This is DAY2, Room B, Session 2B-2: “S-FEM”.

SelectiveCS-FEM-T10 with Radial-type Mesh Subdivision

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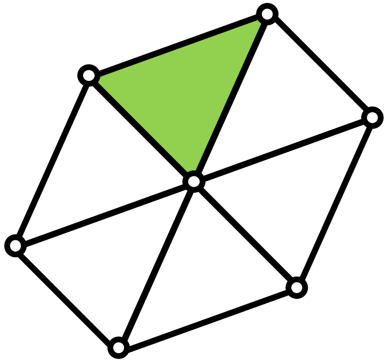
What is S-FEM?

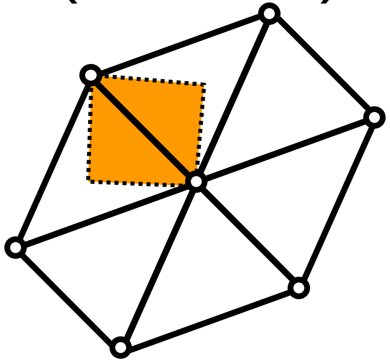
- ***Smoothed*** finite element method (**S-FEM**) is a relatively new FE formulation proposed by Prof. G. R. Liu in 2006.
- S-FEM is one of the **strain smoothing** techniques.
- There are several types of classical S-FEMs depending on the **domains of strain smoothing**.

For example in a 2D triangular mesh:

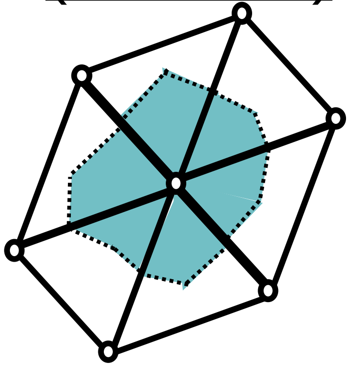
Standard FEM



Edge-based S-FEM (ES-FEM)



Node-based S-FEM (NS-FEM)



What are the major pros of S-FEM?

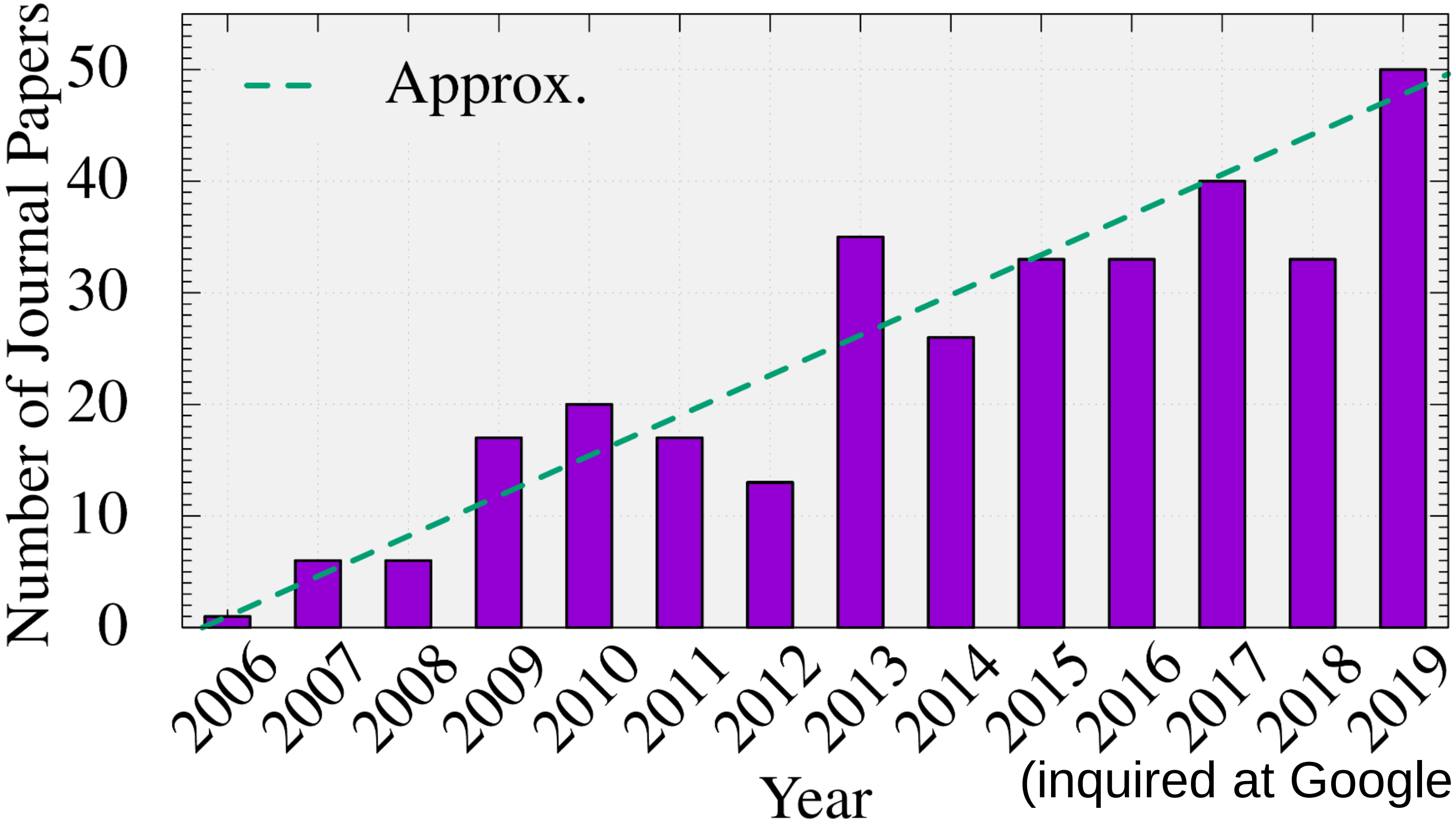
Even when we use 4-node tetrahedral mesh:

1. **Super-linear mesh convergence rate.**
   (Almost same rate as the 2nd-order element.)
2. **Shear locking free with ES-FEM.**
   (Excellent with tetrahedral mesh.)
3. **Little accuracy loss in skewed meshes.**
   (No problem with complex geometry.)

S-FEM is a powerful method
suitable for practical industrial applications.

How popular is S-FEM?

Number of journal papers whose **title** contains "**smoothed finite element**":



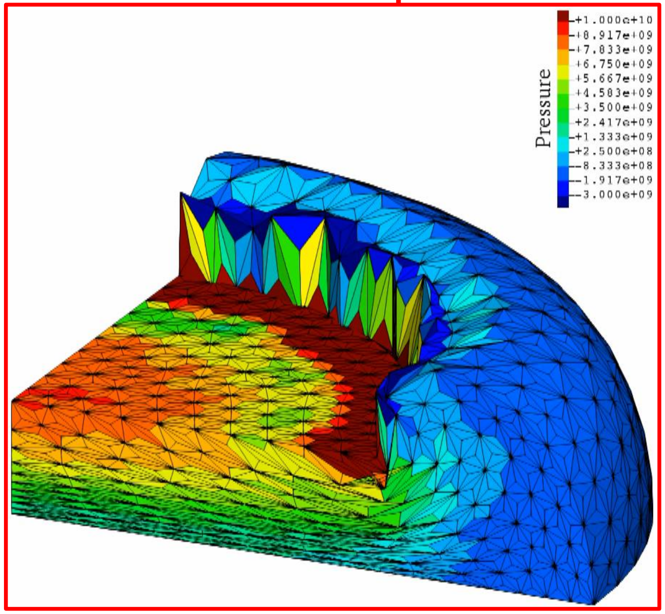
(inquired at Google Scholar)

The attraction of S-FEM is expanding continuously.

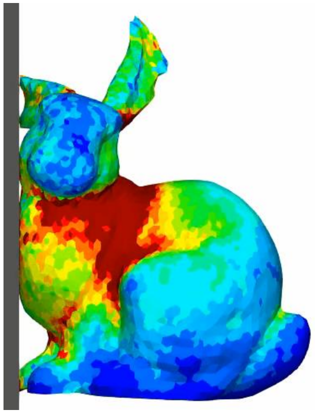
Applications of S-FEMs in Our Lab

- **Solid mechanics (still in lab stage)**

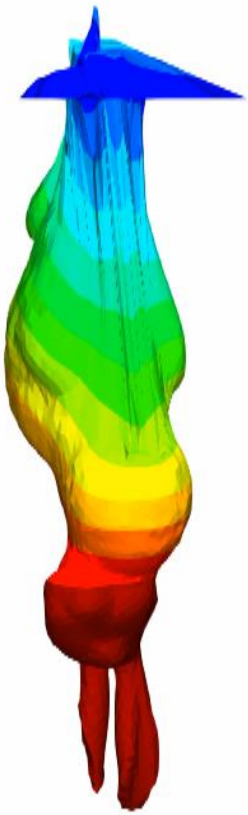
Static Implicit



Dynamic Explicit



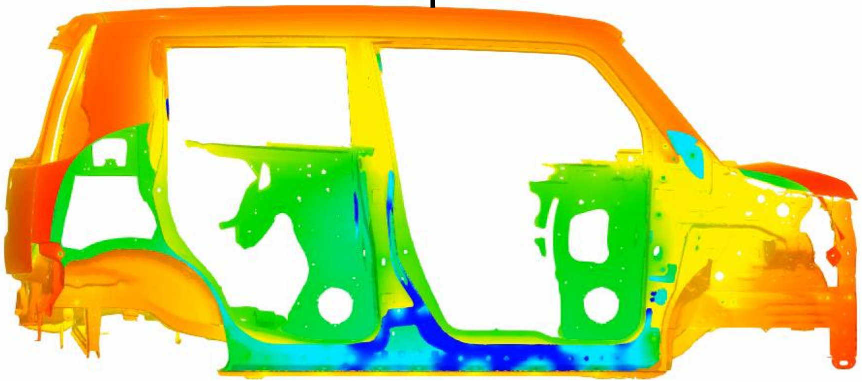
Viscous Implicit



- **Electrostatic**

(already in practice)

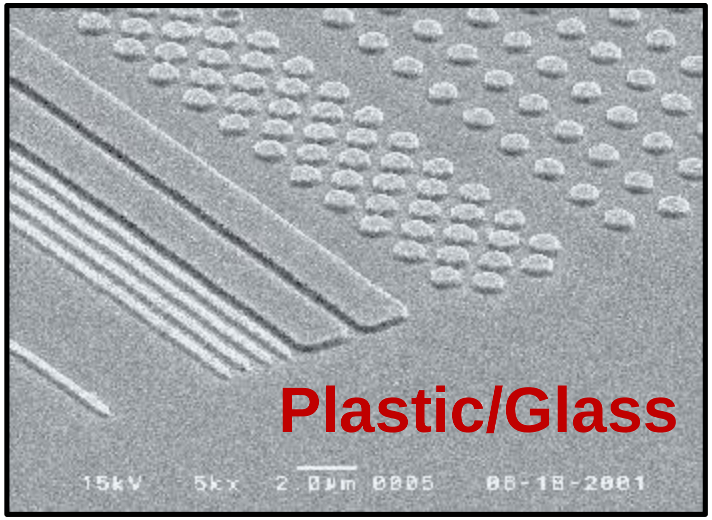
Electro Deposition



Motivation

What we want to do in actuality:

- Solve **hyper large deformation** analyses accurately and robustly.
- Treat complex geometries with **tetrahedral meshes**.
- Consider **nearly incompressible materials** $(\nu \simeq 0.5)$.
- Support **contact** problems.
- Handle **auto re-meshing**.



Issues

Conventional tetrahedral (T4/T10) FE formulations still have issues in accuracy and/or robustness especially in nearly incompressible cases.

- 2nd or higher order elements:
  - ✗ Volumetric locking. Accuracy loss in large strain.
- B-bar/F-bar method, Selective reduced integration (SRI):
  - ✗ Not applicable to tetrahedral element directly.
- F-bar-Patch method:
  - ✗ Difficulty in building good-quality patches.
- **u/p mixed (hybrid) method** (ABAQUS **C3D10MH** etc.):
  - ✗ Early convergence failure. Accuracy loss in large strain.
- **F-bar aided ES-FEM-T4** [Y.Onishi, IJNME, 109 (2017)] :
  - ✓ Accurate & robust ✗ Hard to implement in FEM codes.
- **SelectiveCS-FEM-T10** [Y.Onishi, IJCM, 17 (2020)] :
  - ✓ Accurate, robust & easy to implement. ✗ Not yet optimal.

Objective

To find an optimal formulation of
SelectiveCS-FEM-T10
for severe large deformation analyses.

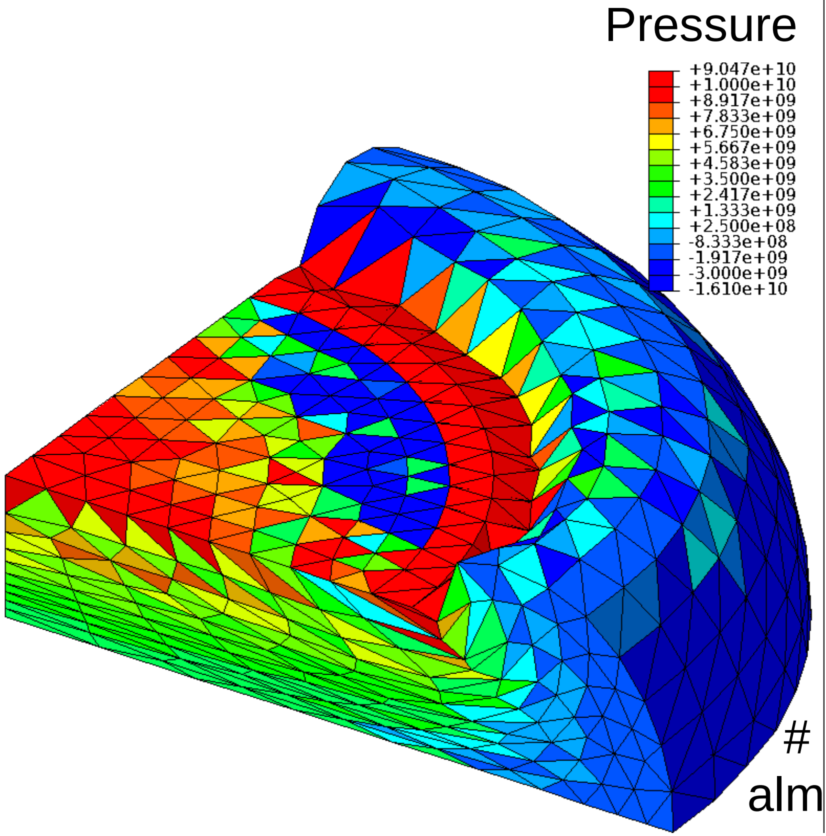
Table of Body Contents

- Quick Review of Issues in Conventional Methods
- Formulation of New SelectiveCS-FEM-T10
- Demonstrations of New SelectiveCS-FEM-T10
- Summary

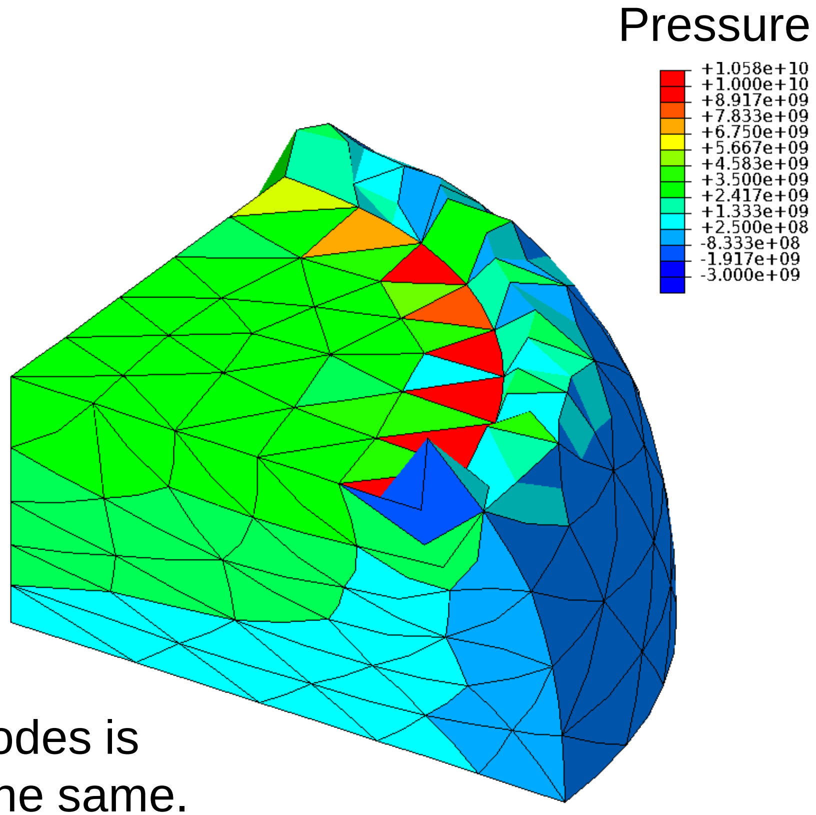
Quick Review of Issues in Conventional Methods

Issues in Barreling Analysis of Rubber Cylinder

Neo-Hookean ***hyperelastic*** body with $\nu_{\text{ini}} = 0.49$



# of Nodes is almost the same.



1st order hybrid T4 (C3D4H)

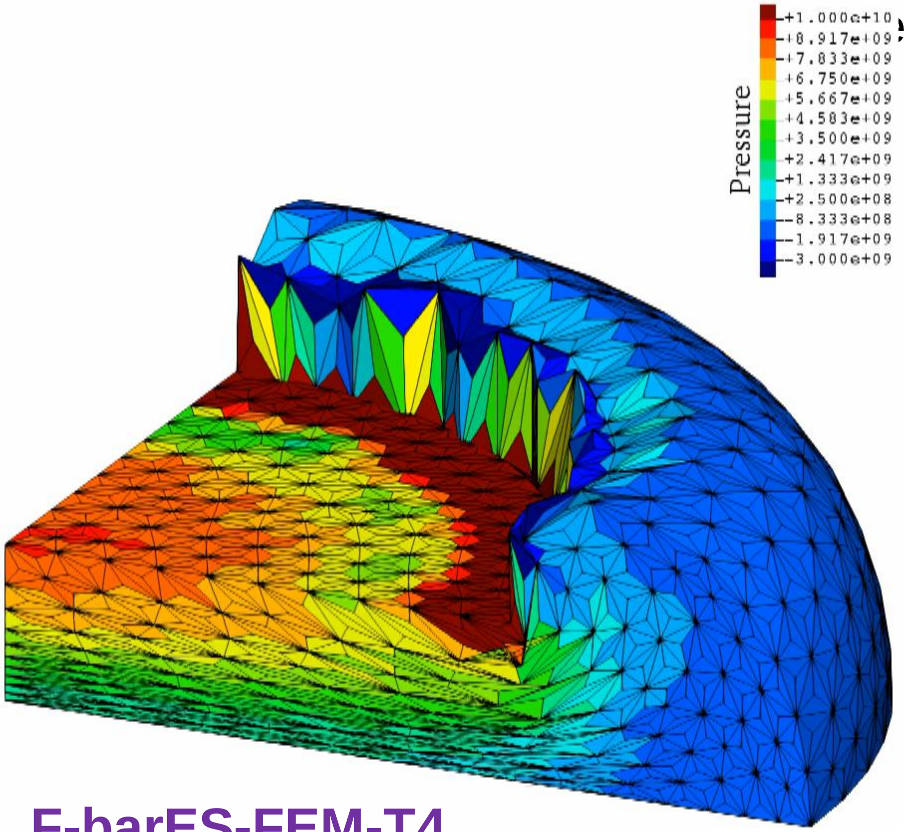
- ✓ No volumetric locking
- ✗ Pressure checkerboarding
- ✗ Shear & corner locking

2nd order modified hybrid T10 (C3D10MH)

- ✓ No shear/volumetric locking
- ✗ Early convergence failure
- ✗ Low interpolation accuracy

Issues in Barreling Analysis of Rubber Cylinder

Neo-Hookean ***hyperelastic*** body with $\nu_{\mathrm{ini}} = 0.49$



Same mesh as C3D4H case.

Although F-barES-FEM-T4 is accurate and robust,

- ✘ it cosumes larger memory & CPU costs.
- ✘ it cannot be implemented in general-purpose FE software due to the adoption of ES-FEM.

Another approach adopting **CS-FEM** with **T10** element would be effective.

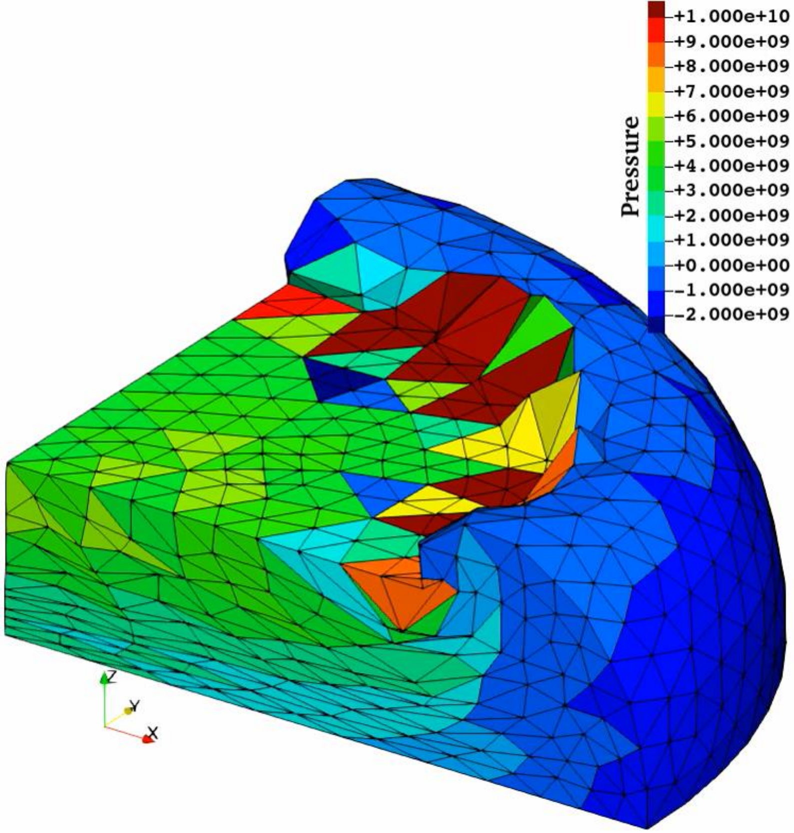
F-barES-FEM-T4

- ✓ No shear/volumetric locking
- ✓ No corner locking
- ✓ No pressure checkerboarding
- ✓ No increase in DOF

Y. Onishi, IJMNE, Vol. 109 (2017).

Issues in Barreling Analysis of Rubber Cylinder

Neo-Hookean ***hyperelastic*** body with $\nu_{\text{ini}} = 0.49$



Same mesh
as C3D10MH
case.

As other S-FEMs,
SelectiveCS-FEM-T10
has many varieties
in the formulation.

The conventional
method was
not an optimal
formulation yet.

SelectiveCS-FEM-T10 (Conventional Ver.)

- ✓ No shar/voluemetric locking
- ✓ Little corner locking
- ✓ Little pressure checkerboarding
- ✓ Same cost & userbility as T10 elements.

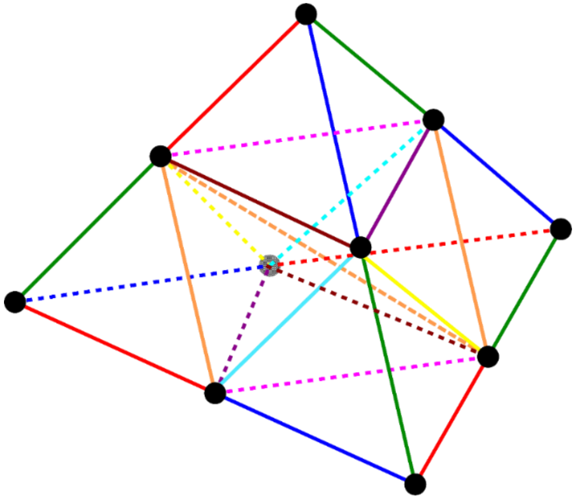
This study proposes
New Selective
CS-FEM-T10.

Y. Onishi, IJCM,
Vol. 17 (2020).

Formulation of
New SelectiveCS-FEM-T10

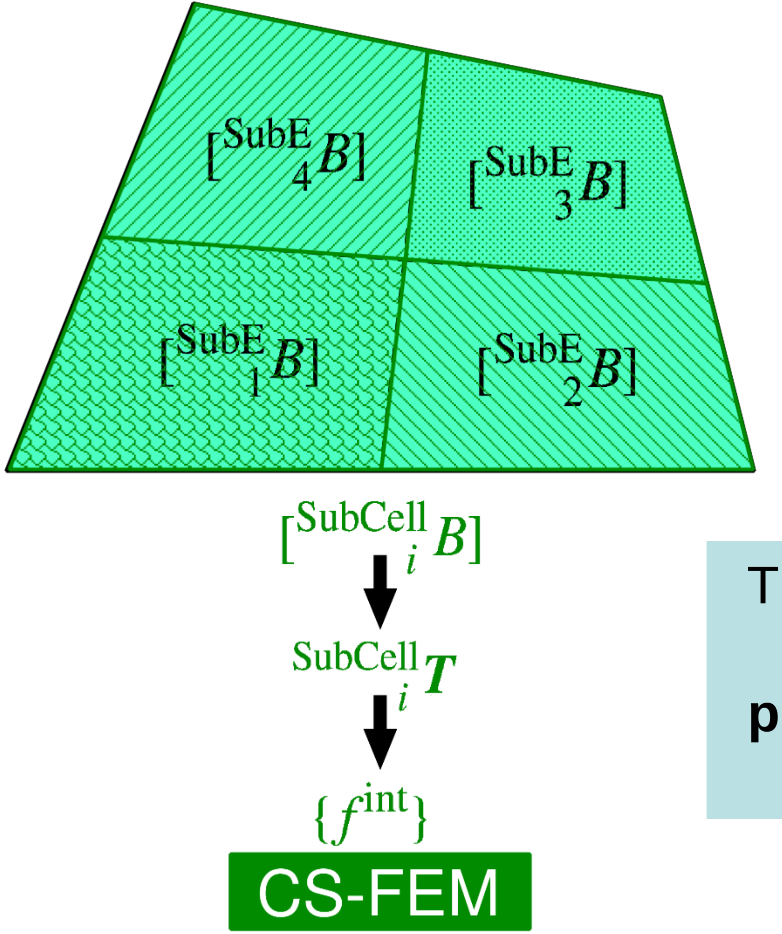
Concepts of SelectiveCS-FEM-T10

- Using T10 element and subdivide it into T4 sub-elements.
  ⟹ Overcomes the weak points of intermediate nodes.
- Adopting CS-FEM having no strain smoothing across multiple elements.
  ⟹ Becomes an independent element of existing FE codes.
- Applying selective reduced integration (SRI).
  ⟹ Overcomes volumetric locking.



Brief of Cell-based S-FEM (CS-FEM)

- Subdivide each element into some sub-element.
- Calculate $[{}^{\rm SubE}B]$ at each sub-element.
- Calculate $\boldsymbol{F}, \boldsymbol{T}, \{f^{\rm int}\}$ etc. in each sub-element.

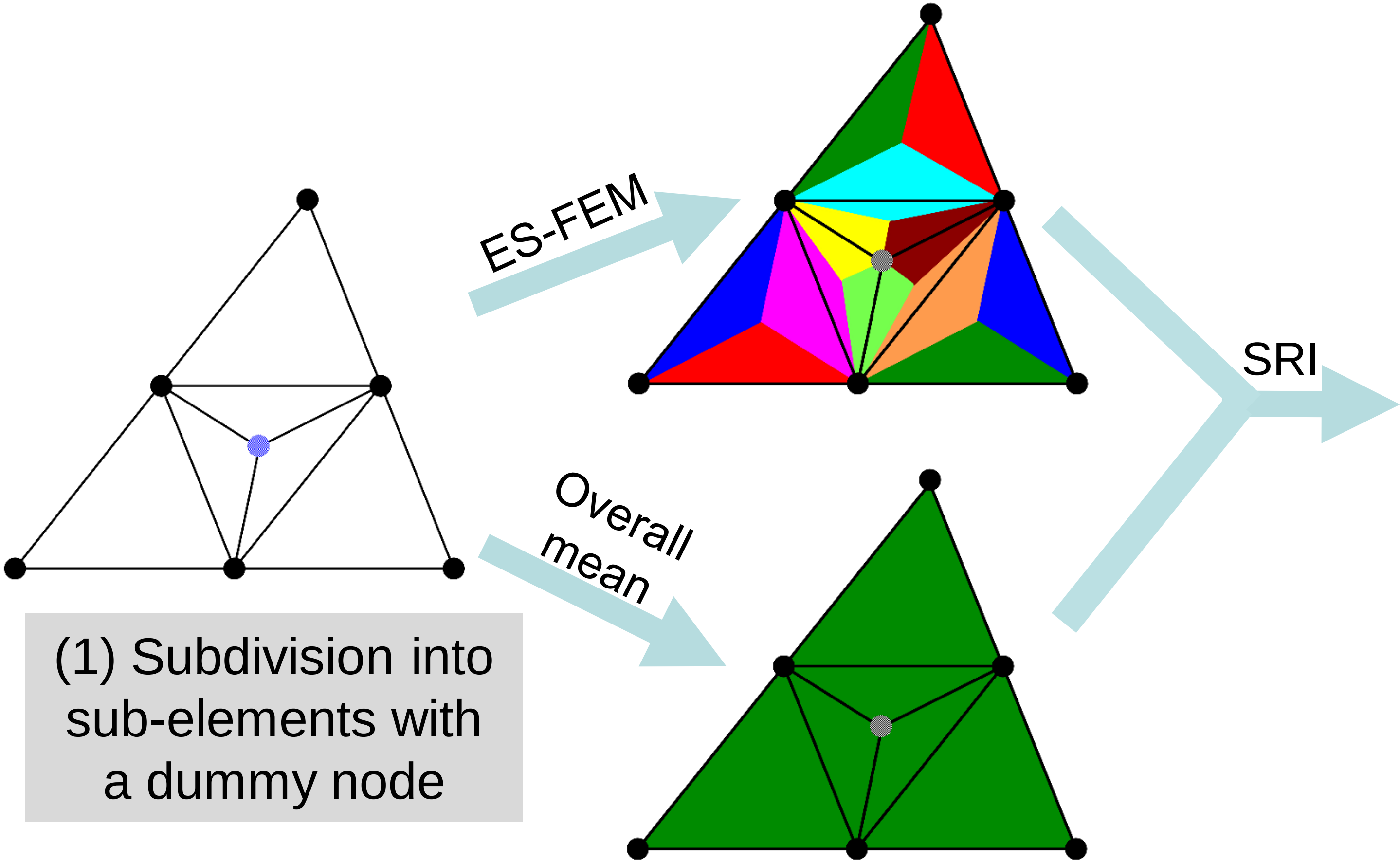


As if putting
an integration point
on each sub-element

The biggest advantage
of CS-FEM is its
portability to existing
FE codes.

Flowchart of New SelectiveCS-FEM-T10

Explanation in 2D (6-node triangular element) for simplicity



(2) Dev. strain smoothing at edges

(1) Subdivision into sub-elements with a dummy node

(3) Vol. strain smoothing with all sub-elements

(4) $\{f^{\mathrm{int}}\}$ and $[K]$

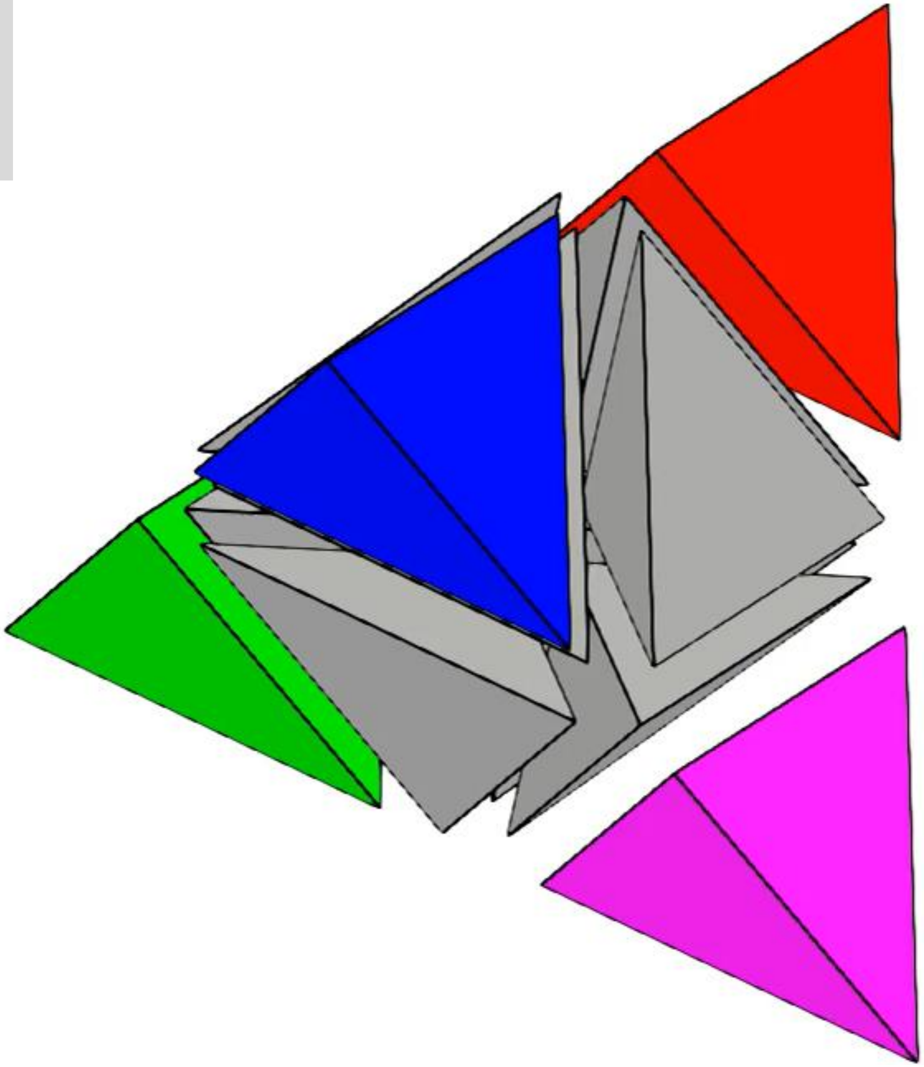
In case of 3D, there are varieties in mesh subdivision.

Mesh Subdivision Types in 3D

Conventional subdivision (30% shrunk mesh)

Each frame edge
is owned by only
one sub-element.

There are
12 sub-elements
in total.



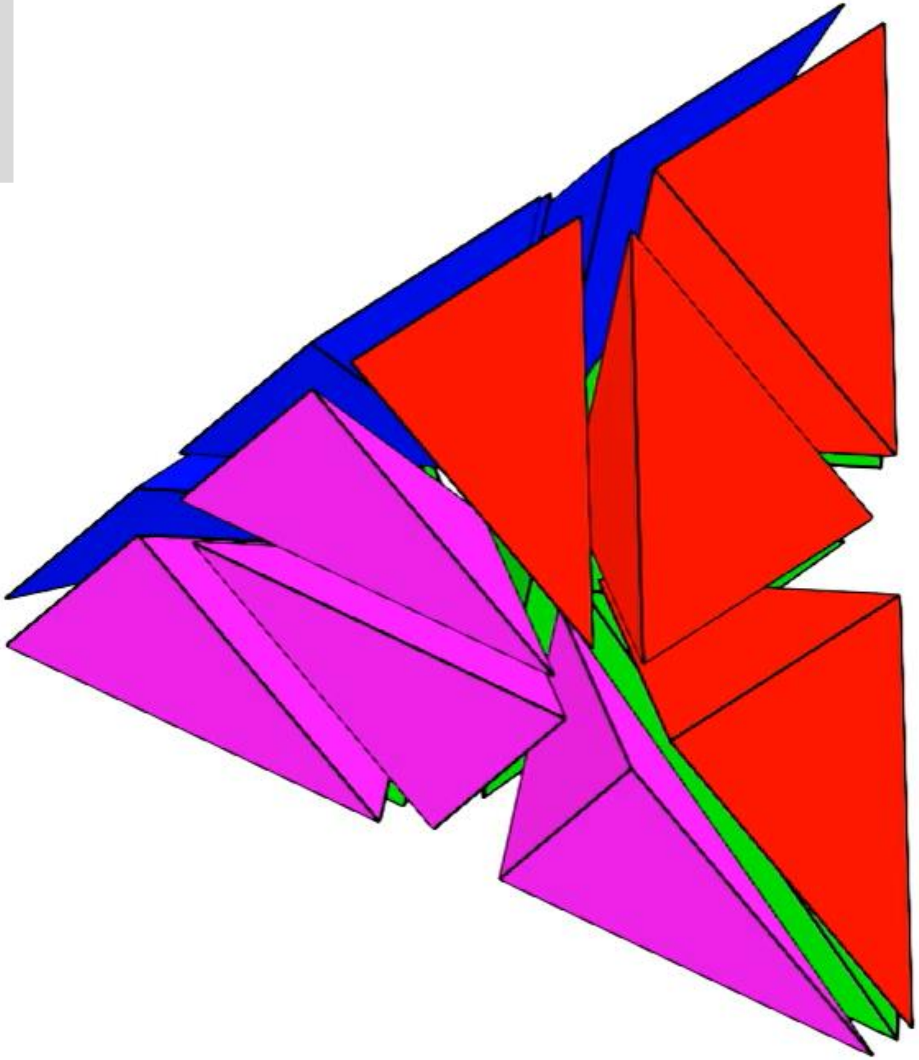
Strain on
frame edges
are NOT
smoothed
by ES-FEM.

Mesh Subdivision Types in 3D

New Radial-type subdivision (30% shrunk mesh)

Each frame edge is owned by two sub-elements.

There are 16 sub-elements in total.



Strain on all edges are smoothed by ES-FEM.

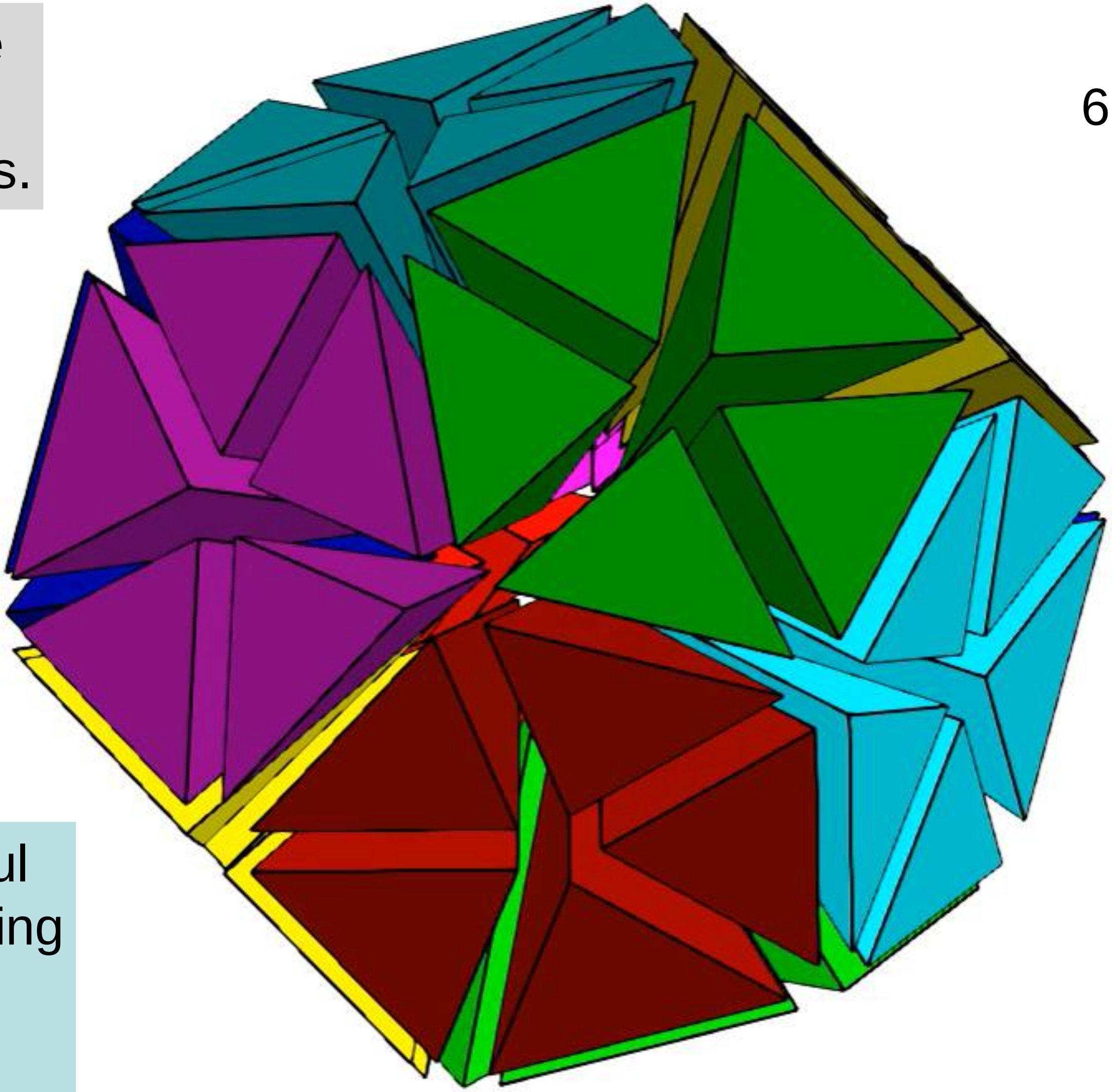
Sub-elements have a little larger skewness.

Mesh Subdivision Types in 3D

Future extension: Radial-type for polyhedral mesh

Each frame edge
is owned by
two sub-elements.

There are
60 sub-elements
in total.



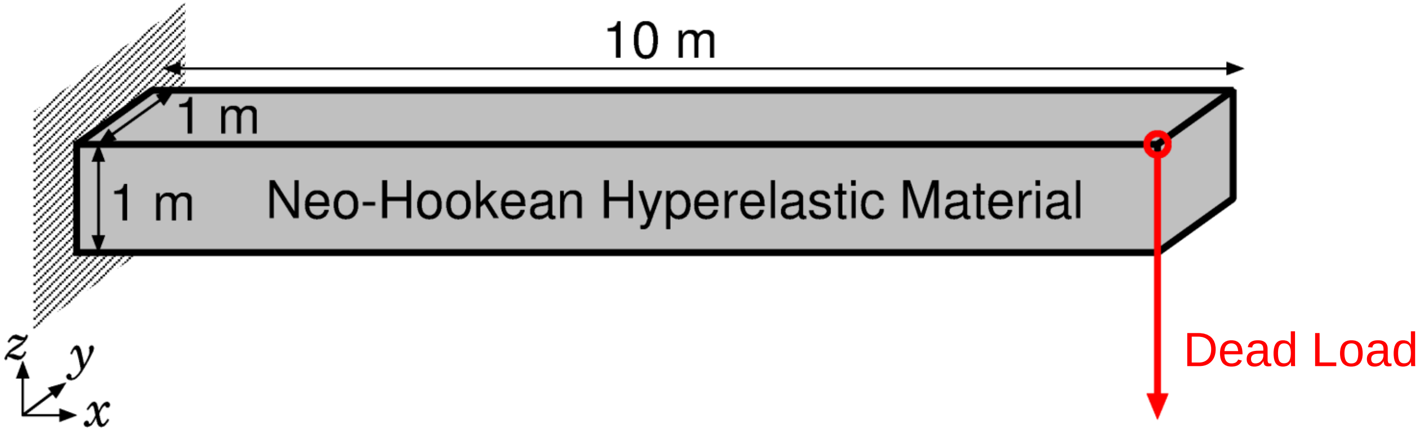
It should be useful
for mixture meshing
such as ANSYS
Mosaic Meshing.

Demonstration of
New SelectiveCS-FEM-T10

Static Implicit

Bending of Hyperelastic Cantilever

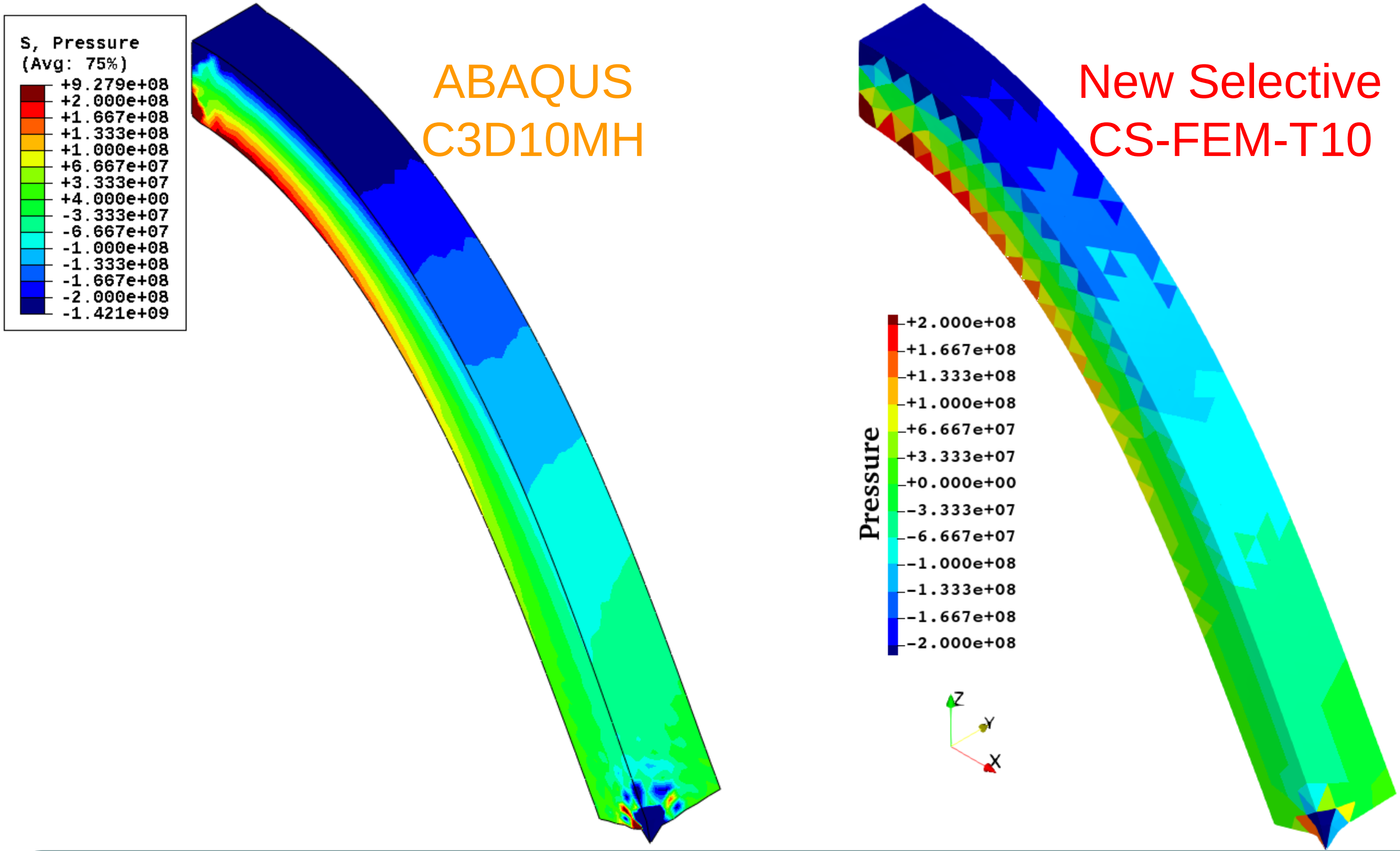
Outline



- Neo-Hookean hyperelastic material
- Initial Poisson’s ratio: $\nu_0 = 0.49$
- Compared to ABAQUS C3D10MH (modified hybrid T10 element) with the same mesh.

Bending of Hyperelastic Cantilever

Comparison of the pressure dist. at the final state

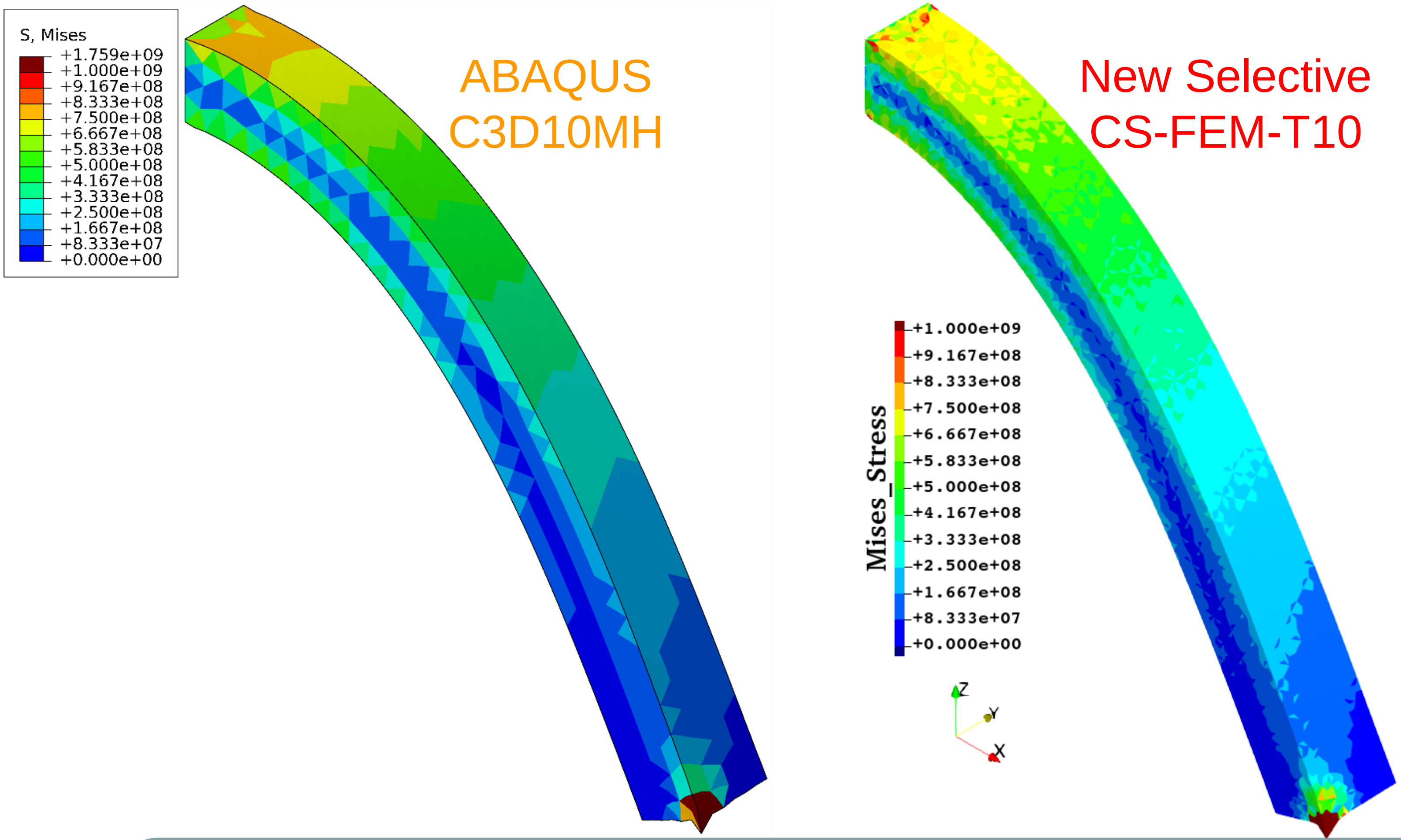


Almost the same pressure distributions
with no checkerboarding. (No locking of course.)

Static
Implicit

Bending of Hyperelastic Cantilever

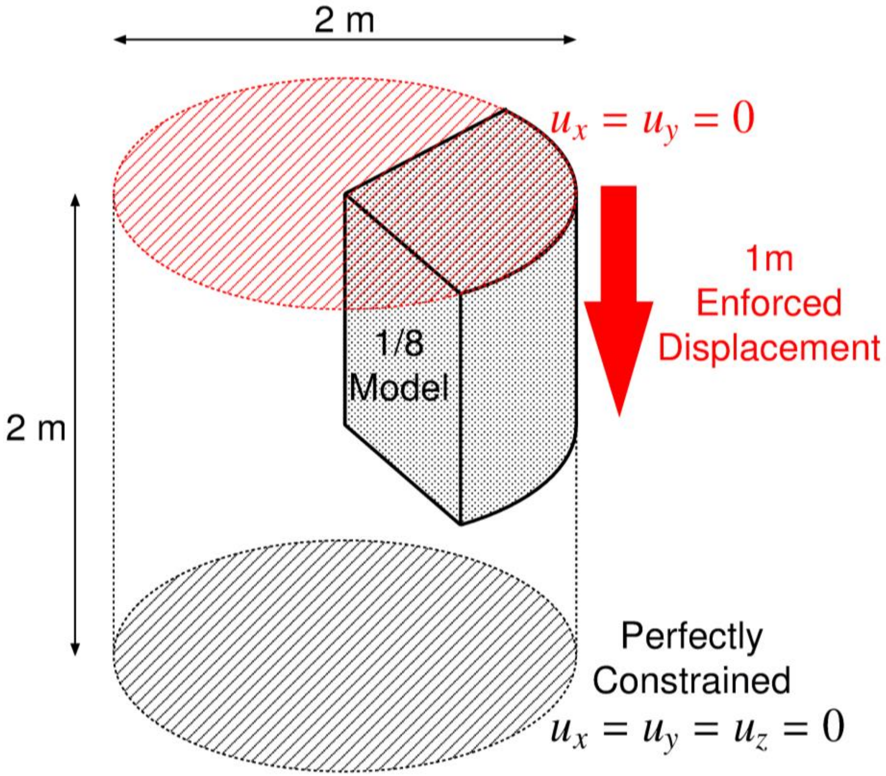
Comparison of the Mises stress dist. at the final state



Almost the same Mises stress distributions.

Barreling of Hyperelastic Cylinder

Outline



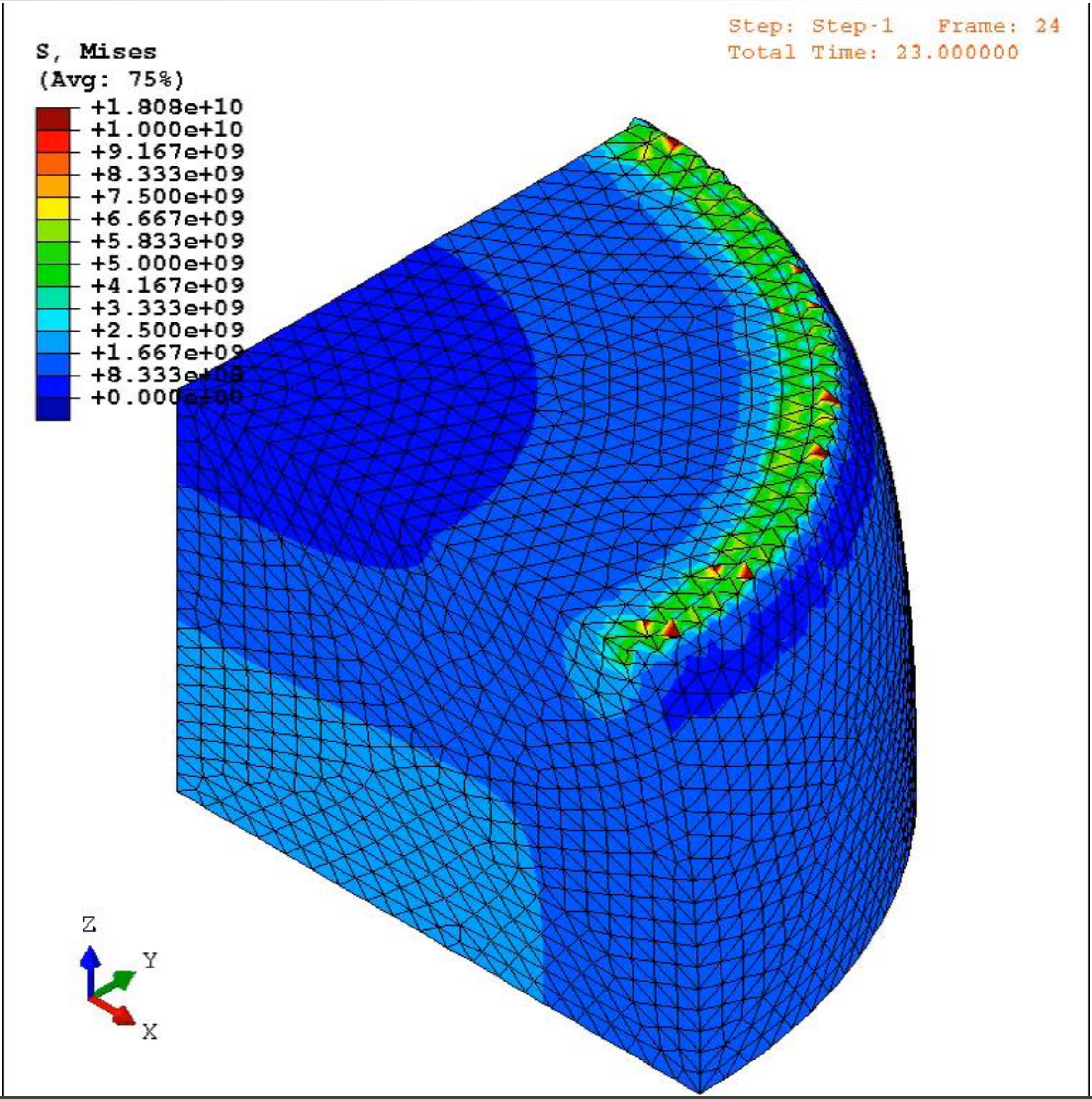
- Enforce axial displacement on the top face.
- Neo-Hookean body with $\nu_{\mathrm{ini}} = 0.49$.
- Compare results with ABAQUS T10 hybrid elements (C3D10H, C3D10MH, C3D10HS) using the same mesh.

Static
Implicit

Barreling of Hyperelastic Cylinder

Animation of Mises stress (ABAQUS C3D10MH)

Convergence failure at **24%** compression



Unnaturally oscillating distributions are obtained around the rim.

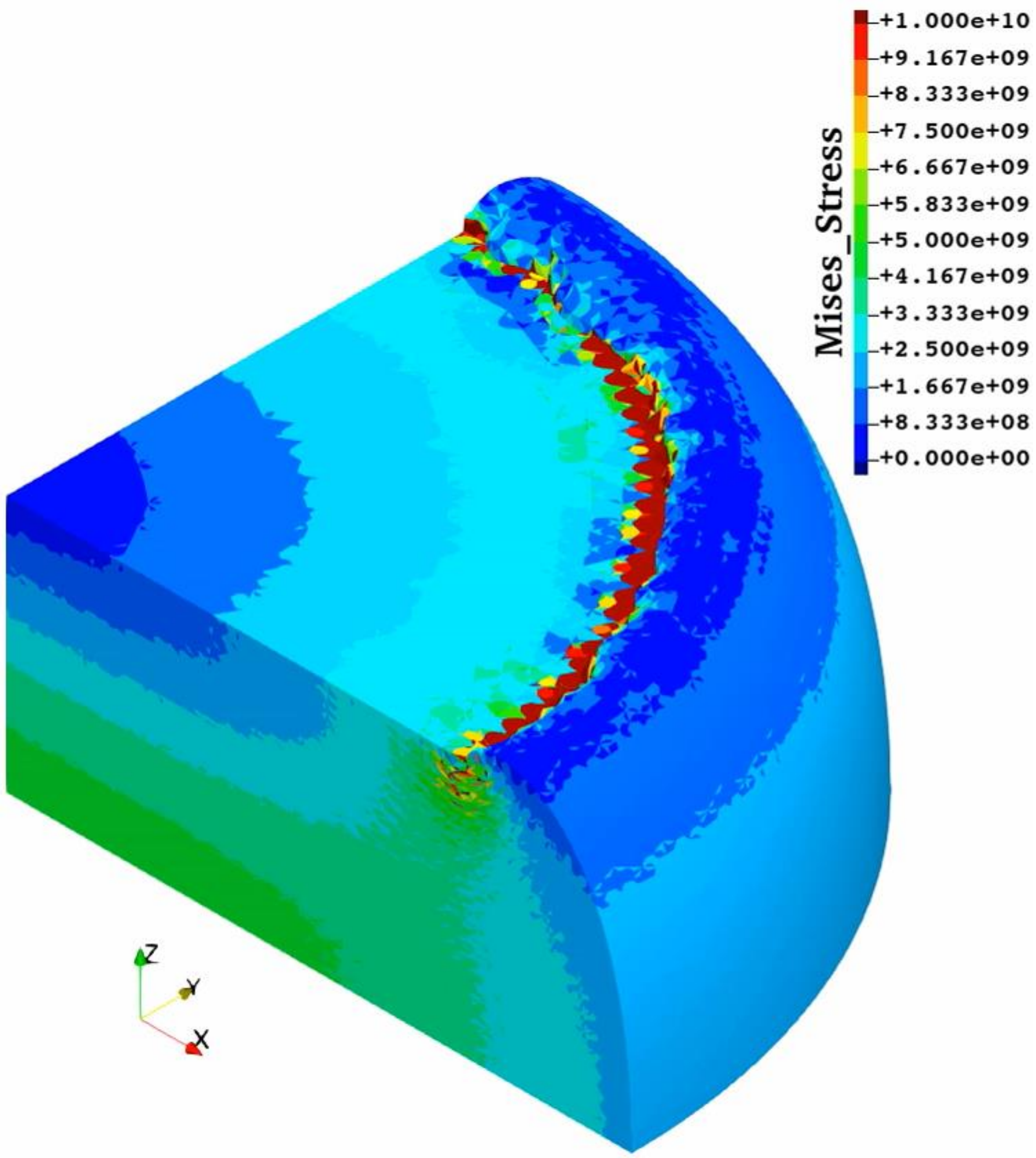
Static Implicit

Barreling of Hyperelastic Cylinder

***Animation of Mises stress* (*New Selective CS-FEM-T10*)**

Convergence failure at **47%** compression

The present element is more **long-lasting (robust)** than ABAQUS C3D10MH

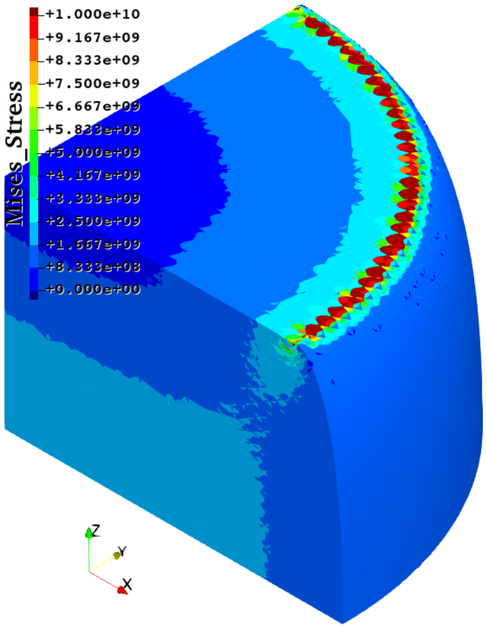


Smooth distributions are obtained except around the rim.

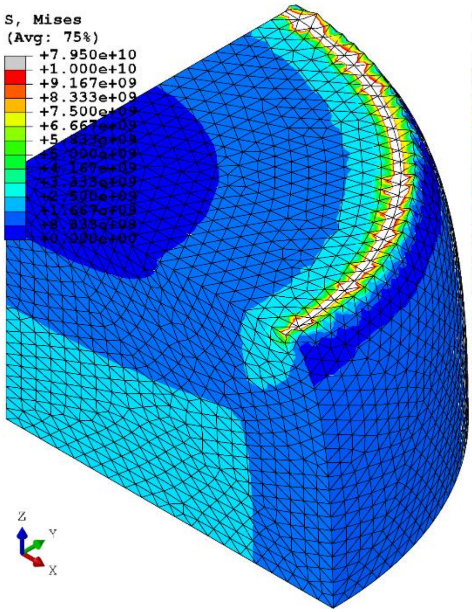
Static Implicit

Barreling of Hyperelastic Cylinder

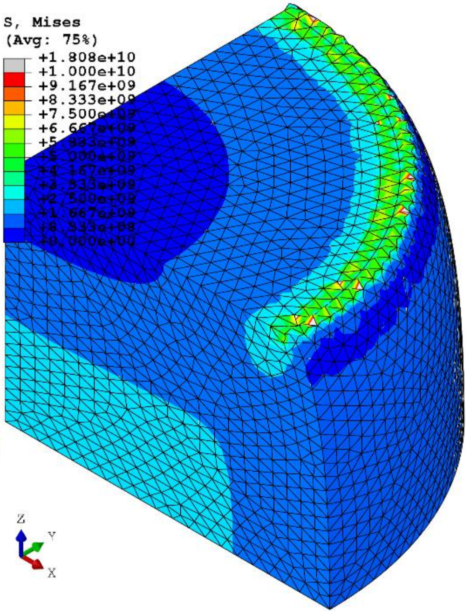
Comparison of Mises stress at 24% comp.



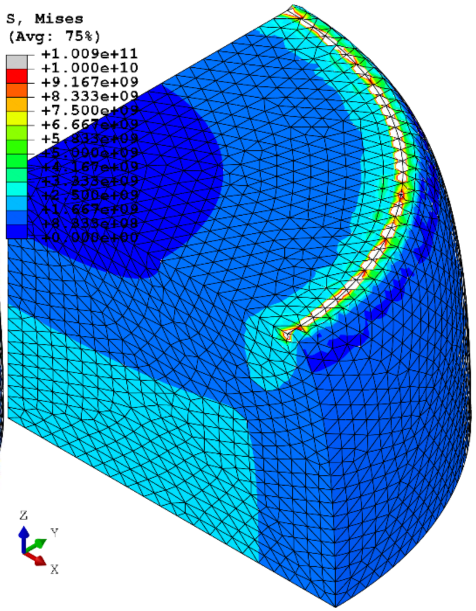
New Selective
CS-FEM-T10



ABAQUS
C3D10H



ABAQUS
C3D10MH



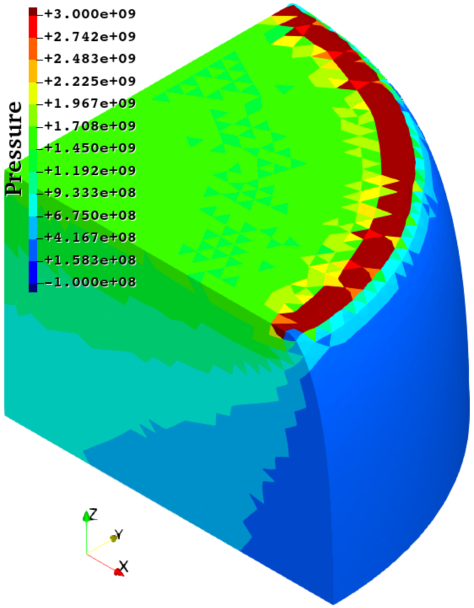
ABAQUS
C3D10HS

All results are similar to each other
except around the rim having stress singularity.

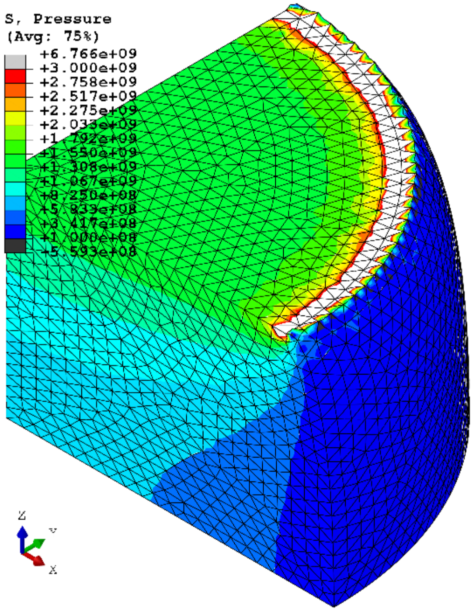
Static
Implicit

Barreling of Hyperelastic Cylinder

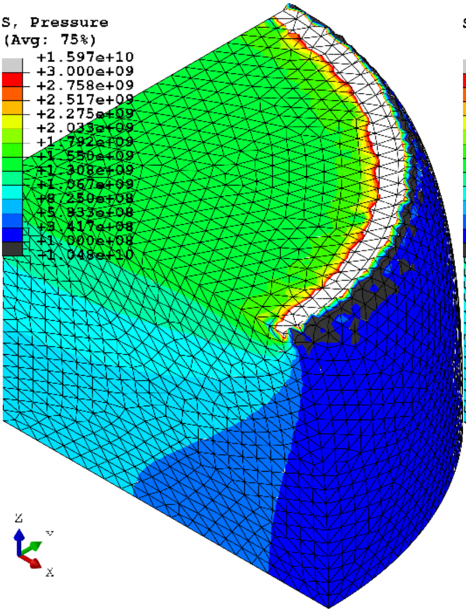
Comparison of pressure at 24% comp.



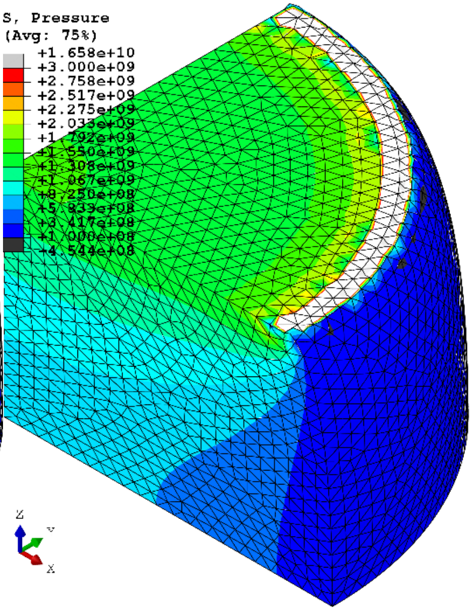
New Selective
CS-FEM-T10



ABAQUS
C3D10H



ABAQUS
C3D10MH



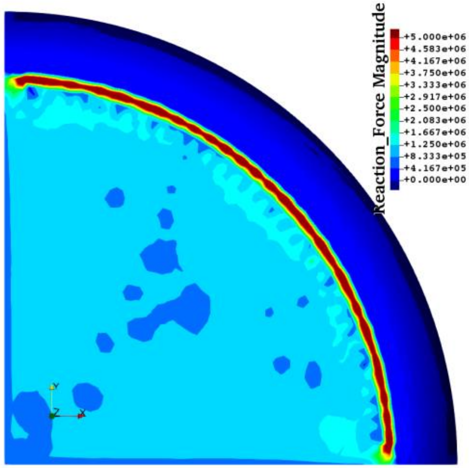
ABAQUS
C3D10HS

All results are similar to each other
except around the rim having stress singularity.

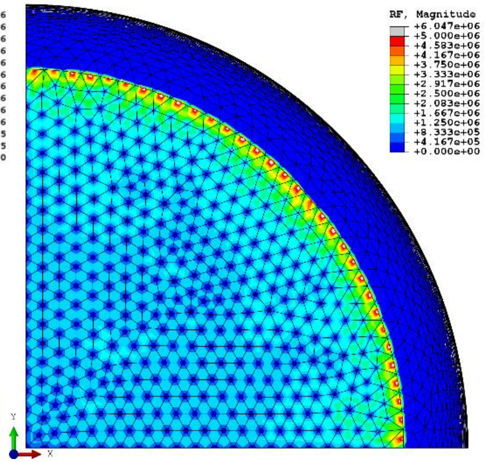
Static
Implicit

Barreling of Hyperelastic Cylinder

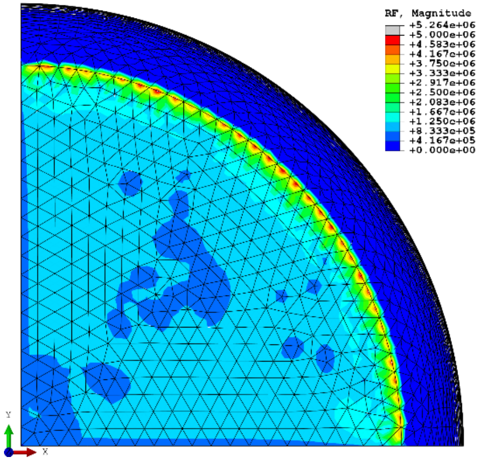
<u>Comparison of nodal reaction force at 24% comp.</u>



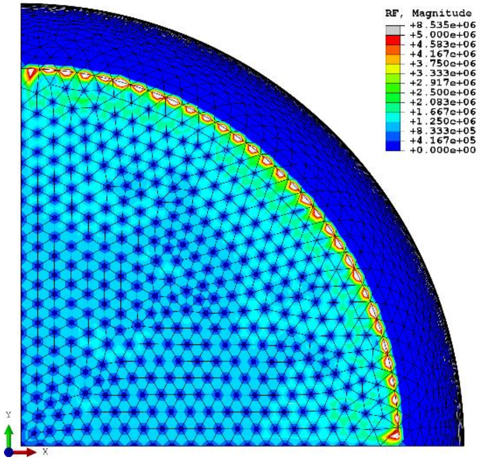
New Selective
CS-FEM-T10



ABAQUS
C3D10H



ABAQUS
C3D10MH



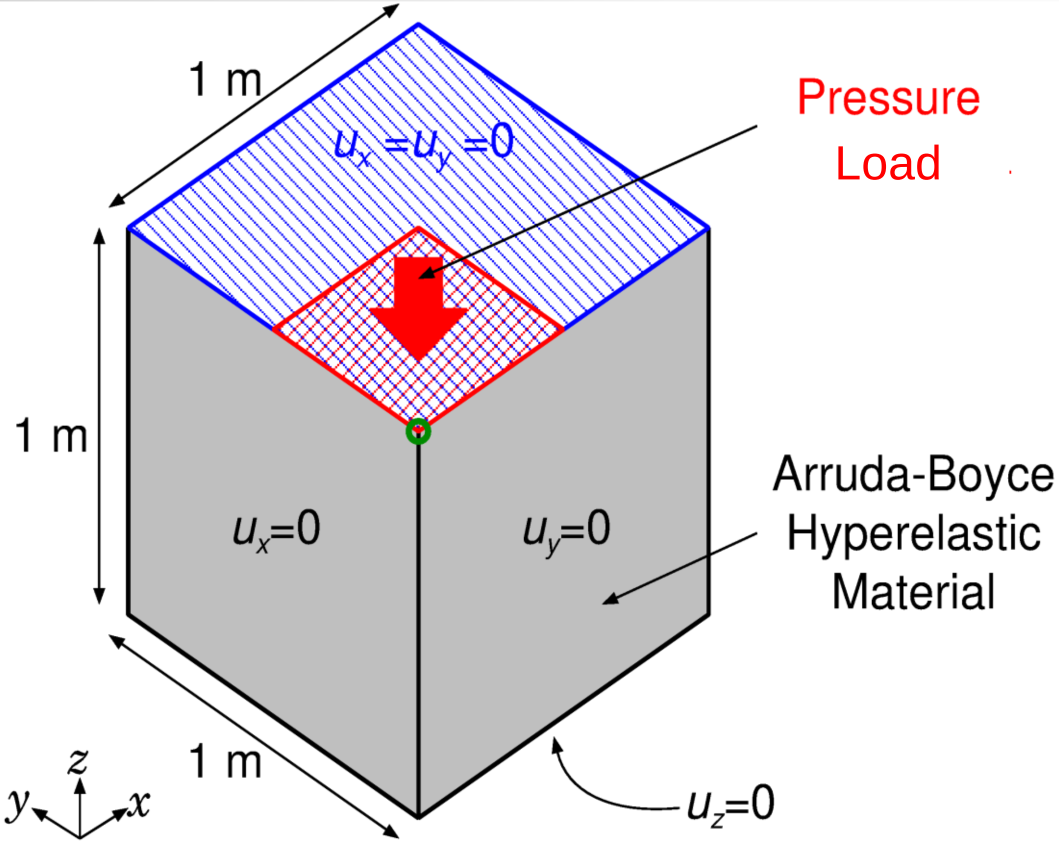
ABAQUS
C3D10HS

ABAQUS C3D10H and C3D10HS
suffer from nodal force oscillation.

Static Implicit

Compression of Hyperelastic Block

Outline

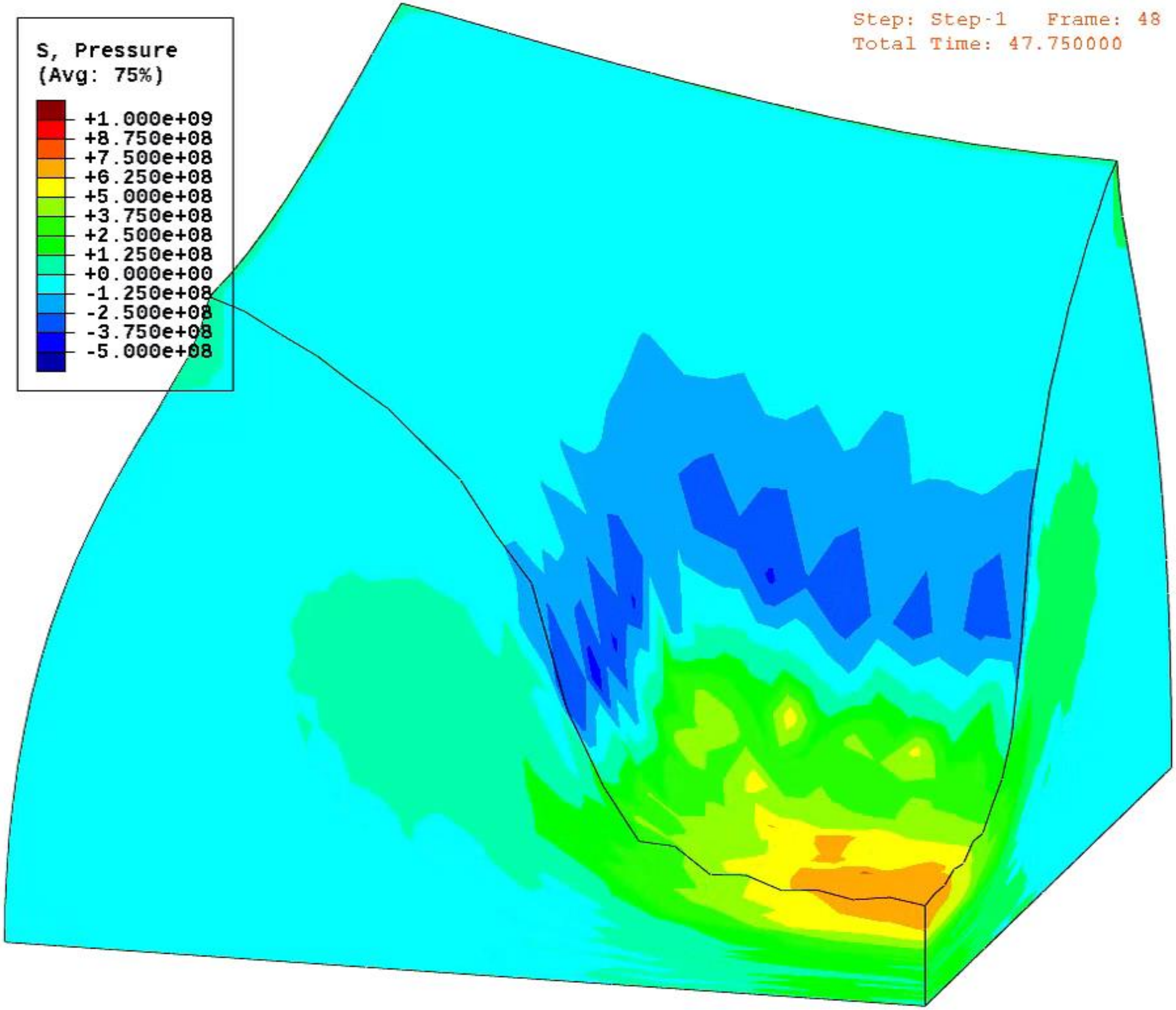


- Arruda-Boyce hyperelastic material ($\nu_{\text{ini}} = 0.499$).
- Applying pressure on ¼ of the top face.
- Compared to ABAQUS C3D10MH with the same unstructured T10 mesh.

Static
Implicit

Compression of Hyperelastic Block

Animation of pressure dist. (ABAQUS C3D10MH)

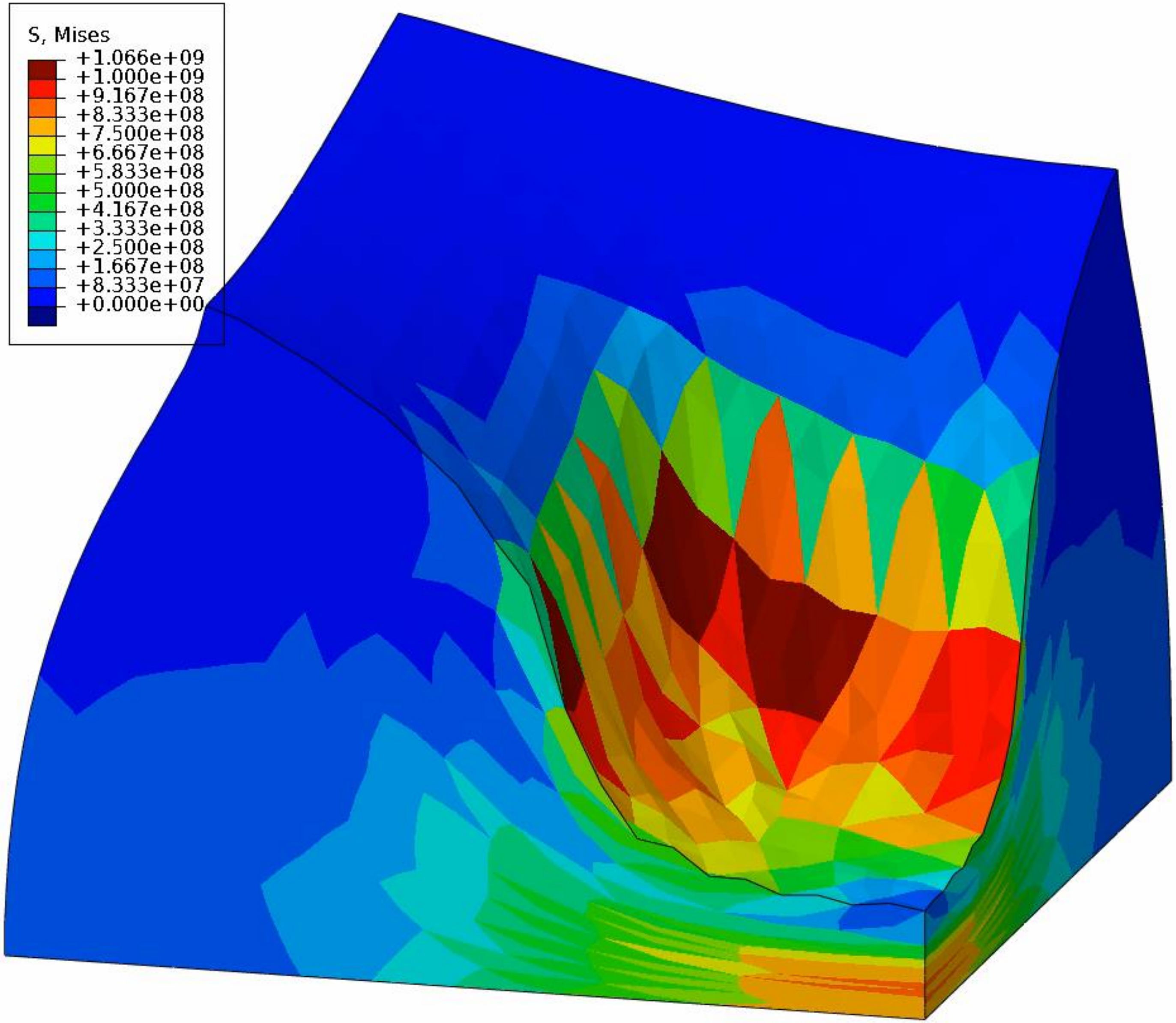


Convergence failure at **0.7 GPa** pressure

Static
Implicit

Compression of Hyperelastic Block

<u>Animation of Mises stress dist.</u>
<u>(ABAQUS C3D10MH)</u>



Convergence failure at **<u>0.7 GPa</u>** pressure

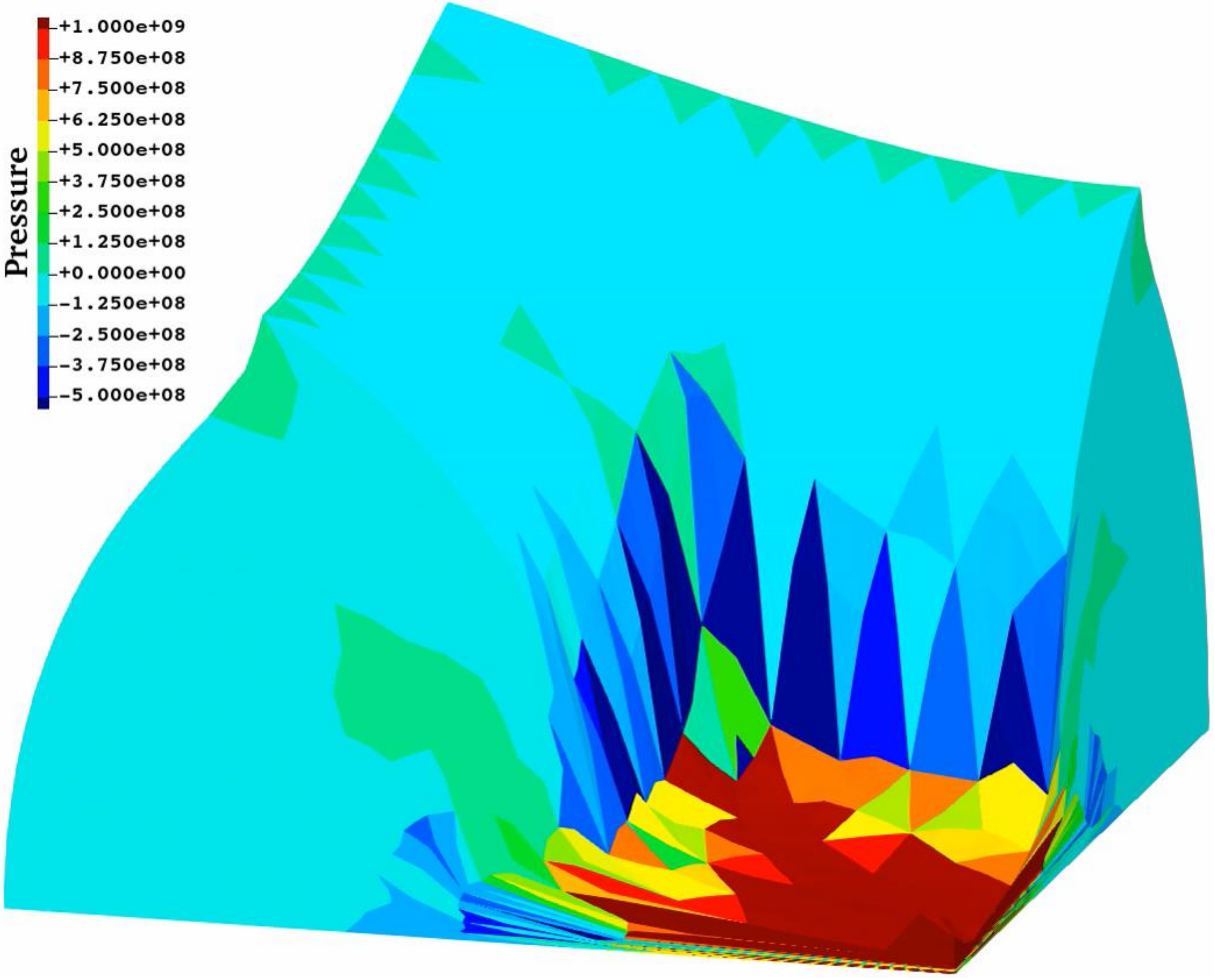
Static
Implicit

Compression of Hyperelastic Block

Animation of pressure dist. (New Selective CS-FEM-T10)

Convergence failure at **3.3GPa** pressure

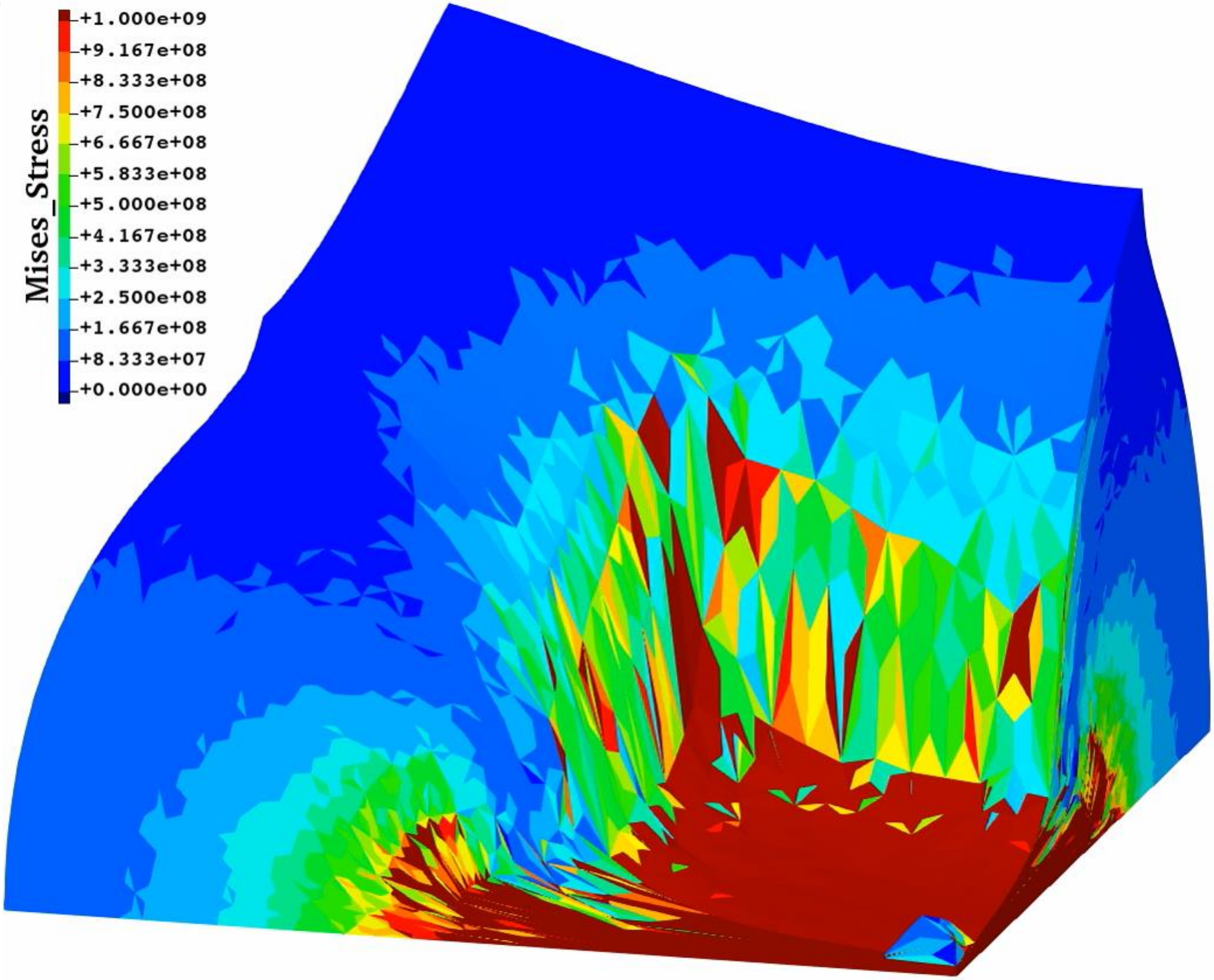
The present element is more **long-lasting (robust)** than ABAQUS C3D10MH



Static
Implicit

Compression of Hyperelastic Block

Animation of Mises stress dist. (New Selective CS-FEM-T10)

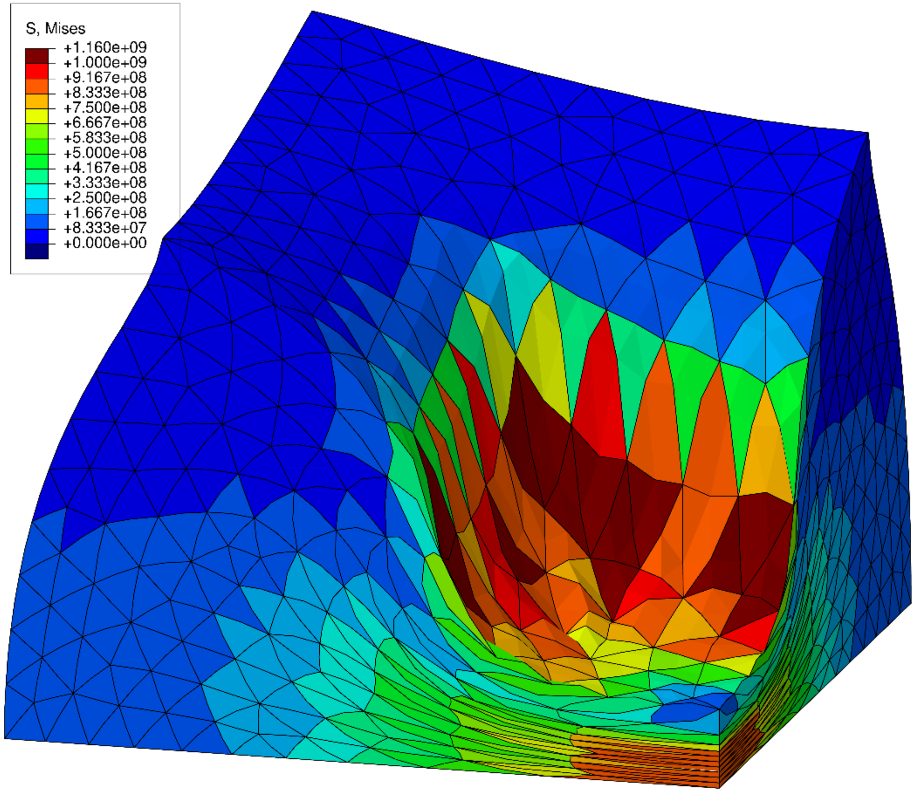


The present element presents Mises stress oscillation.

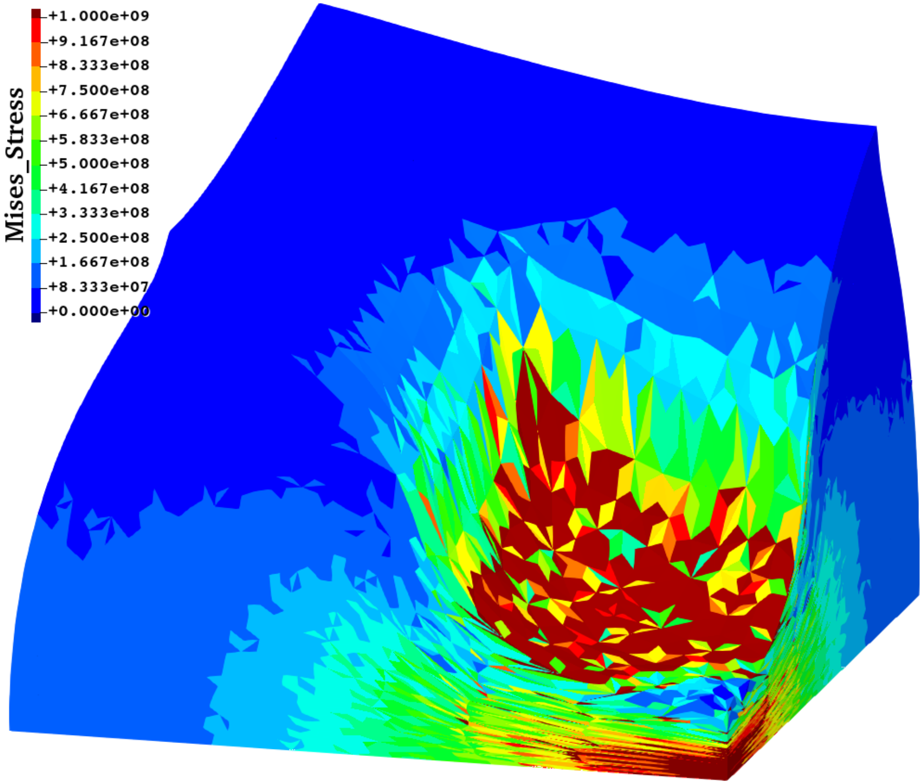
Static
Implicit

Compression of Hyperelastic Block

Misess stress dist. at 0.7 GPa pressre



ABAQUS C3D10MH



New SelectiveCS-FEM-T10

Less smooth Mises stress is observed
in SelectiveCS-FEM-T10 compared to C3D10MH.
Further improvement is still required.

Summary

Characteristics of SelectiveCS-FEM-T10

Benefits

- ✓ Accurate
  (no locking, no checkerboarding, no force oscillation).
- ✓ Robust (long-lasting in large deformation).
- ✓ No increase in DOF (No static condensation).
- ✓ Same memory & CPU costs as the other T10 elements.
- ✓ Implementable to commercial FE codes
  (e.g., ABAQUS UEL).

Drawbacks

- ✗ Mises stress oscillation in some extreme analyses.
- ✗ No longer a T4 formulation.

SelectiveCS-FEM-T10 is competitive
with the best ABAQUS T10 element, C3D10MH.

Summary

Summary

- **The present method (New SelectiveCS-FEM-T10) is more robust than the conventional one.**
- **The present method is already very good enough for practical use as compared to ABAQUS Tet elements.**

Take-home message

Please consider implementing
New SelectiveCS-FEM-T10 to your in-house code.
It's supremely useful & easy to code!!

FYI

You can download my slides at
http://www.a.sc.e.titech.ac.jp/~yonishi/

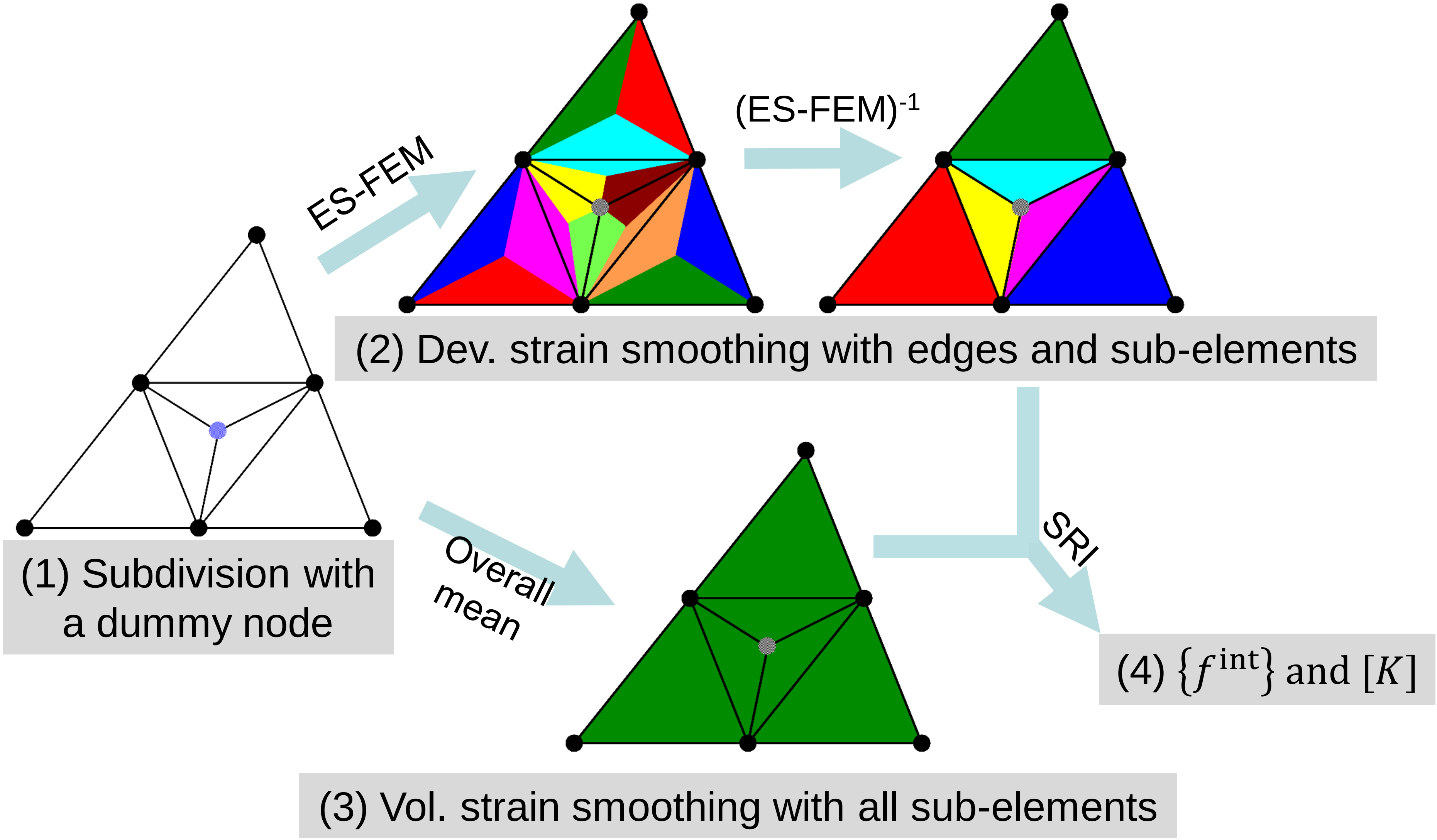
Please contact me on yonishi@a.sc.e.titech.ac.jp.

Thank you for your kind attention!

Appendix

Flowchart of Old SelectiveCS-FEM-T10

Explanation in 2D (6-node triangular element) for simplicity



(1) Subdivision with a dummy node

(2) Dev. strain smoothing with edges and sub-elements

(3) Vol. strain smoothing with all sub-elements

(4) $\{f^{\text{int}}\}$ and $[K]$

Differences between Old and New

1. The new formulation adopts radial-type mesh subdivision.
    - Strain smoothing on all edges including frame edges.
    - Larger skewness of sub-elements .
2. The new formulation has No ES-FEM^{-1} after ES-FEM.
    - Strain & stress evaluation at edges (NOT at sub-elements).

Discussions

- The old formulation is shorter-lasting than the new one probably because of the low-energy modes induced by the multiple smoothing (too much smoothing).
- The new formulation does not need multiple smoothing because any edge is owned by multiple sub-elements.