A large deformation tetrahedral smoothed finite element formulation for nearly incompressible solids based on the strain smoothed element (SSE) technique

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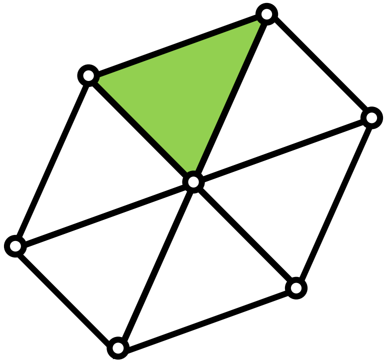
What is S-FEM?

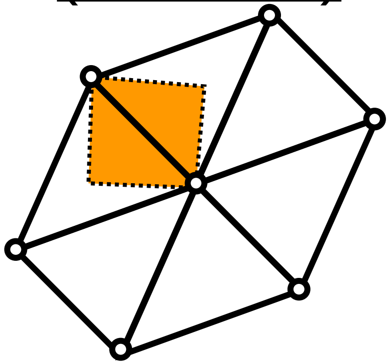
- ***Smoothed*** finite element method (**S-FEM**) is a relatively new FE formulation proposed by Prof. G. R. Liu in 2006.
- S-FEM is one of the **strain smoothing** techniques.
- There are several types of classical S-FEMs depending on the **domains of strain smoothing**.

For example, in a 2D triangular mesh:

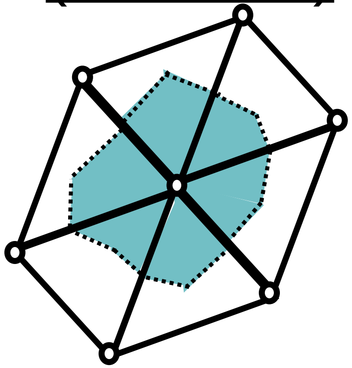
Standard FEM



Edge-based S-FEM (ES-FEM)



Node-based S-FEM (NS-FEM)



What are the major benefits of S-FEM?

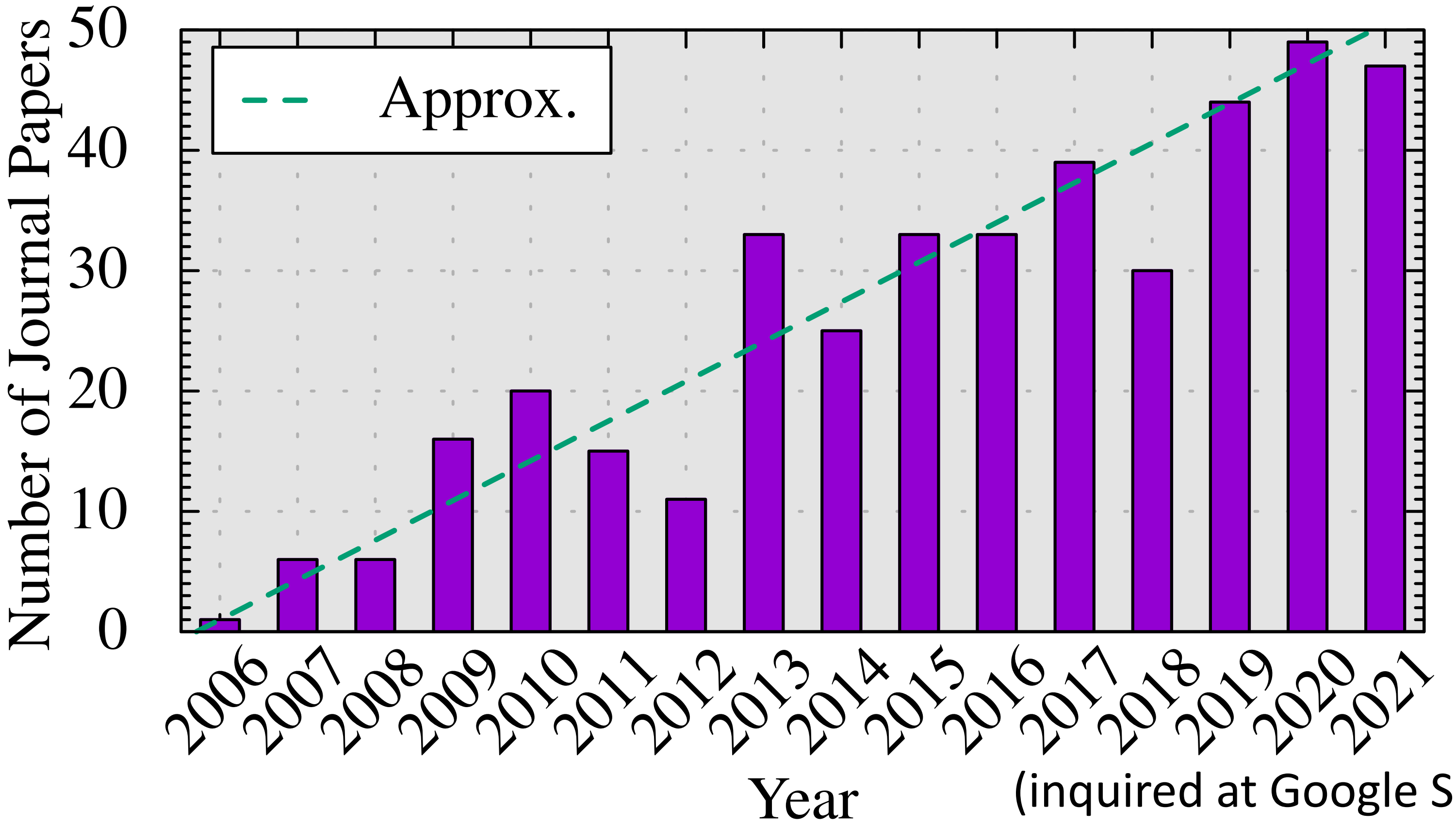
1. **Super-linear mesh convergence rate.**
   (Almost same rate as 2nd-order elements with T4 mesh.)
2. **Shear locking free with ES-FEM.**
   (Excellent accuracy with T4 mesh.)
3. **Little accuracy loss with skewed meshes.**
   (No problem with complex geometry or severe deformation.)

T4: 4-node Tetrahedra

S-FEM is a powerful method
suitable for practical industrial applications.

How popular is S-FEM?

Number of journal papers whose **title** contains "**smoothed finite element**":



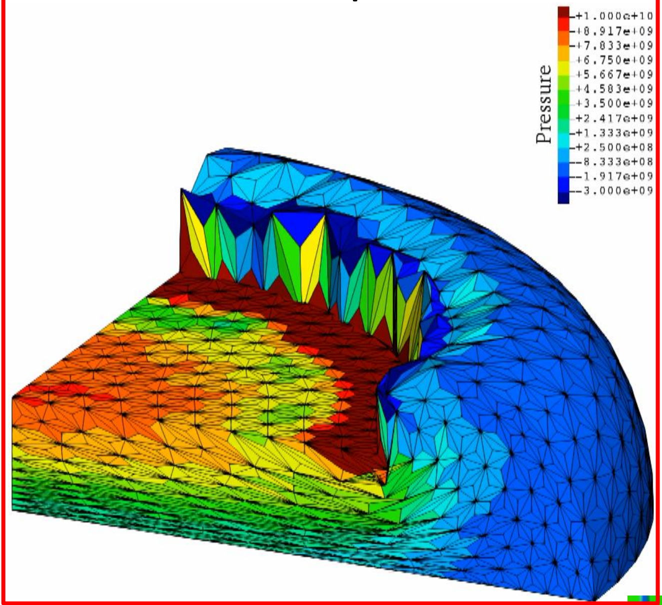
(inquired at Google Scholar)

The attraction of S-FEM is expanding continuously.

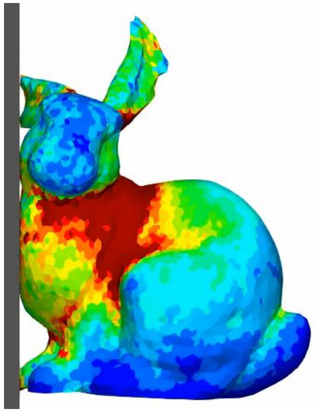
Applications of S-FEM in Our Lab

- **Large deformation solid mechanics (still in academic)**

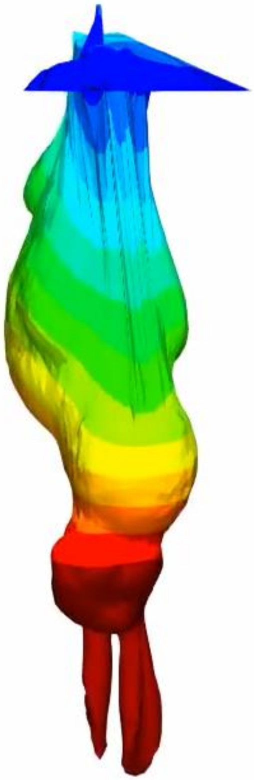
Static Implicit



Dynamic Explicit

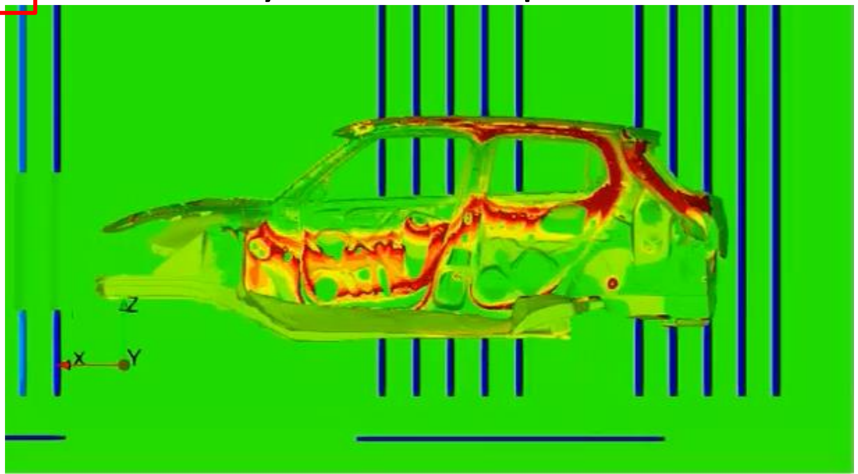


Viscous Implicit



- **Electrostatic**
  (already in practice)

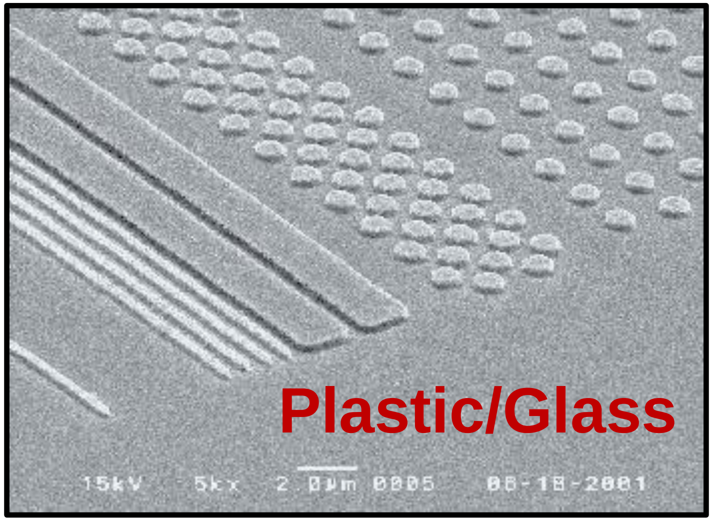
Carbody Electro-Deposition



Motivation

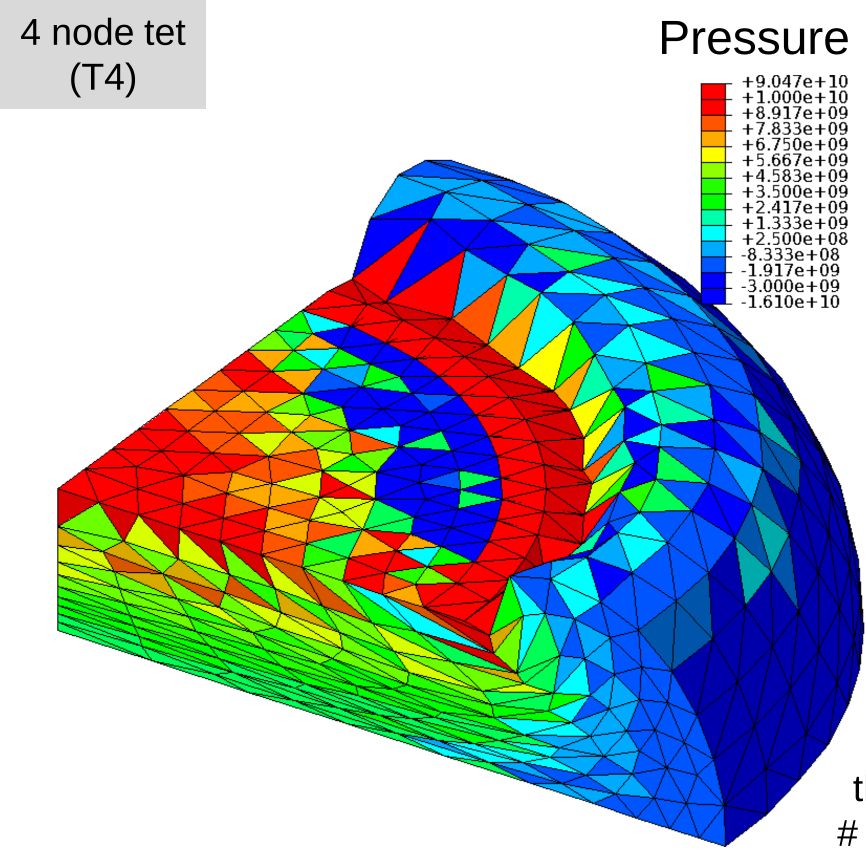
What we want to do:

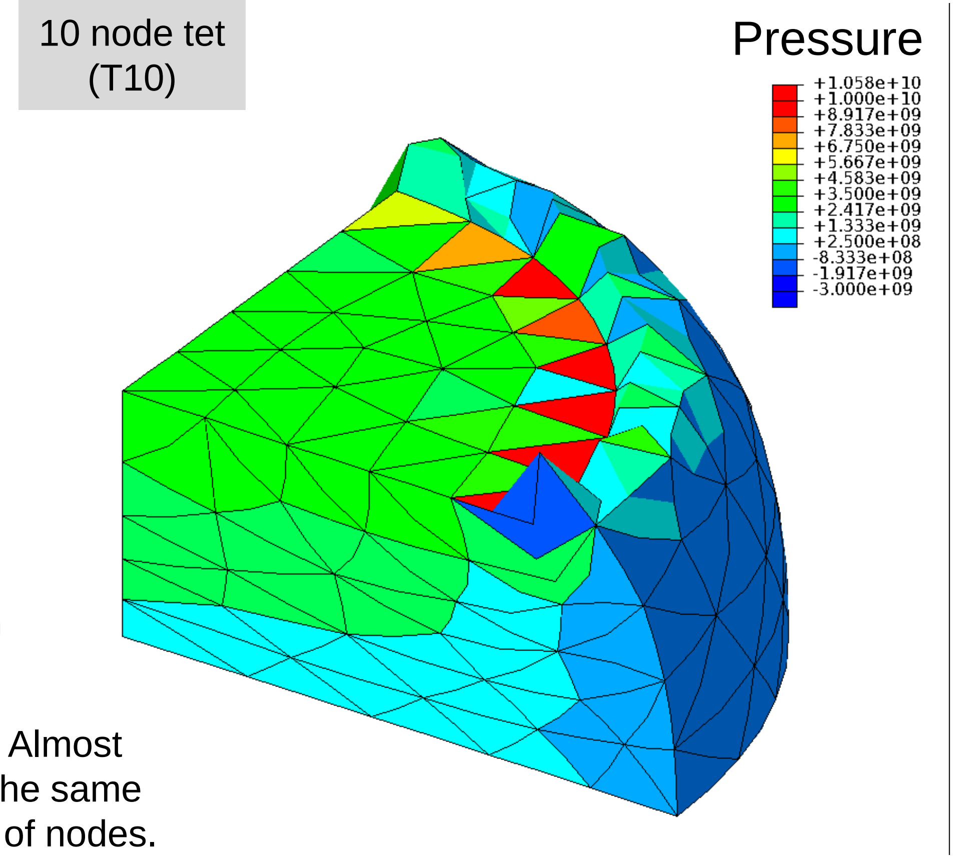
- Solve **severe large deformation** analyses accurately and robustly.
- Treat complex geometries with **tetrahedral meshes**.
- Consider **nearly incompressible materials** $(\nu \simeq 0.5)$.
- Support **contact** problems.
- Handle **auto re-meshing**.



Issues in Conventional FE (ABAQUS)

Neo-Hookean ***hyperelastic*** body with $\nu_{\text{ini}} = 0.49$


4 node tet (T4)


10 node tet (T10)

Almost
the same
# of nodes.

<u>ABQUS C3D4H</u>

- ✓ No volumetric locking.
- ✗ Pressure checkerboarding.
- ✗ Shear locking & Corner locking.

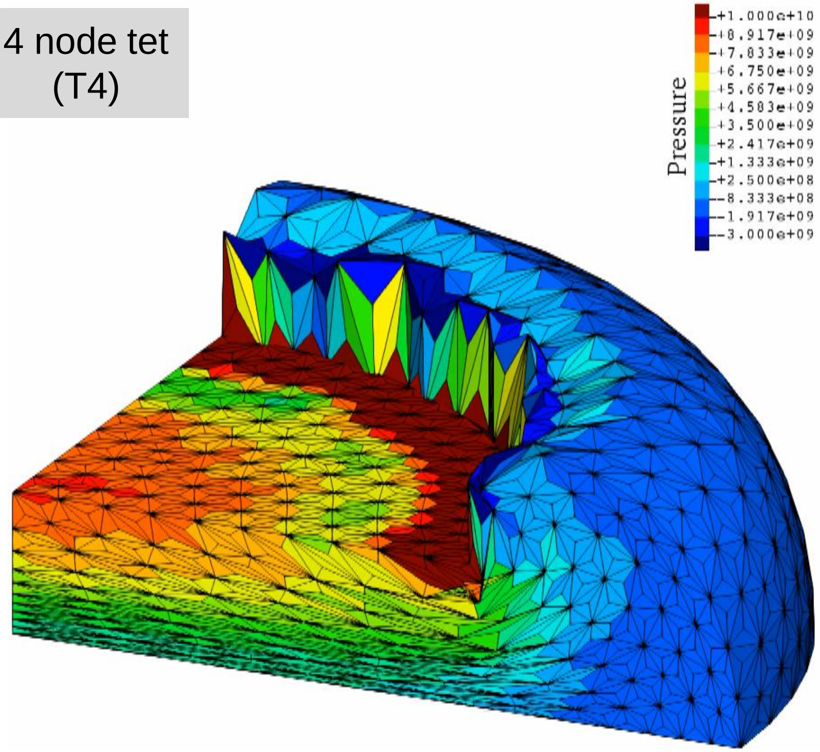
<u>ABAQUS C3D10MH</u>

- ✓ No shear/volumetric locking.
- ✗ Short lasting (weak to severe deformation).
- ✗ Low interpolation accuracy.

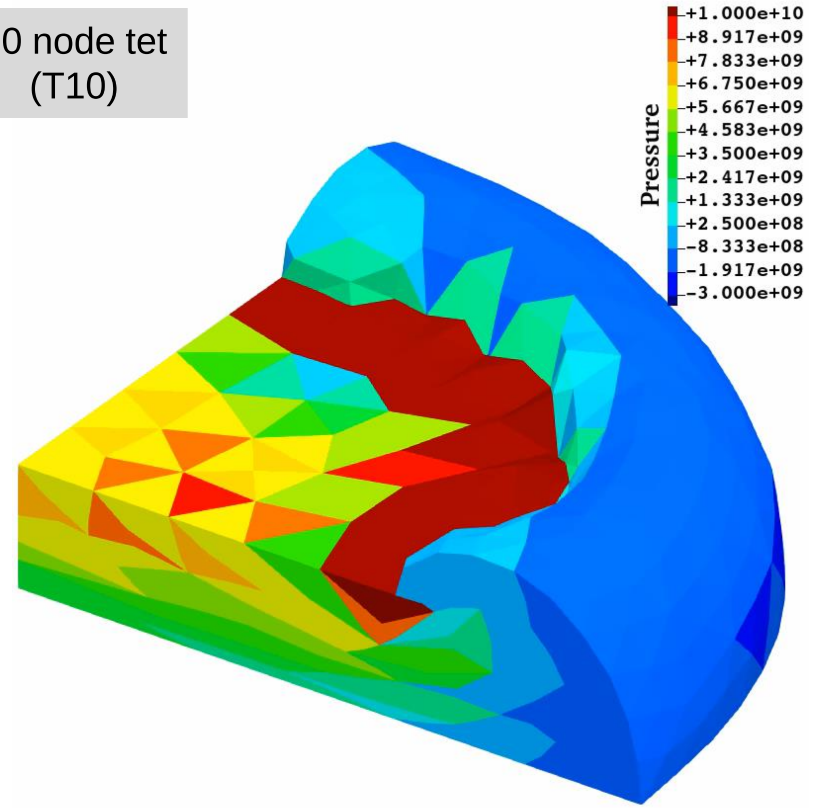
Our Approach using S-FEM

Neo-Hookean ***hyperelastic*** body with $\nu_{\mathrm{ini}} = 0.49$

4 node tet
(T4)



10 node tet
(T10)



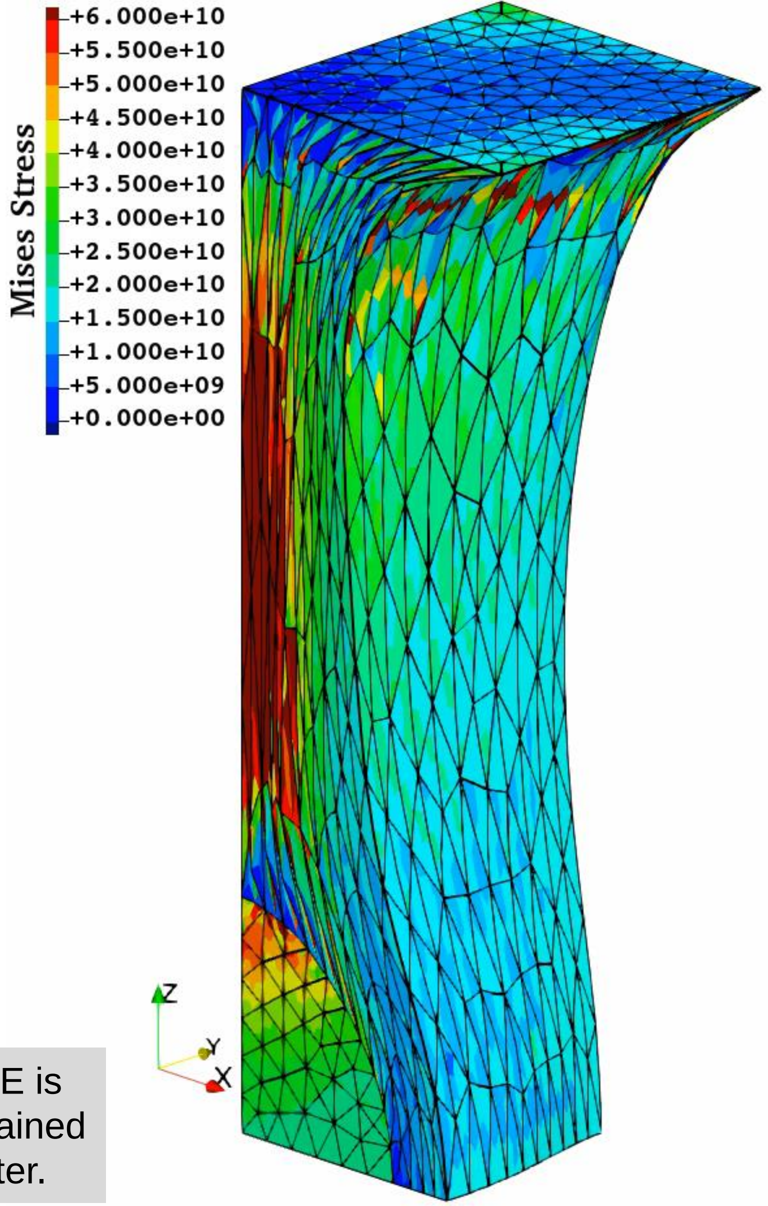
F-barES-FEM-T4 (2017)

✓ No shear/volumetric locking.
✓ Less pressure checkerboarding.
✓ Less corner locking. Long lasting.
✗ Long CPU time. Incompatible w/ FE.

SelectiveCS-FEM-T10 (2021)

✓ No shear/volumetric locking.
✓ Less pressure checkerboarding.
✓ Less corner locking. Long lasting.
✓ Same CPU time. Compatible w/ FE.

Issue in SelectiveCS-FEM-T10



Deviatoric stress oscillation

is observed in some cases…

e.g.) Tension of filler-rubber composite:

- A hard iron filler (1/8 sphere)
- Poisson’s ratio of rubber (soft) is 0.49.

✓ Excellent in robustness.
Up to 200% stretch!!

✓ Excellent in nodal displacement/
force and pressure accuracy.

✗ Poor in deviatoric stress accuracy.
Due to the lack of deviatoric strain smoothing?

Can we resolve this issue
by using the approach of
Strain Smoothed Element (SSE),
which adopts multi-smoothing?

SSE is explained later.

Objective

Development of a new T10 element formulation
"SelectiveCSSE-T10"
introducing the concept of SSE
to reduce the deviatoric stress oscillation.

Table of contents:

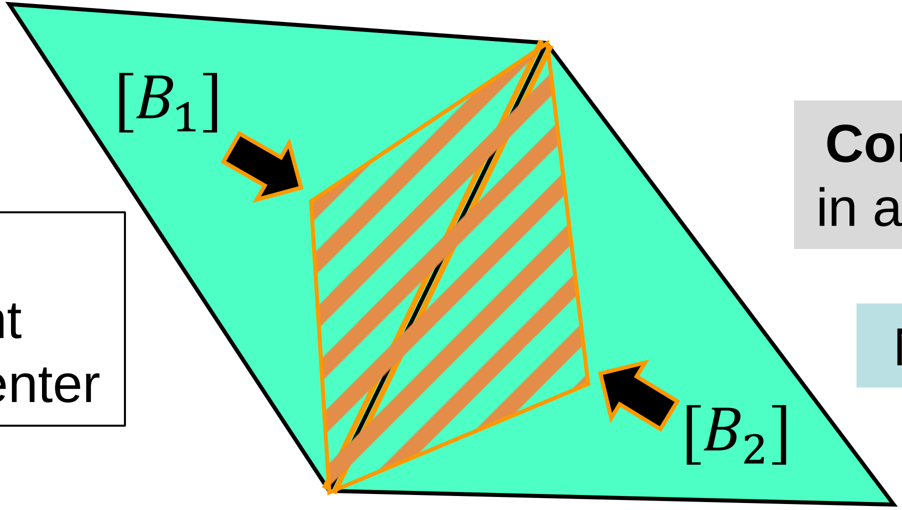
- Method: brief of ES-FEM, SSE, SelectiveCS-FEM-T10, and SelectiveCSSE-T10
- Results: an example of analysis
- Summary

Method

Brief of ES-FEM-T3

Let us consider two 3-node triangular elements in 2D, for simplicity.

- Calculate $[B](=\mathrm{d}\boldsymbol{N}/\mathrm{d}\boldsymbol{x})$ at each element as usual.
- Distribute each $[B]$ to the connecting edge with an area weight and build $[{}^{\mathrm{Edge}}B]$.
- Calculate deformation gradient $(\boldsymbol{F})$, Cauchy stress $(\boldsymbol{\sigma})$ and nodal internal force $\{f^{\mathrm{int}}\}$ in each edge smoothing domain with $[{}^{\mathrm{Edge}}B]$.



As if putting
a Gauss point
on each edge center

Constant stress/strain
in a smoothing domain.

No shear locking.

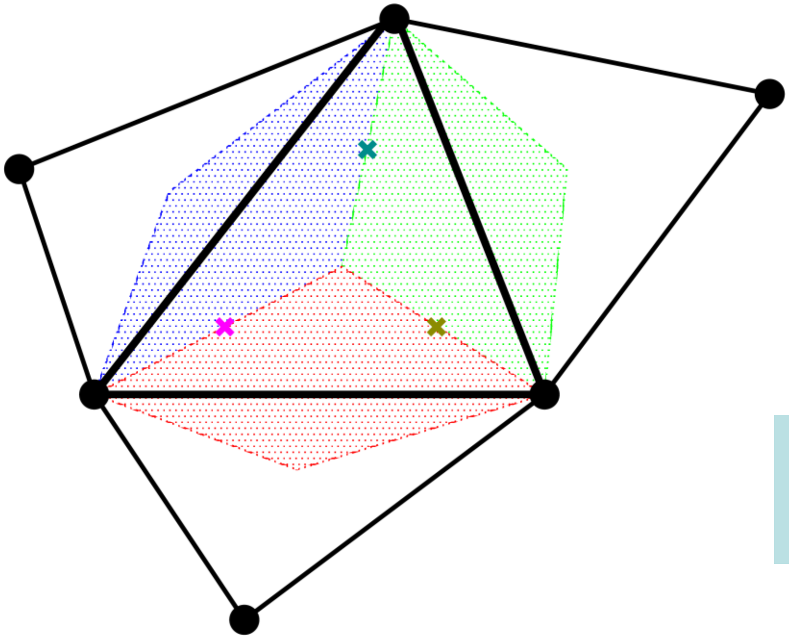
$[{}^{\mathrm{Edge}}B]$

↳ ${}^{\mathrm{Edge}}\boldsymbol{F}$, ${}^{\mathrm{Edge}}\boldsymbol{\sigma}$, $\{{}^{\mathrm{Edge}}f^{\mathrm{int}}\}$ etc.

Brief of SSE-T3

Let us consider four 3-node triangular elements in 2D, for simplicity.

- Calculate $[B](= \mathrm{d}\boldsymbol{N}/\mathrm{d}\boldsymbol{x})$ at each element as usual.
- Calculate $[{}^{\mathrm{Edge}}B]$ at each edge in the same way as ES-FEM-T3.
- Distribute each $[{}^{\mathrm{Edge}}B]$ to the three Gauss points with an equal weight and build $[{}^{\mathrm{Gaus}}B]$.
- Calculate $\boldsymbol{F}$, $\boldsymbol{\sigma}$, $\{f^{\mathrm{int}}\}$ etc. with $[{}^{\mathrm{Gaus}}B]$s in the same way as FEM-T6.



Strain smoothing is performed twice before evaluating the stress/strain.

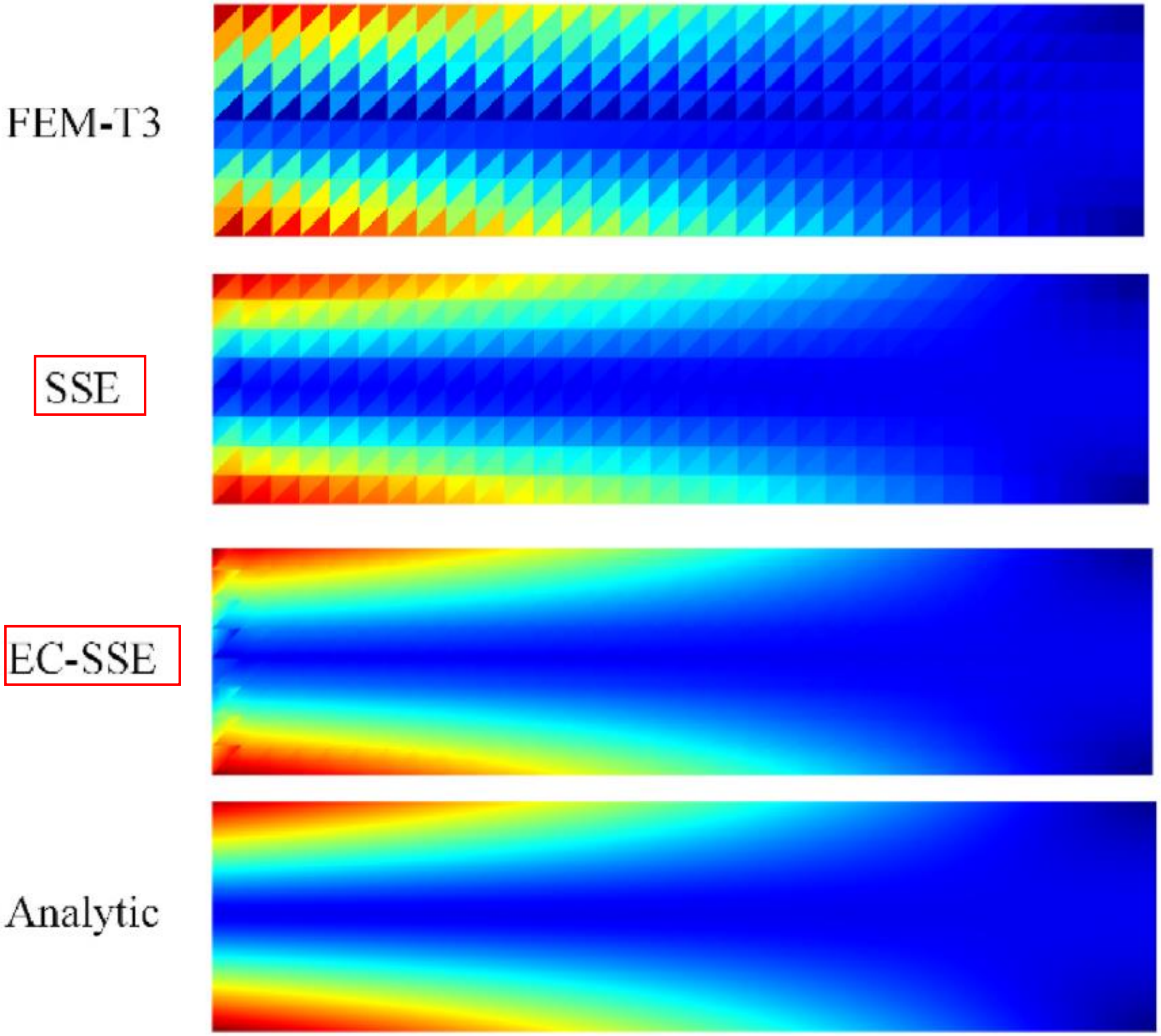
Linear stress/strain in an element.

No shear locking.

Faster rate of mesh convergence.

Performance of SSE-T3

Comparison of Mises stress dist. in cantilever bending analysis



- Step-like stress dist. (poor).
- Shear locking.

- Linear stress dist. (good).
- No shear locking.

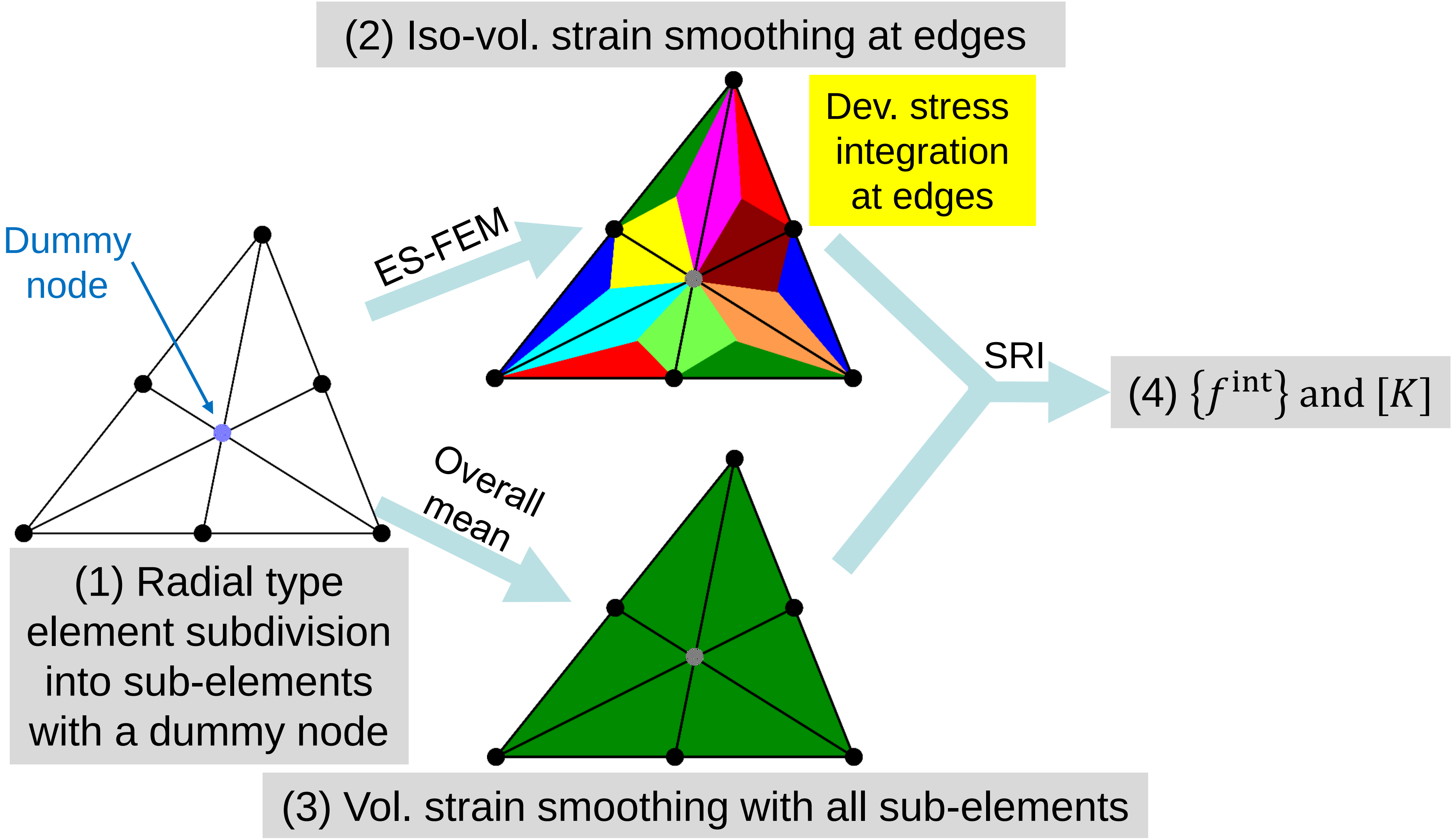
- Linear stress dist. (fine).
- No shear locking.

But,
- SSE has volumetric locking.
- SSE cannot be implemented as an element of standard FE codes.

T. Jinsong *et al*., Euro. J. Mech. /A, v95, 2022.

Brief of SelectiveCS-FEM-T10 (Old)

Explanation in 2D (6-node triangular element), for simplicity



(1) Radial type element subdivision into sub-elements with a dummy node

(2) Iso-vol. strain smoothing at edges

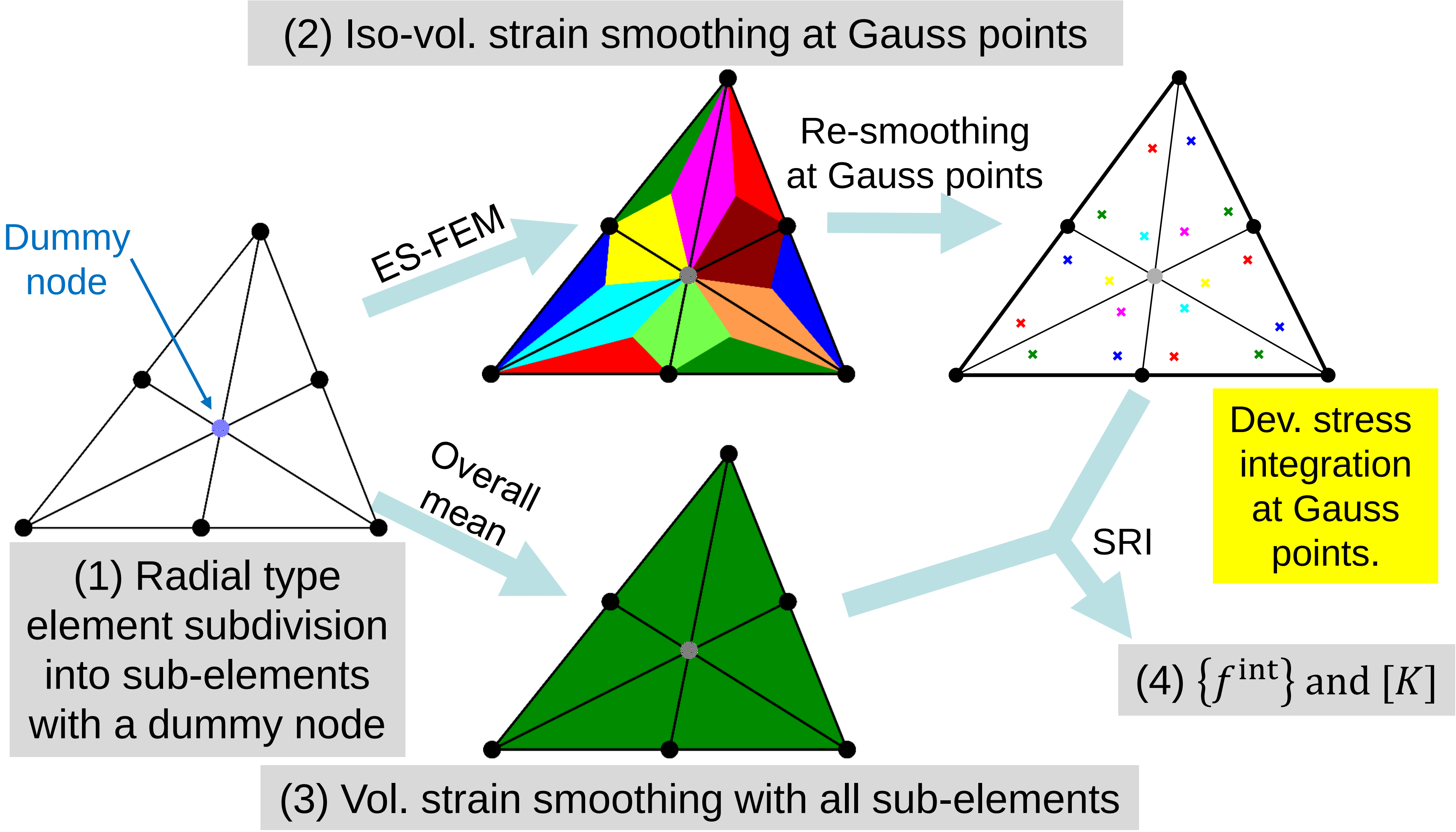
Dev. stress integration at edges

(3) Vol. strain smoothing with all sub-elements

(4) $\{f^{\text{int}}\}$ and $[K]$

Brief of SelectiveCSSE-T10 (New)

Explanation in 2D (6-node triangular element), for simplicity



(2) Iso-vol. strain smoothing at Gauss points

Dev. stress integration at Gauss points.

(1) Radial type element subdivision into sub-elements with a dummy node

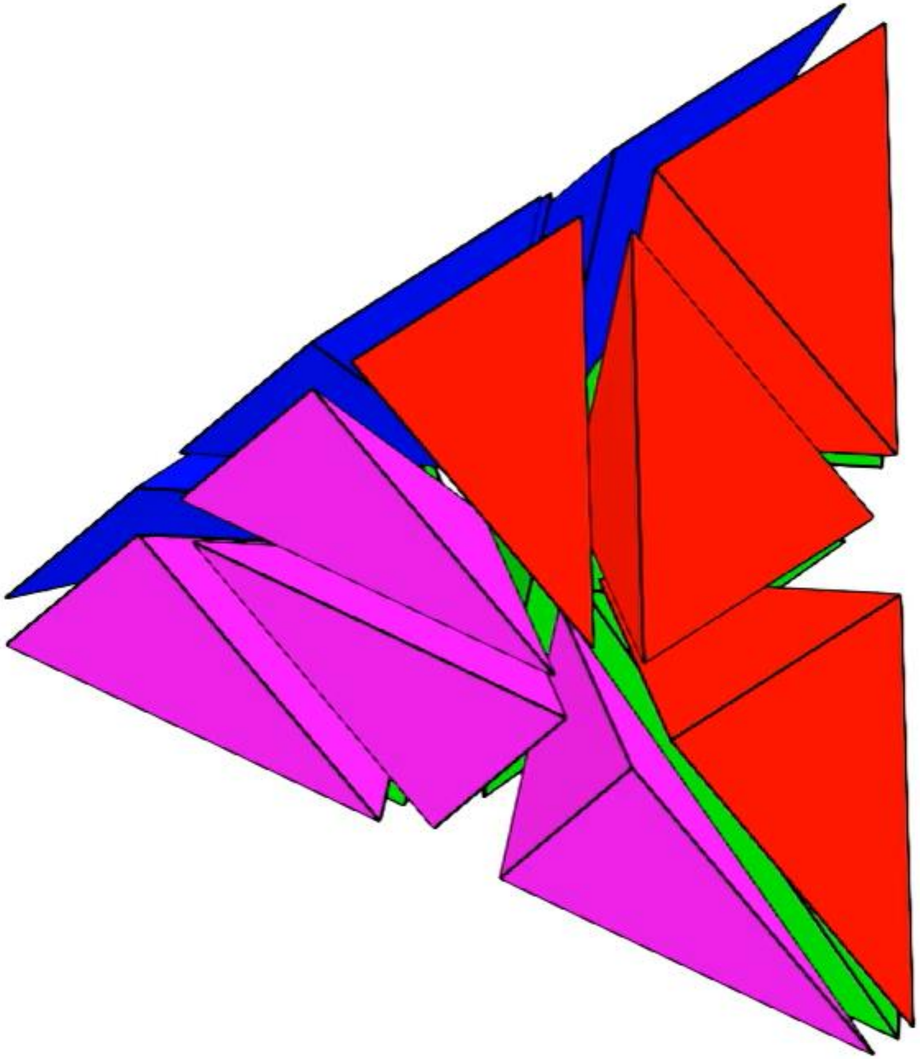
(4) $\{f^{\mathrm{int}}\}$ and $[K]$

(3) Vol. strain smoothing with all sub-elements

Brief of SelectiveCSSE-T10 in 3D

Radial subdivision of a T10 element (30% shrunk mesh)

There are 16
T4 sub-elements
in total.



1st strain
smoothing
on 34 edges
(ES-FEM).

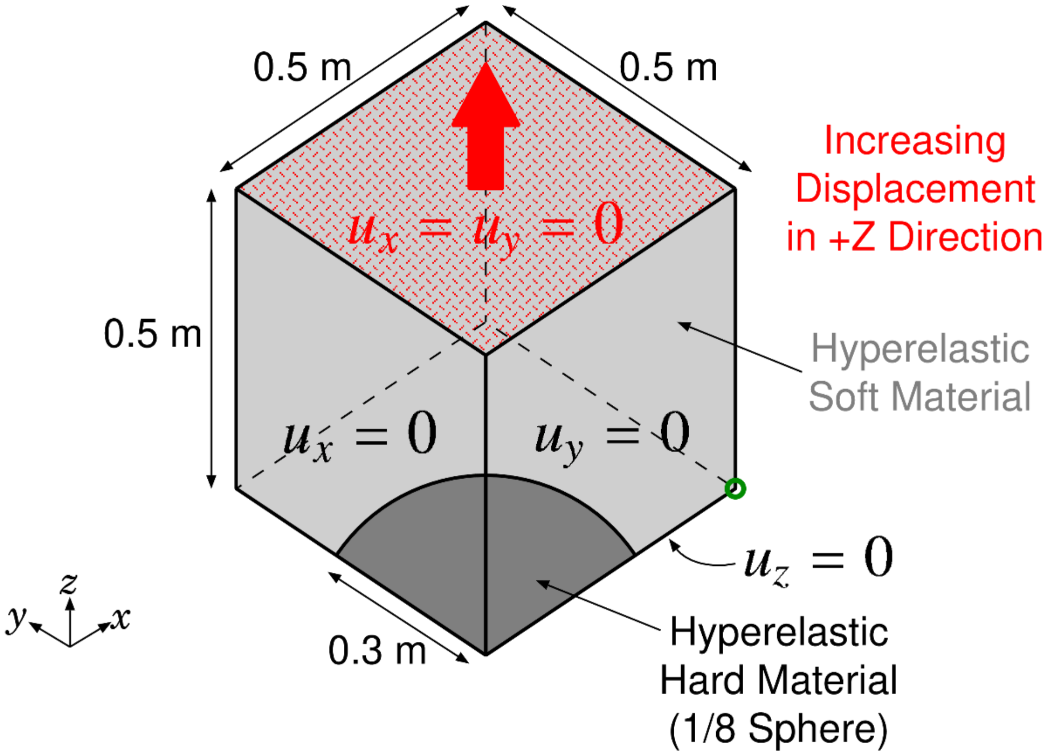
2nd strain
smoothing
on 64 Gauss
points
(SSE).

Results

Static Implicit

Tension of Filler-Rubber Composite

Outline

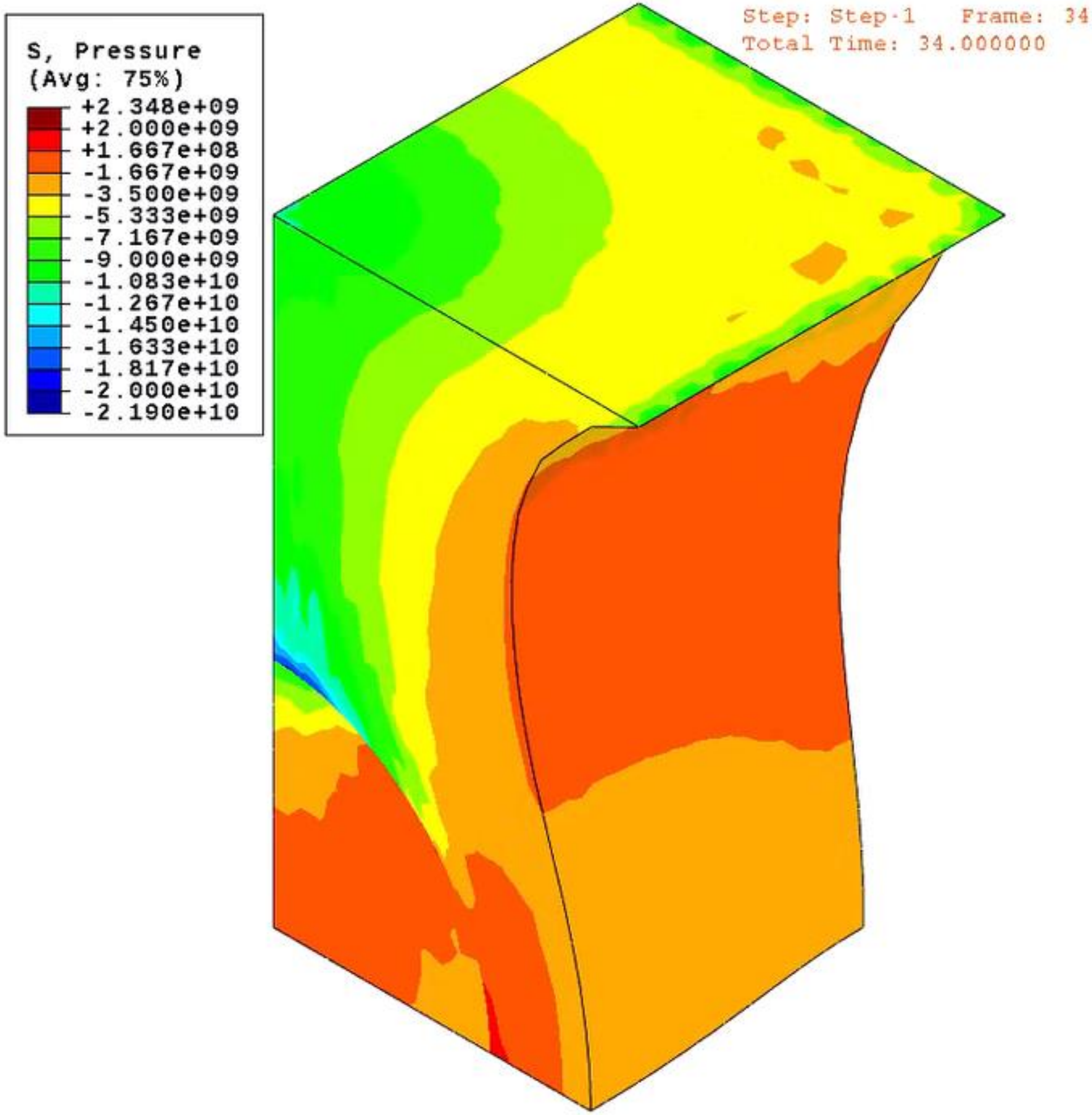


- Filler: Neo-Hookean hyperelastic ($E_{\text{ini}} = 260\ \text{GPa}, \boldsymbol{\nu_{\text{ini}} = 0.3}$)
- Rubber: Neo-Hookean hyperelastic ($E_{\text{ini}} = 6\ \text{GPa}, \boldsymbol{\nu_{\text{ini}} = 0.49}$)
- T10 mesh (about 11,000 nodes & 7,000 elements)
- Compared to ABAQUS C3D10MH (the best T10 element in ABAQUS) with the same mesh.

Static Implicit

Tension of Filler-Rubber Composite

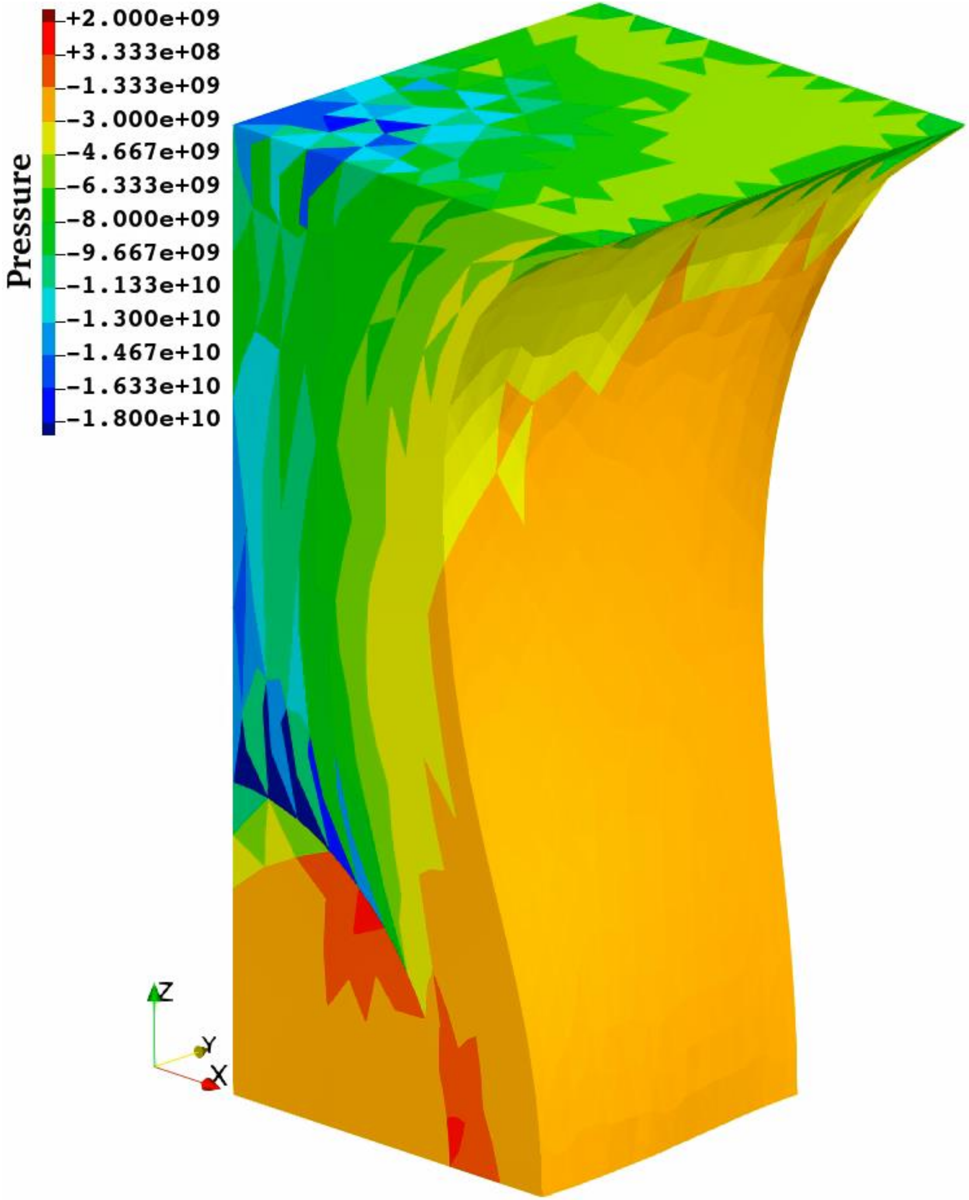
Result of
ABAQUS
C3D10MH
(Pressure
Dist.)



Convergence failure at ~70% stretch.

Tension of Filler-Rubber Composite

Result of Selective CSSE-T10 (Pressure Dist.)

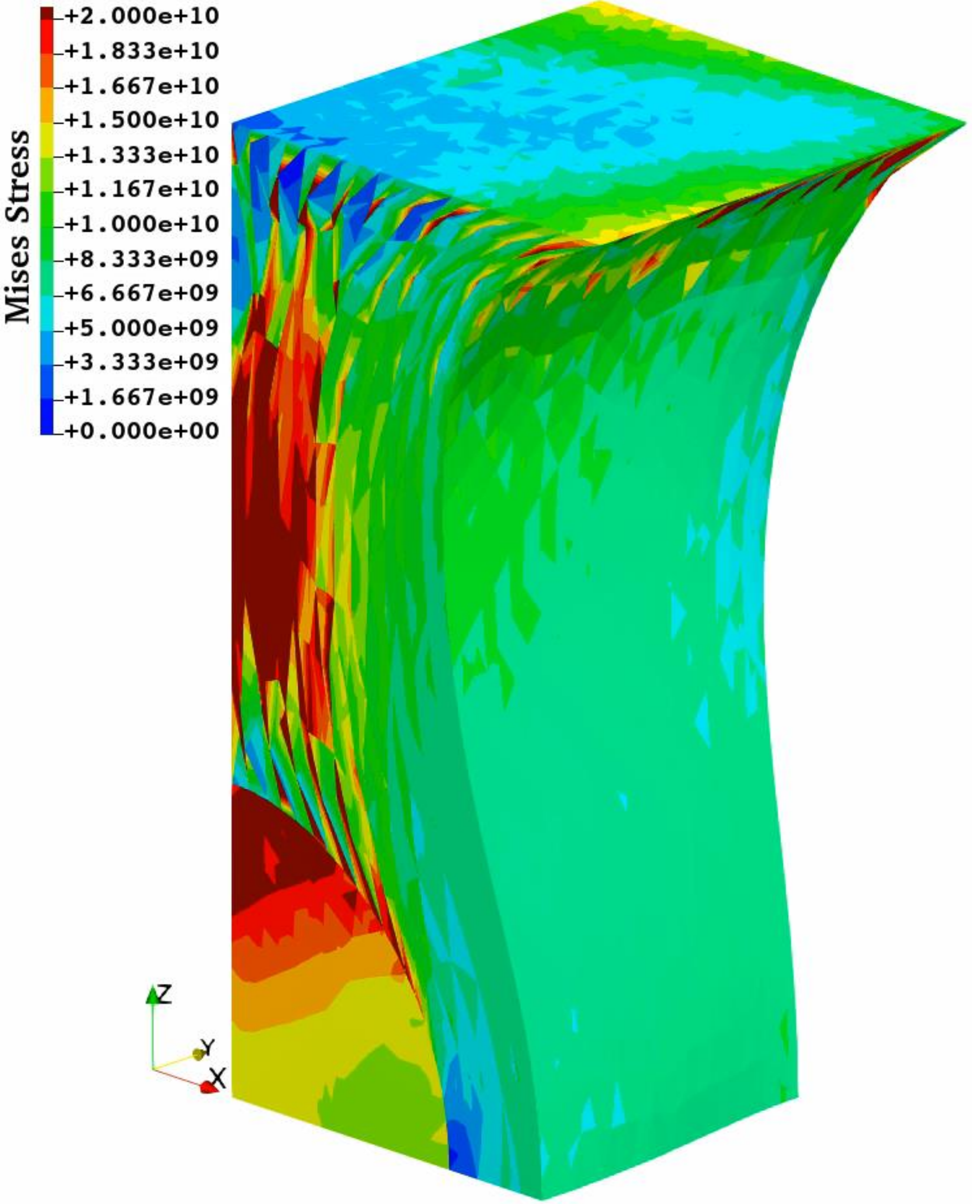


Convergence failure at ~100% stretch.
⇓
Less robust compared to Selective CS-FEM-T10 (up tp 200% stretch.)

Static Implicit

Tension of Filler-Rubber Composite

Result of Selective CSSE-T10 (Mises Stress Dist.)

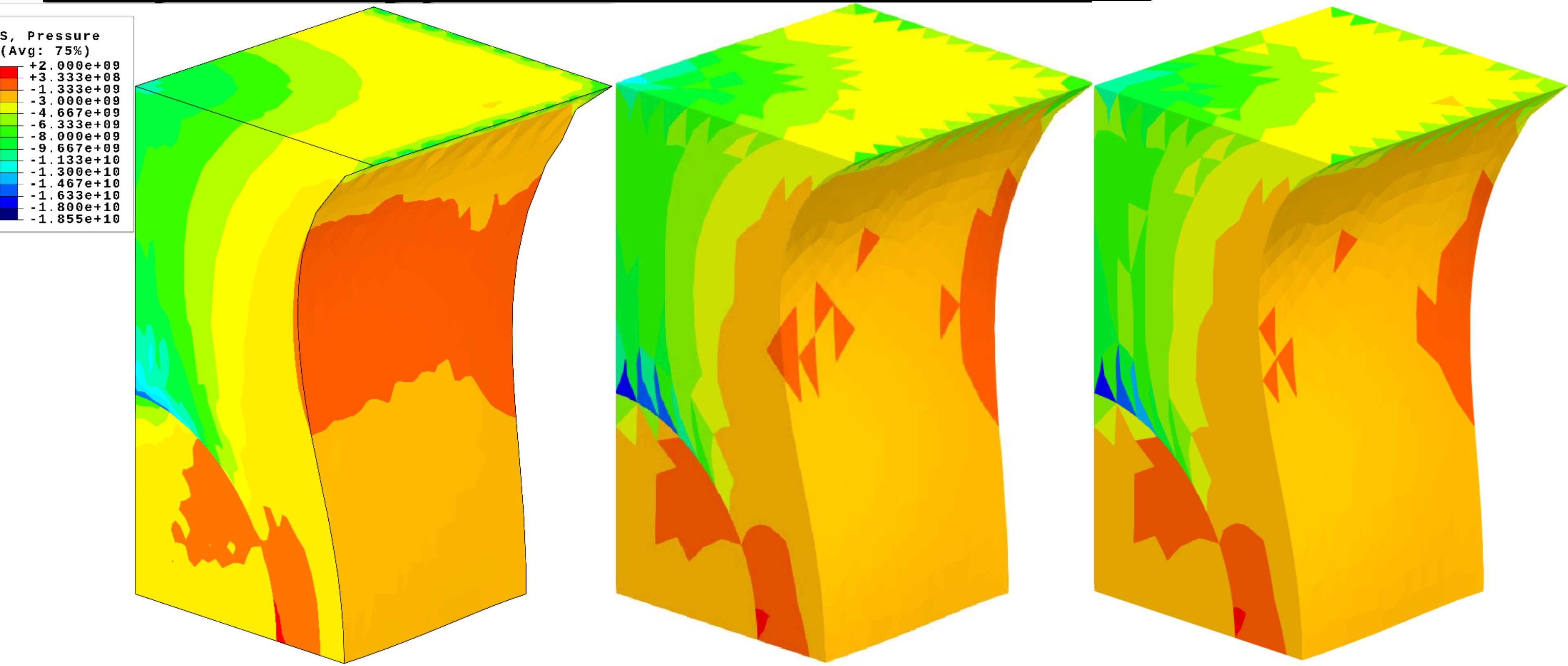


The issue of deviatoric stress oscillation was hardly solved…

Static
Implicit

Tension of Filler-Rubber Composite

Comparison of pressure dist. at 60% stretch.



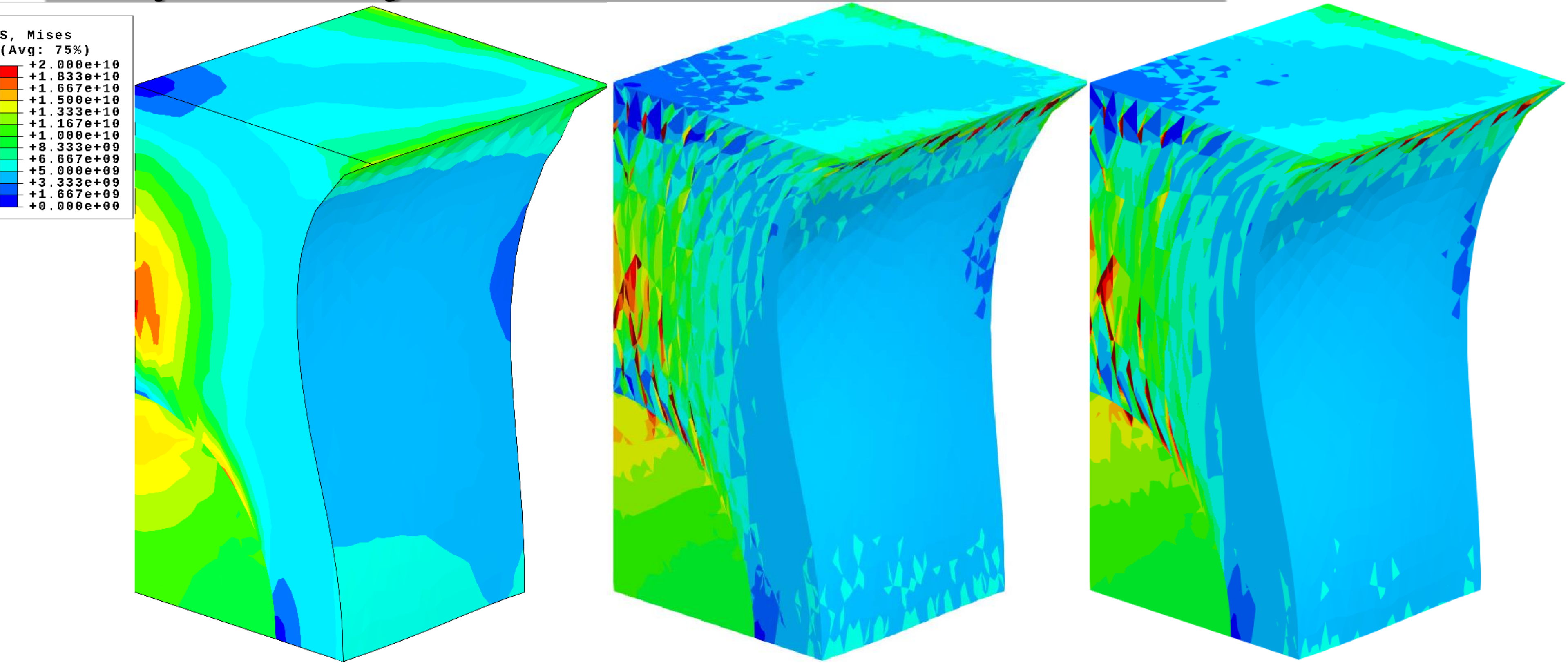
ABAQUS C3D10MH　SelectiveCS-FEM-T10　SelectiveCSSE-T10

SelectiveCSSE-T10 has sufficient accuracy in displacement/force and pressure.

Static
Implicit

Tension of Filler-Rubber Composite

Comparison of Mises stress dist. at 60% stretch



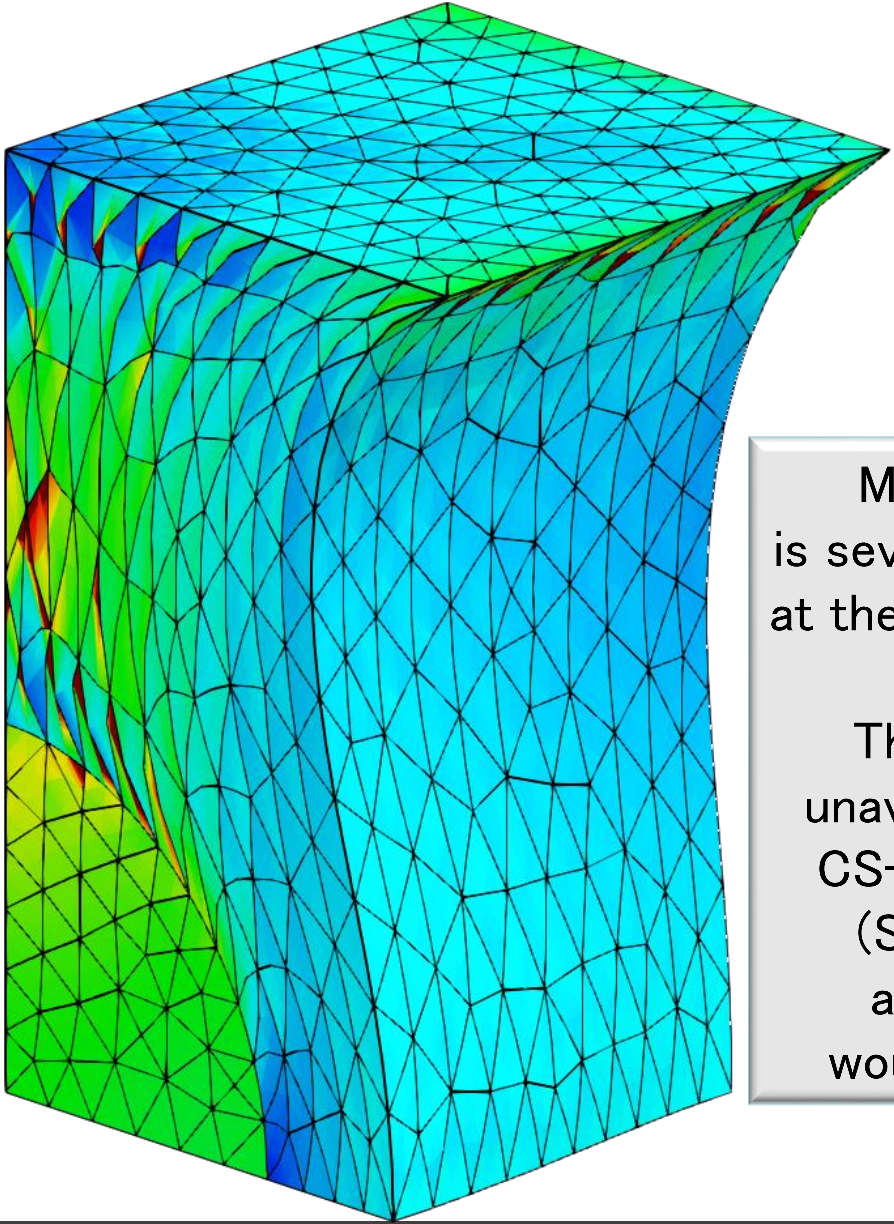
ABAQUS C3D10MH SelectiveCS-FEM-T10 SelectiveCSSE-T10

SelectiveCSSE-T10 resolves the oscillation issue only a little bit.
Unfortunately, there is no fundamental improvement...

Static
Implicit

Tension of Filler-Rubber Composite

Mises stress
Dist. of
SelectiveCSSE-T10
at 60% stretch



Mises stress dist.
is severely discontinuous
at the element interfaces.
⇓
This issue may be
unavoidable as long as
CS-FEM is adopted…
(Strain smoothing
across elements
would be essential?)

Summary

Summary

- A new CS-FEM formulation, SelectiveCSSE-T10, was developed.
- SelectiveCSSE-T10 introduces the idea of the strain smoothing element (SSE) to our conventional method (SelectiveCS-FEM-T10) to resolve the issue of deviatoric stress oscillation.
- Unfortunately, SelectiveCSSE-T10 cannot resolve the oscillation issue fundamentally...

☐ Another approach should be taken to achieve a volumetric locking-free SSE formulation, which is our future work.

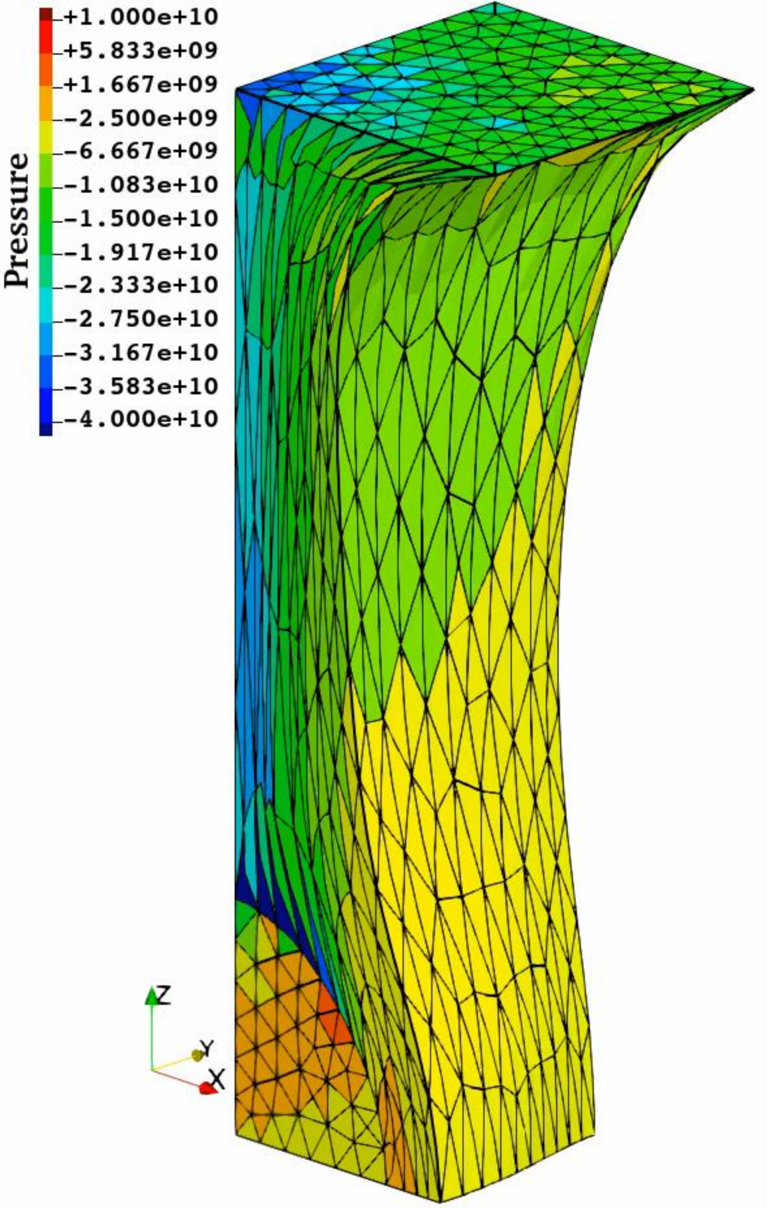
Thank you for your kind attention!

Appendix

Static
Implicit

Tension of Filler-Rubber Composite

Result of Selective CS-FEM-T10 (Pressure Dist.)



Convergence failure at ~200% stretch.