressure
Blue: negative pressure



Clear
stripe
pattern

SelectiveCS-FEM-T10



Unclear
stripe
pattern

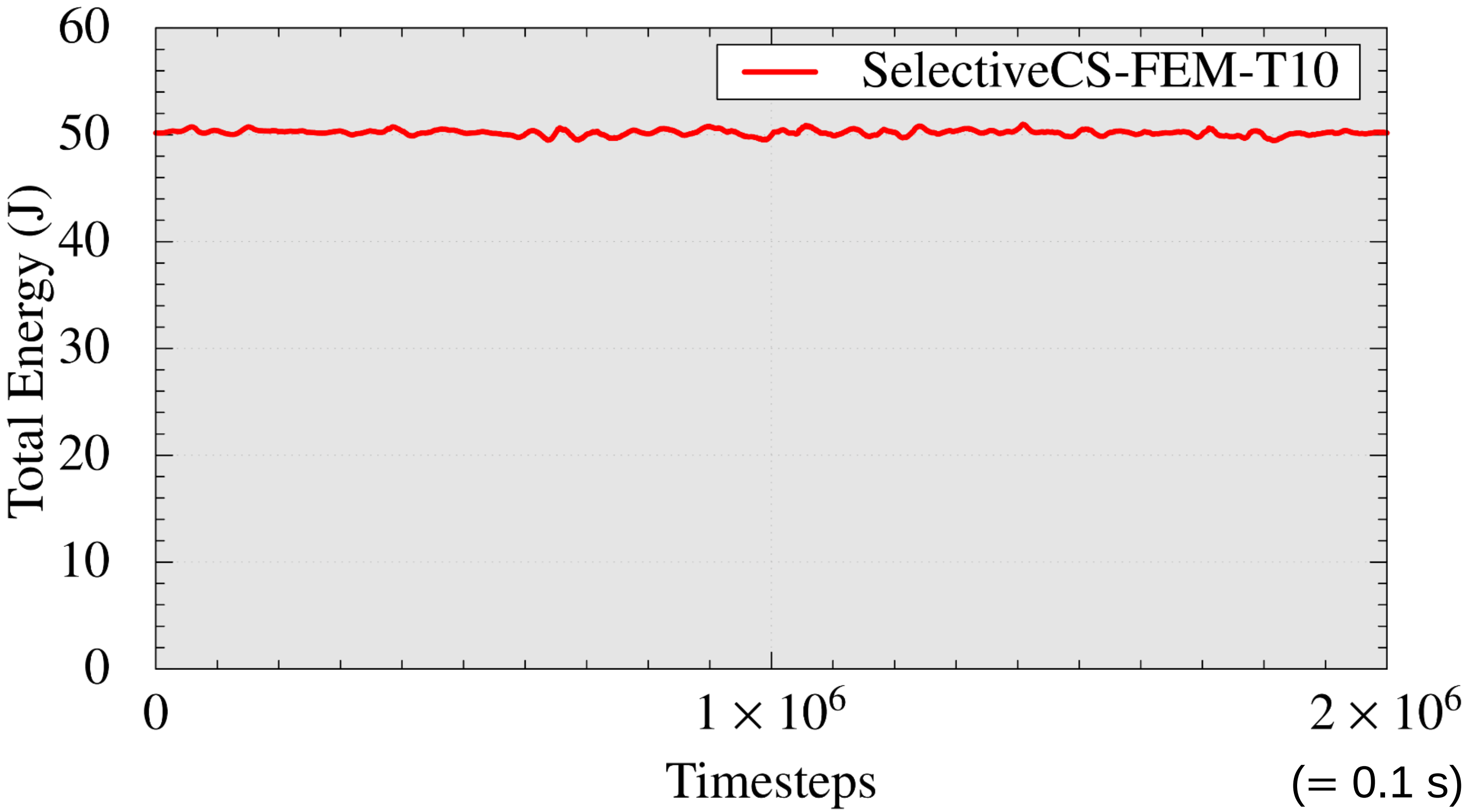
ABAQUS C3D10M

SelectiveCS-FEM-T10 seems to calculate the initial pressure wave propagation more correctly than ABAQUS C3D10M.

Dynamic Explicit

Swing of Bunny Ears

Timestep-history of total energy (= kinetic + strain)



SelectiveCS-FEM-T10 has enough energetic stability in dynamic analysis.

Summary

Summary

Summary

- A new S-FEM was proposed, which is called **SelectiveCS-FEM-T10**:
  - **More robust to severe large deformation** than the conventional T10s.
  - **Enough accuracy for practical use** as compared to ABAQUS's best T10.
  - Slower than conventional T10s only in dynamic explicit analysis.
- More severe large deformation dynamic analyses should be performed for evaluation.

Take-home message

If you are interested in large deformation analysis,

please consider implementing SelectiveCS-FEM-T10 to your FE code.

It's supremely useful & easy to code!!

Thank you for your kind attention!

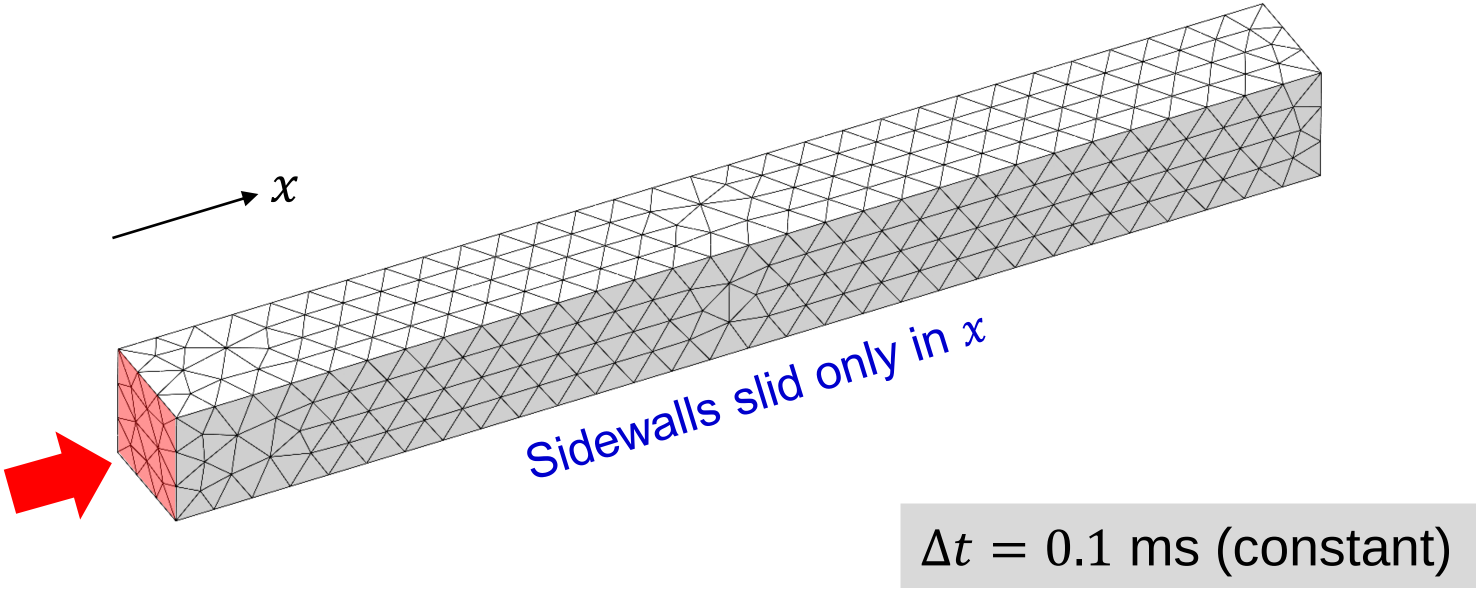
Appendix

Computational Cost

- In static, modal, dynamic implicit analyses:
  - CPU time: almost the same as the standard T10.
    ∵ Time to solve the matrix equation (i.e., $[K]\{u\} = \{f\}$) dominates the CPU time.
  - Memory size: several times larger than the standard T10.
    ∵ Memory to store $\boldsymbol{F}$ and $\boldsymbol{\sigma}$ at Gauss points occupies a main part of the memory size.
    SelectiveCS-FEM-T10 has 34 edges. Standard T10 has 4 Gauss Points.
- In dynamic explicit analysis:
  - CPU time: several times longer than the standard T10.
    ∵ Time to build internal force vector $\{f^{\mathrm{int}}\}$ occupies a main part of the CPU time.
  - Memory size: several times larger than the standard T10.
    ∵ Same reason above.

Trade-off between robustness and costs

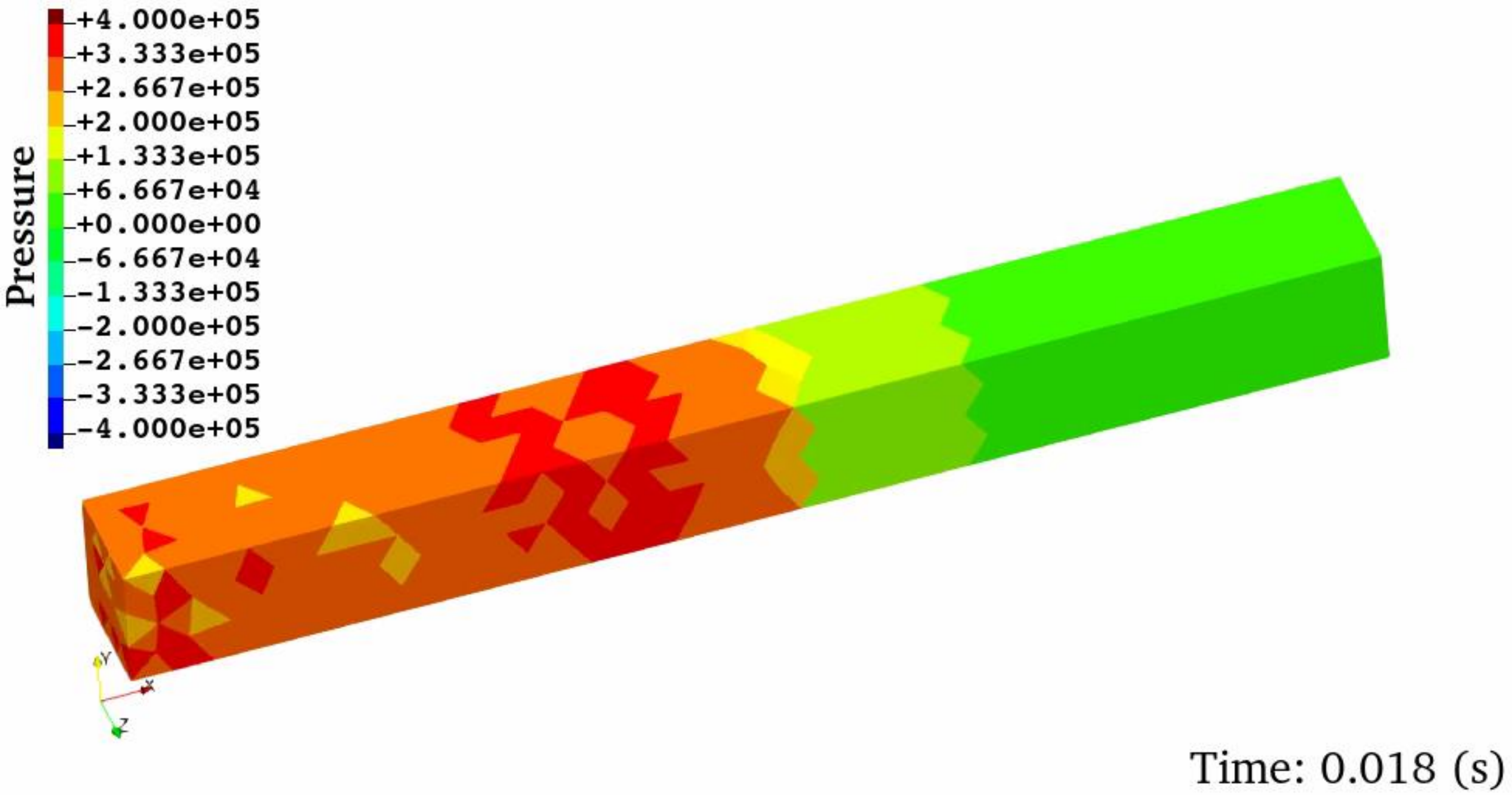
Wave Propagation in a Long Bar



- 10 m × 1 m × 1 m.
- Neo-Hookean, $E_{\mathrm{ini}} = 1\ \mathrm{MPa}$, $\boldsymbol{\nu_{\mathrm{ini}} = 0.49}$, $\rho = 920\ \mathrm{kg/m^3}$.
- Lateral confinement on the sidewalls.
- Discretized into T10 mesh. (about 4,000 nodes and 2,000 elements)
- Compared to analytical solution.

Wave Propagation in a Long Bar

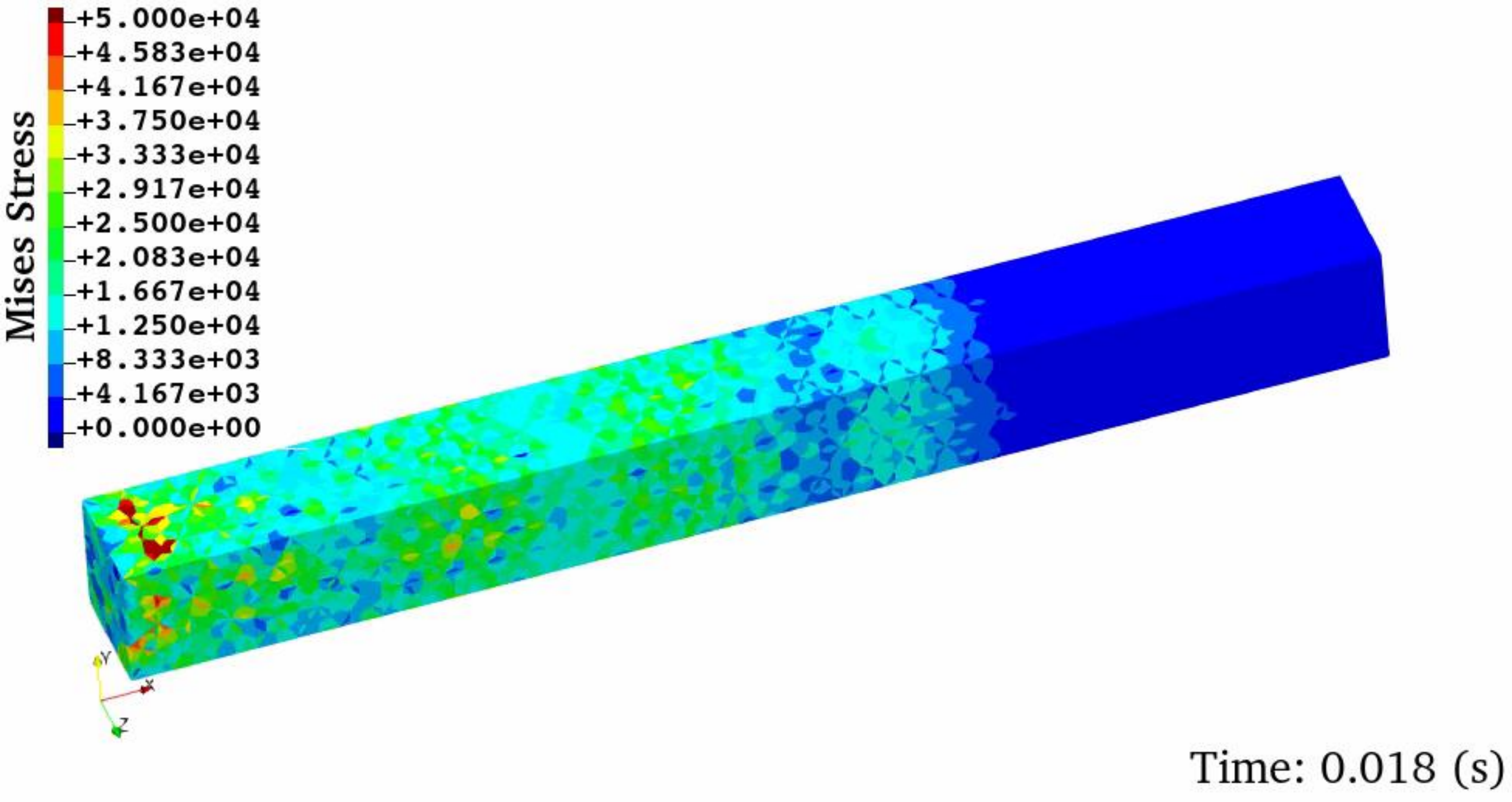
Animation of pressure



SelectiveCS-FEM-T10 seem to has good pressure accuracy.

Wave Propagation in a Long Bar

Animation of Mises stress

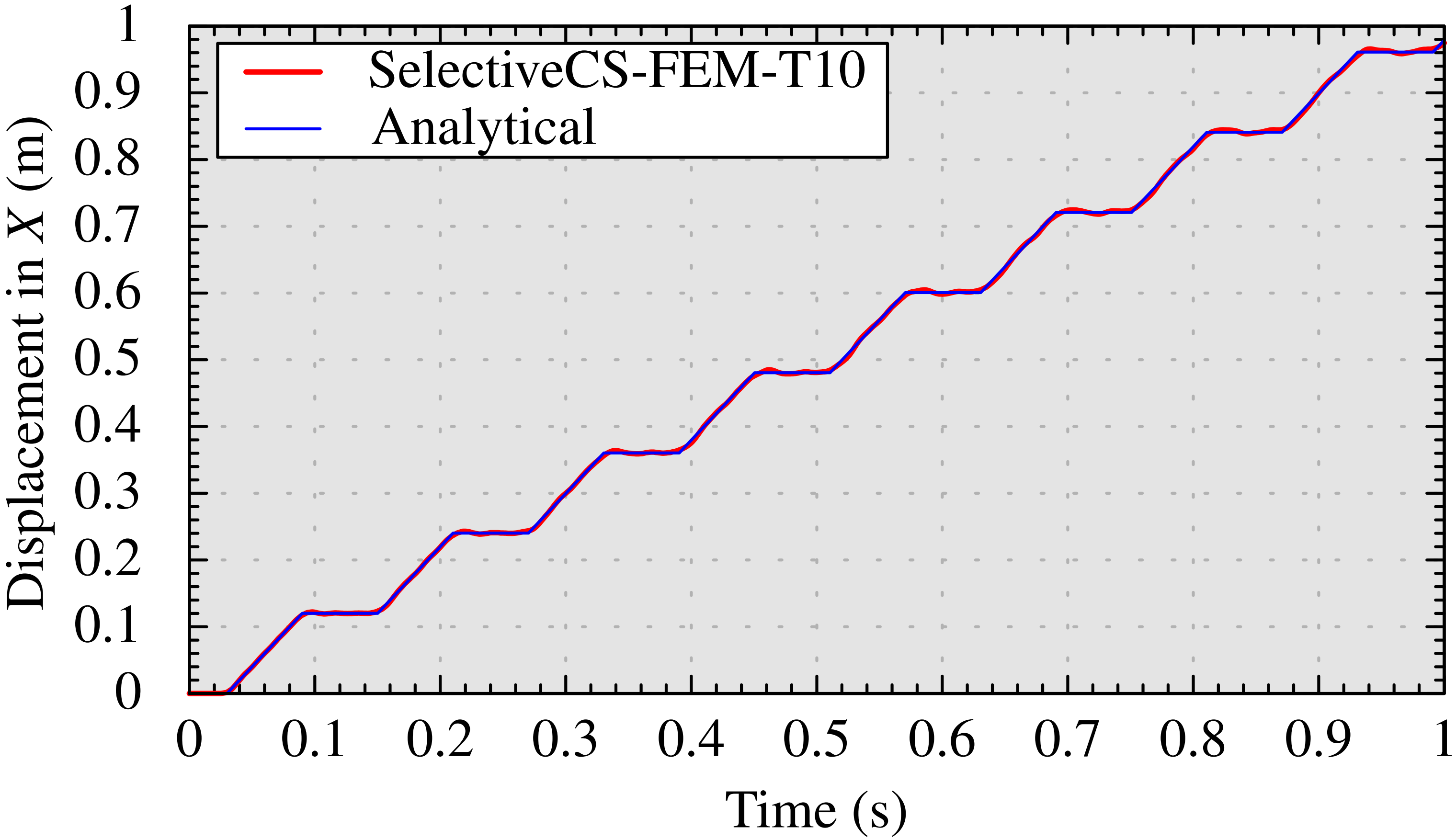


We need more careful investigation.

Dynamic Explicit

Wave Propagation in a Long Bar

Time-history of displacement at the right end

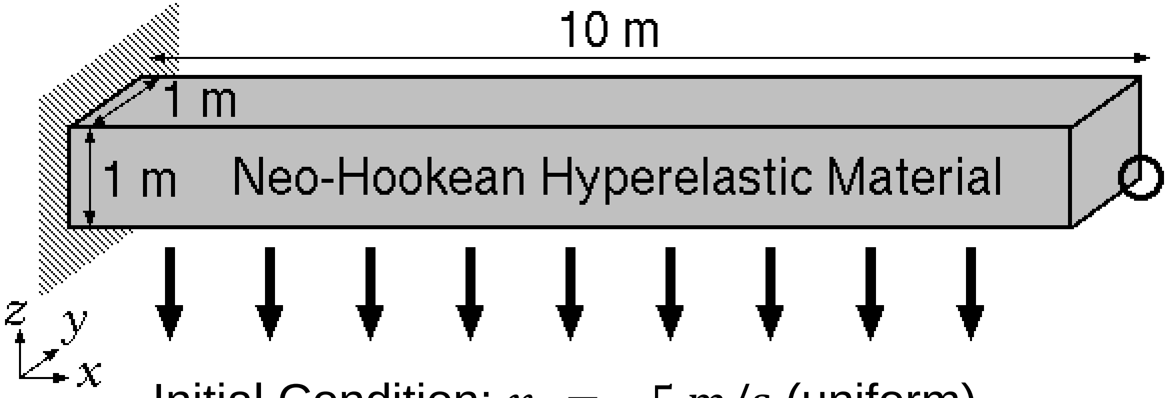


Analytical pressure wave speed:

$$c = \sqrt{\frac{\lambda + 2\mu}{\rho}}$$

SelectiveCS-FEM-T10 has enough accuracy in 1D pressure wave propagation analysis.

Dynamic Bending of Cantilever



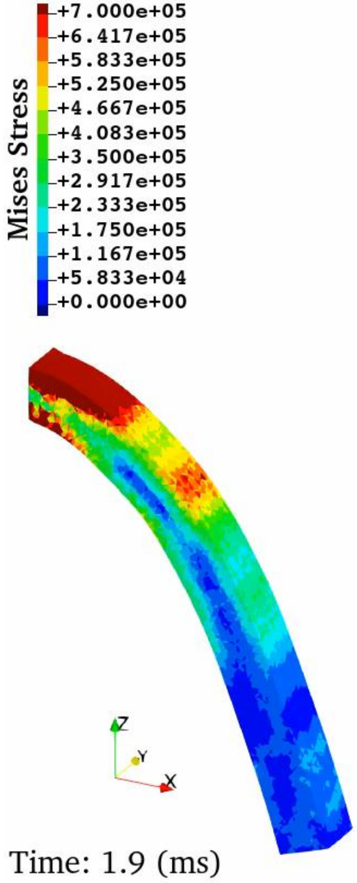
Initial Condition: $v_z = -5\ \mathrm{m/s}$ (uniform)

- Neo-Hookean, $E_{\mathrm{ini}} = 6.0\ \mathrm{MPa}$, $\nu_{\mathrm{ini}} = 0.49$, $\rho = 920\ \mathrm{kg/m^3}$
- Initial velocity: $v_z = -5\ \mathrm{m/s}$ for all nodes of cantilever
- Discretized into T10 mesh. (about 4,000 nodes and 2,000 elements)
- Compared to ABAQUS/Explicit C3D10M (NOT C3D10MH) with the same mesh and $\Delta t\ (= 0.1\ \mathrm{ms})$.

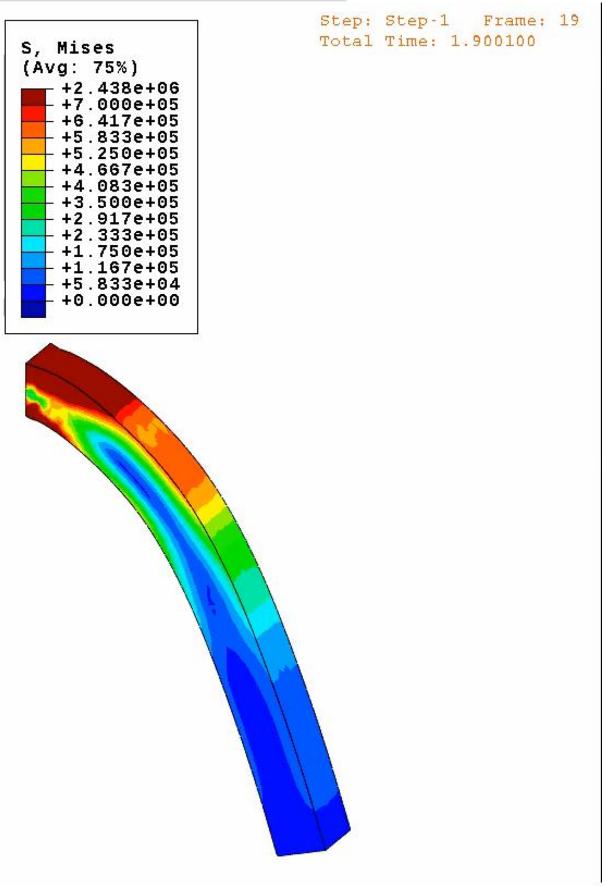
Dynamic Explicit

Dynamic Bending of Cantilever

Comparison of animation of Mises stress



SelectiveCS-FEM-T10/Explicit



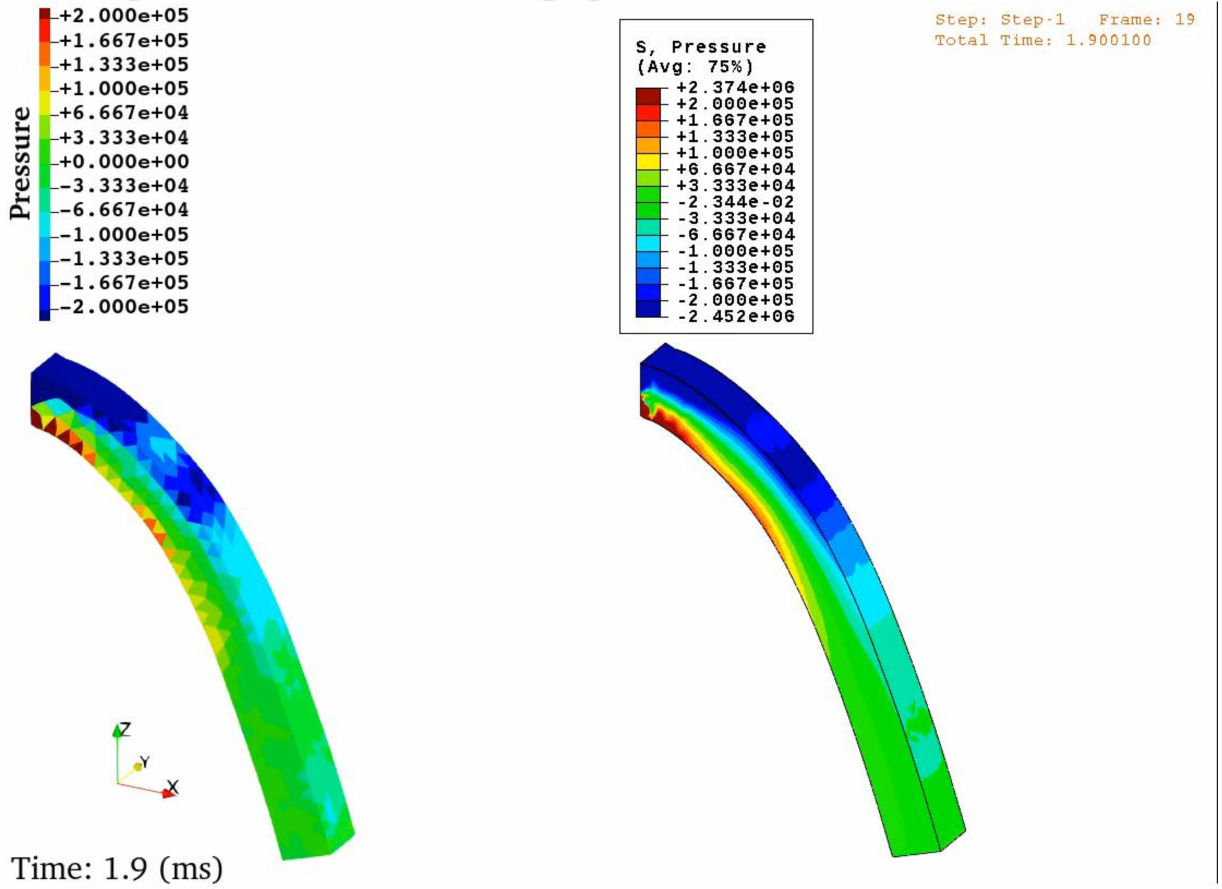
ABAQUS/Explicit C3D10M

SelectiveCS-FEM-T10 has similar accuracy
in displacement and Mises stress to ABAQUS C3D10M.

Dynamic Explicit

Dynamic Bending of Cantilever

Comparison of animation of pressure



SelectiveCS-FEM-T10/Explicit

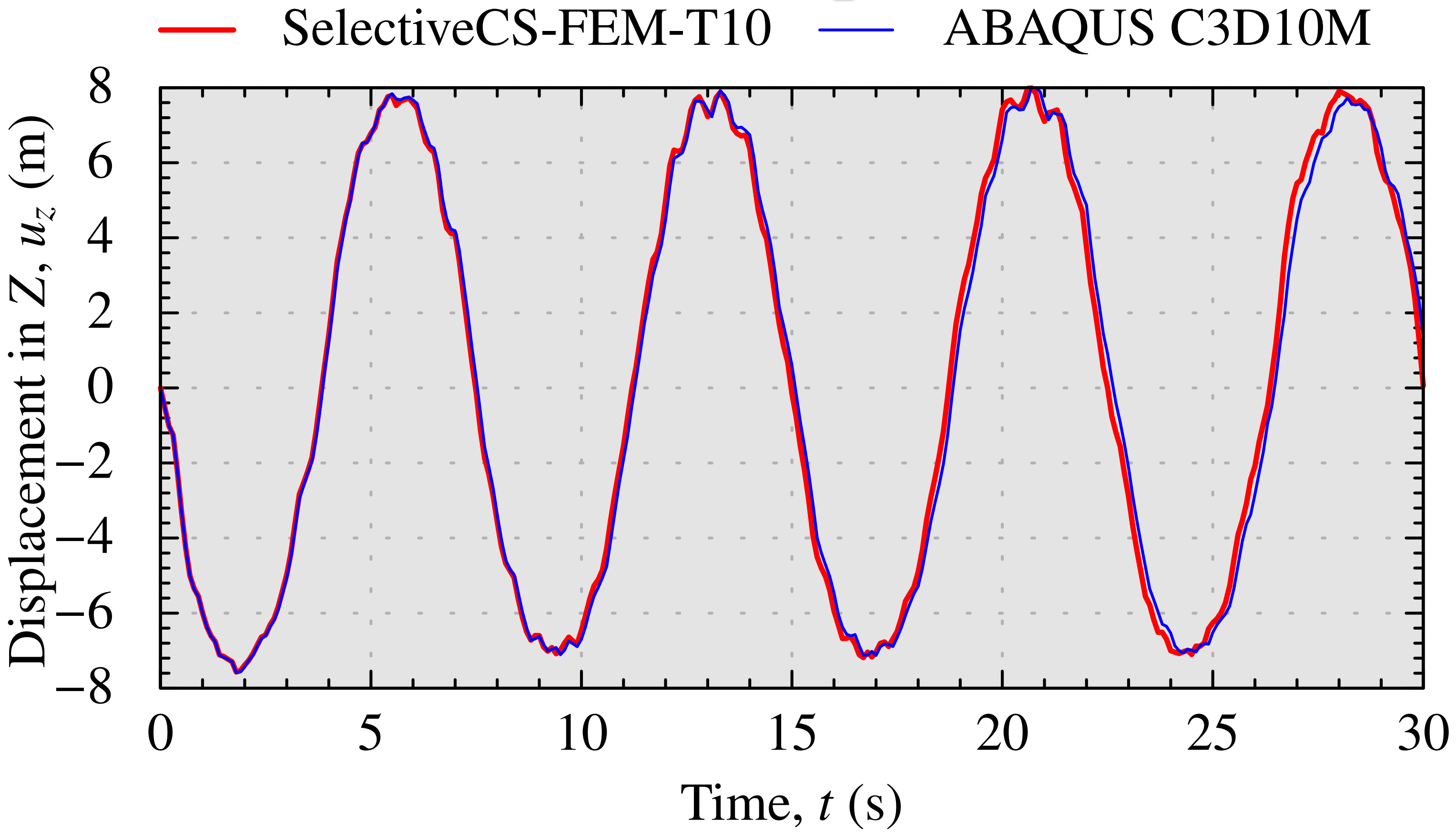
ABAQUS/Explicit C3D10M

SelectiveCS-FEM-T10 has similar accuracy
in displacement and pressure to ABAQUS C3D10M.

Dynamic Explicit

Dynamic Bending of Cantilever

Comparison of time-history of u_z at the tip node



SelectiveCS-FEM-T10 has similar accuracy in displacement to ABAQUS C3D10M.