An Optimal Multiple Smoothing Scheme of Selective Cell-based Smoothed Finite Element Methods with 10-node Tetrahedral Elements for Large Deformation of Nearly Incompressible Solids

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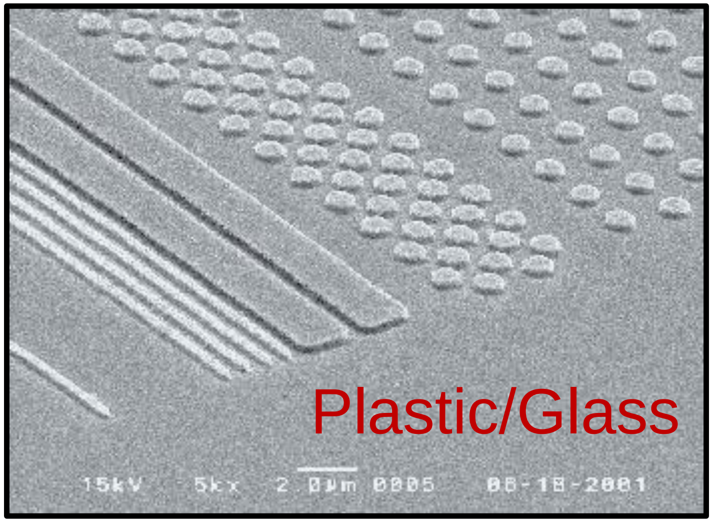
Motivation

What we want to do:

- Solve **hyper large deformation** analyses accurately and stably.
- Treat complex geometries with **tetrahedral meshes**.
- Consider **nearly incompressible materials** $(\nu \simeq 0.5)$.
- Support **contact** problems.
- Handle **auto re-meshing**.



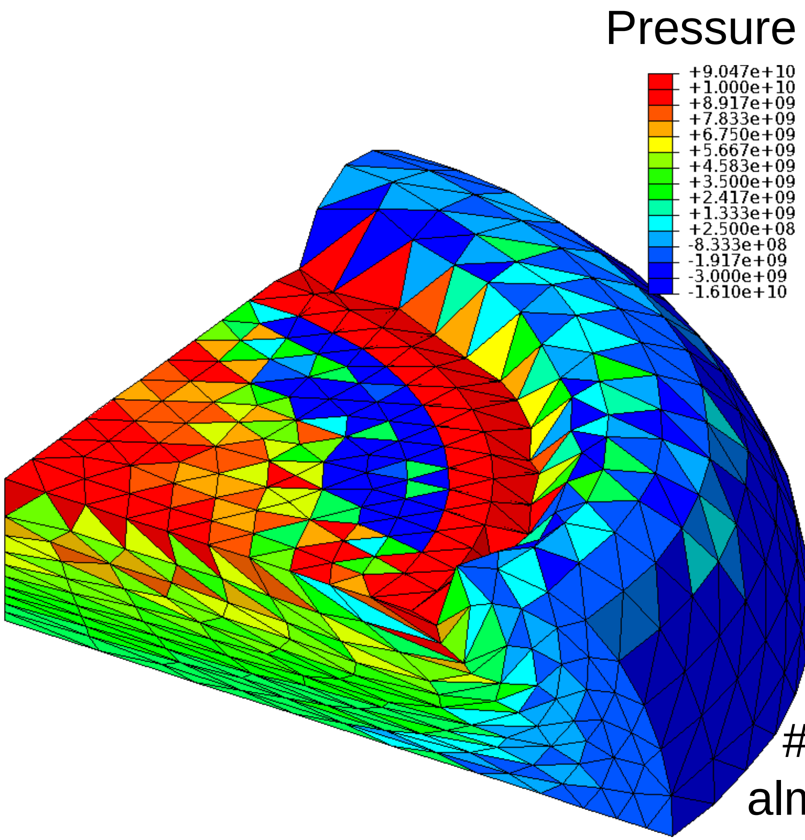
Issues

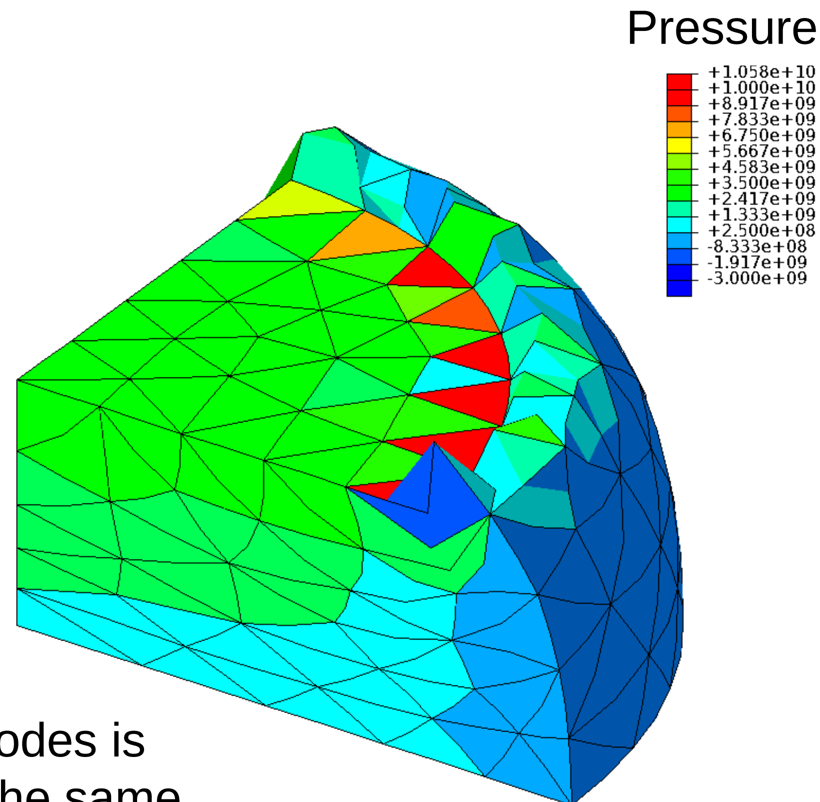
Conventional tetrahedral (T4/T10) FE formulations still have issues in accuracy or stability especially in nearly incompressible cases.

- 2nd or higher order elements:
  - ✗ Volumetric locking.
  - Accuracy loss in large strain due to intermediate nodes.
- B-bar method, F-bar method, Selective reduced integration:
  - ✗ Not applicable to tetrahedral element directly.
- F-bar-Patch method:
  - ✗ Difficulty in building good-quality patches.
- **u/p mixed (hybrid) method:**
  (e.g., ABAQUS/Standard **C3D4H** and **C3D10MH**)
  - ✗ Pressure checkerboarding, Early convergence failure etc..
- **F-bar type smoothed FEM (F-barES-FEM-T4):**
  - ✓ Accurate & stable ✗ Hard to implement in FEM codes.

Issues (cont.)

E.g.) Compression of neo-Hookean ***hyperelastic*** body with $\nu_{\mathrm{ini}} = 0.49$





# of Nodes is almost the same.

1st order hybrid T4 (C3D4H)

- ✓ No volumetric locking
- ✗ Pressure checkerboarding
- ✗ Shear & corner locking

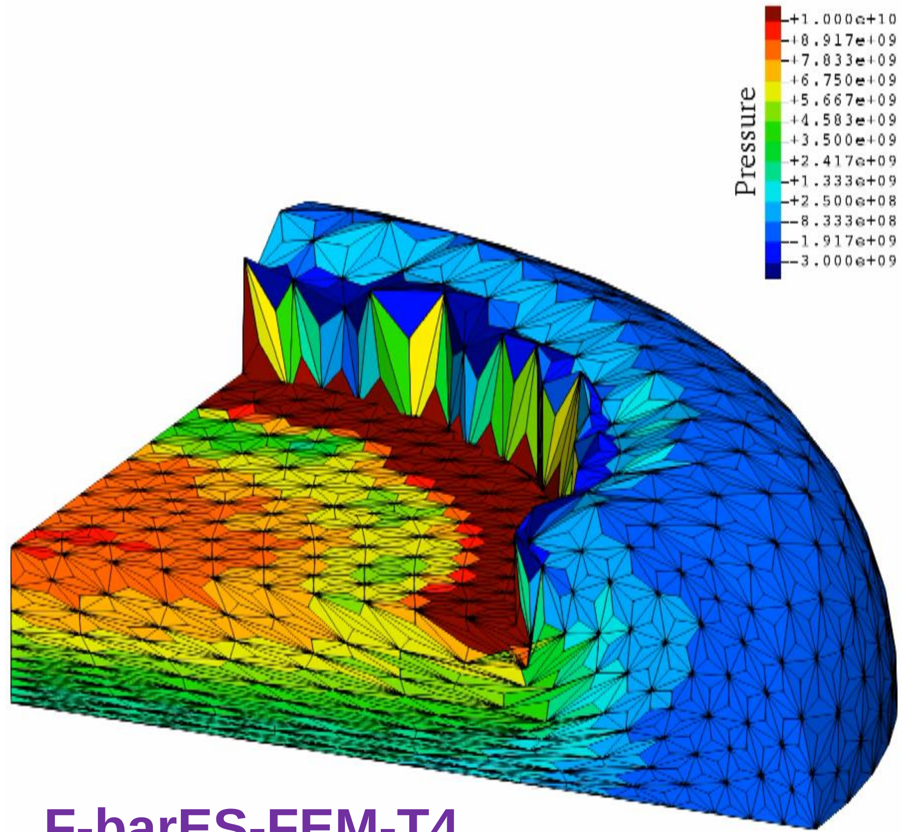
2nd order modified hybrid T10 (C3D10MH)

- ✓ No shear/volumetric locking
- ✗ Early convergence failure
- ✗ Low interpolation accuracy

Issues (cont.)

E.g.) Compression of neo-Hookean ***hyperelastic*** body with $\nu_{\mathrm{ini}} = 0.49$

Same mesh as C3D4H case.



F-barES-FEM-T4

- ✓ No shear/volumetric locking
- ✓ No corner locking
- ✓ No pressure checkerboarding
- ✓ No increase in DOF

Although F-barES-FEM-T4 is accurate and stable, ✗ it cannot be implemented in general-purpose FE software due to the adoption of ES-FEM. Also, it cosumes larger memory & CPU costs.

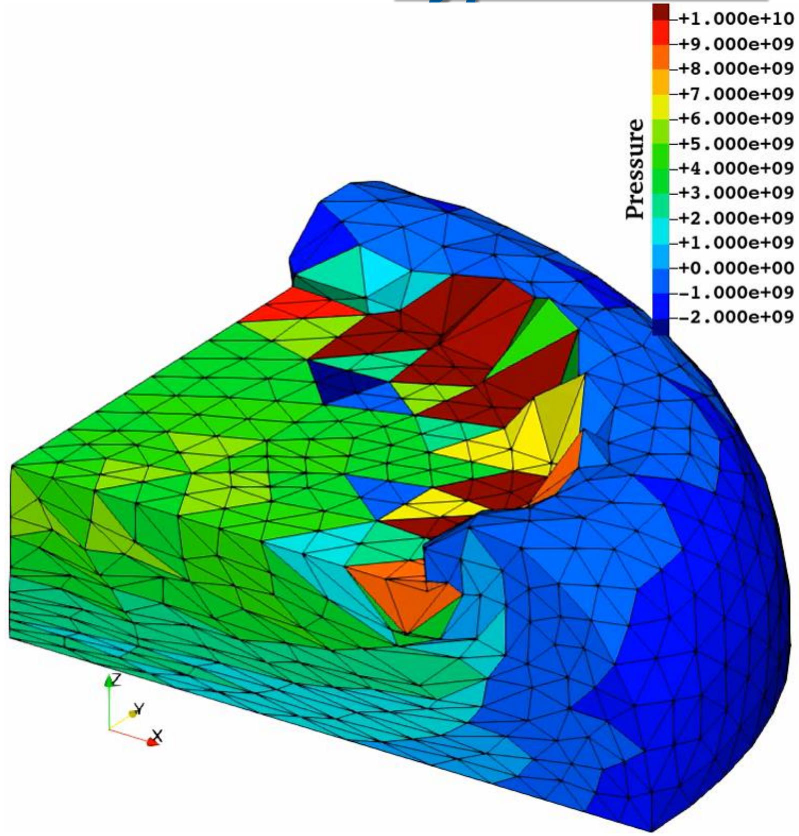
⇩

Another approach adopting CS-FEM with T10 element would be effective.

Issues (cont.)

E.g.) Compression of neo-Hookean ***hyperelastic*** body with $\nu_{\mathrm{ini}} = 0.49$

Same mesh as C3D10MH case.



As other S-FEMs, SelectiveCS-FEM-T10 has many varieties in the formulation.

The proposed method last year was **not an optimal formulation yet.**

SelectiveCS-FEM-T10 (Old Ver.)

- ✓ No shar/voluemetric locking
- ✓ Little corner locking
- ✓ Little pressure checkerboarding
- ✓ Same cost & userbility as T10 elements.

Objective

To find out an optimal formulation of SelectiveCS-FEM-T10

<u>Table of Body Contents</u>

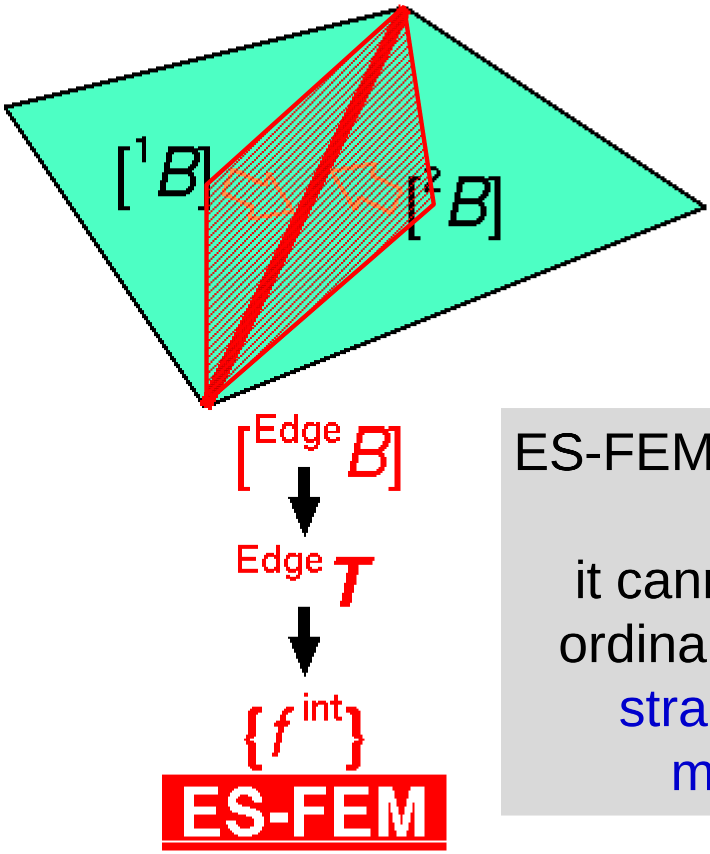
- Quick Introduction of ES-FEM, CS-FEM, and Old SelectiveES-FEM-T10
- Formulation of New SelectiveCS-FEM-T10
- Demonstrations of New SelectiveCS-FEM-T10
- Summary

Quick Introduction of ES-FEM, CS-FEM, and Old SelectiveCS-FEM-T10

Brief Review of Edge-based S-FEM (ES-FEM)

- Calculate $[B]$ at each element as usual.
- Distribute $[B]$ to the connecting edges with area weight and build $[{}^{\mathrm{Edge}}B]$.
- Calculate $\boldsymbol{F}, \boldsymbol{T}, \{f^{\mathrm{int}}\}$ etc. in each edge smoothing domain.

As if putting an integration point on each edge center

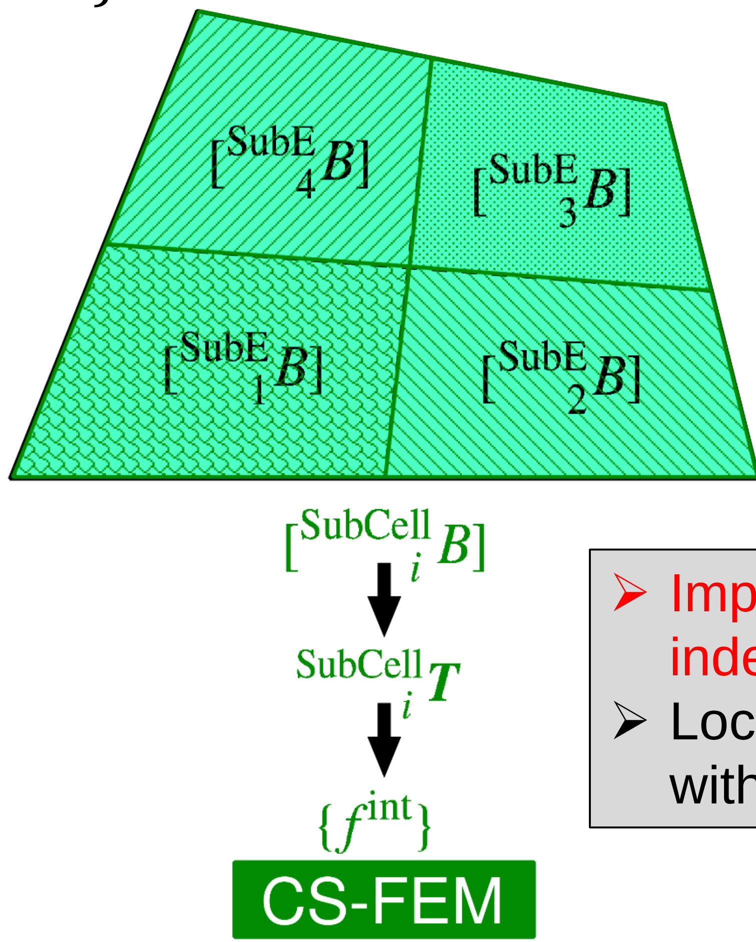


ES-FEM can avoid shear locking.
However,
it cannot be implemented in ordinary FE codes due to the strain smoothing across multiple elements...

Brief Review of Cell-based S-FEM (CS-FEM)

- Subdivide each element into some sub-element.
- Calculate $[{}^{\mathrm{SubE}}B]$ at each sub-element.
- Calculate $\boldsymbol{F}, \boldsymbol{T}, \{f^{\mathrm{int}}\}$ etc. in each sub-element.

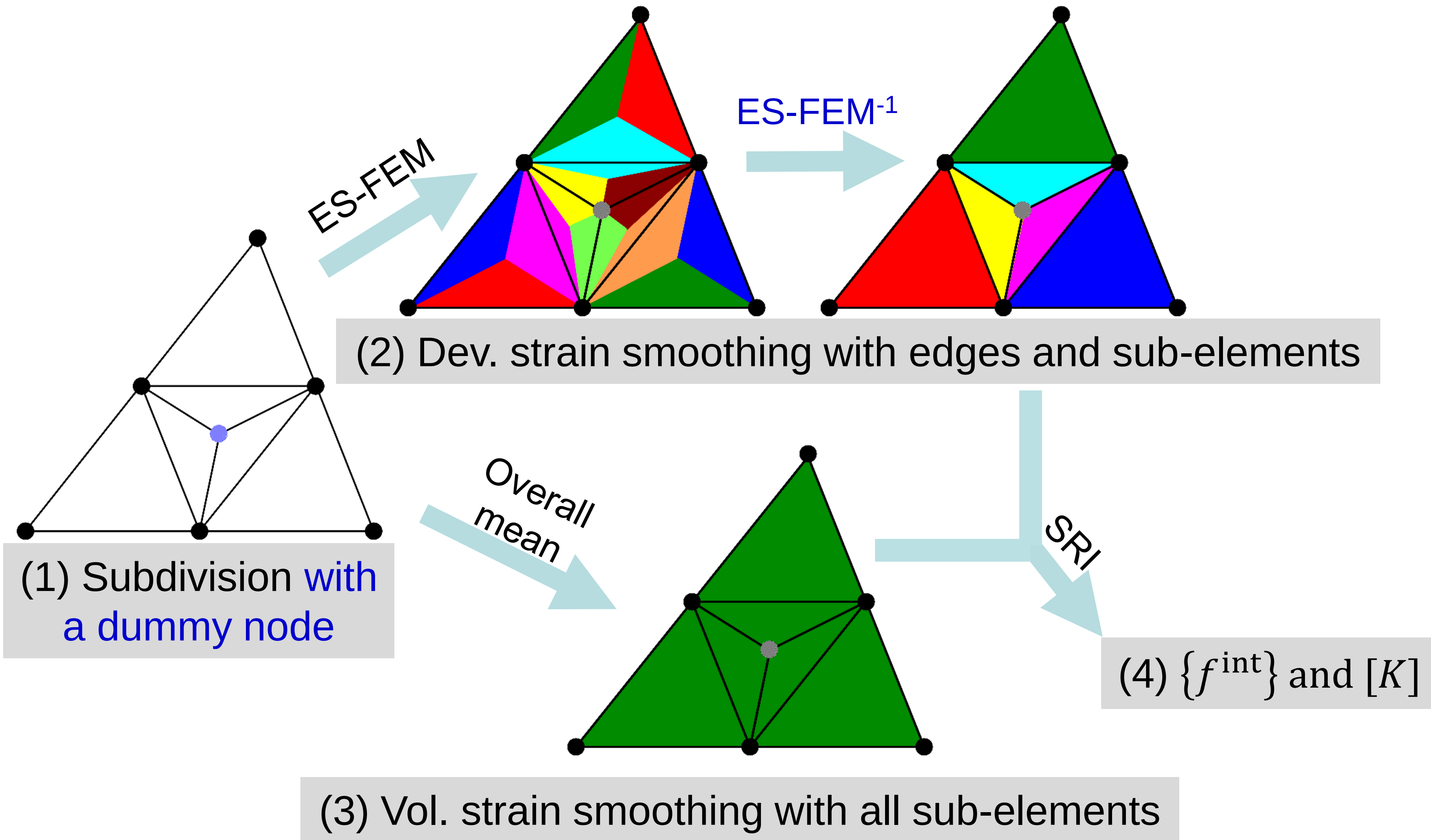
As if putting
an integration point
on each sub-element



- Implementable as an independent finite element.
- Locking can be avoided with SRI etc..

Flowchart of Old SelectiveCS-FEM-T10

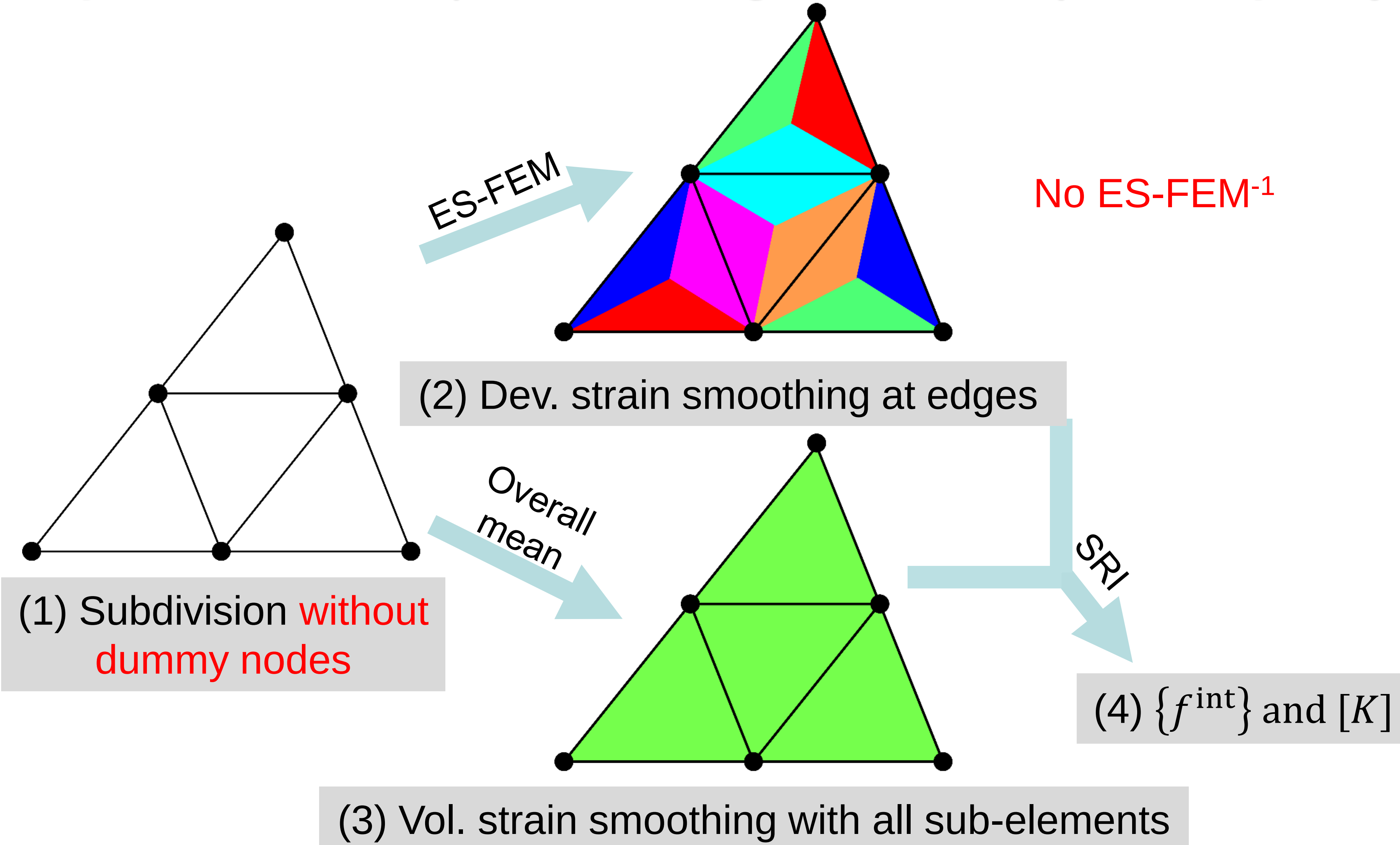
Explanation in 2D (6-node triangular element) for simplicity



Formulation of New SelectiveCS-FEM-T10

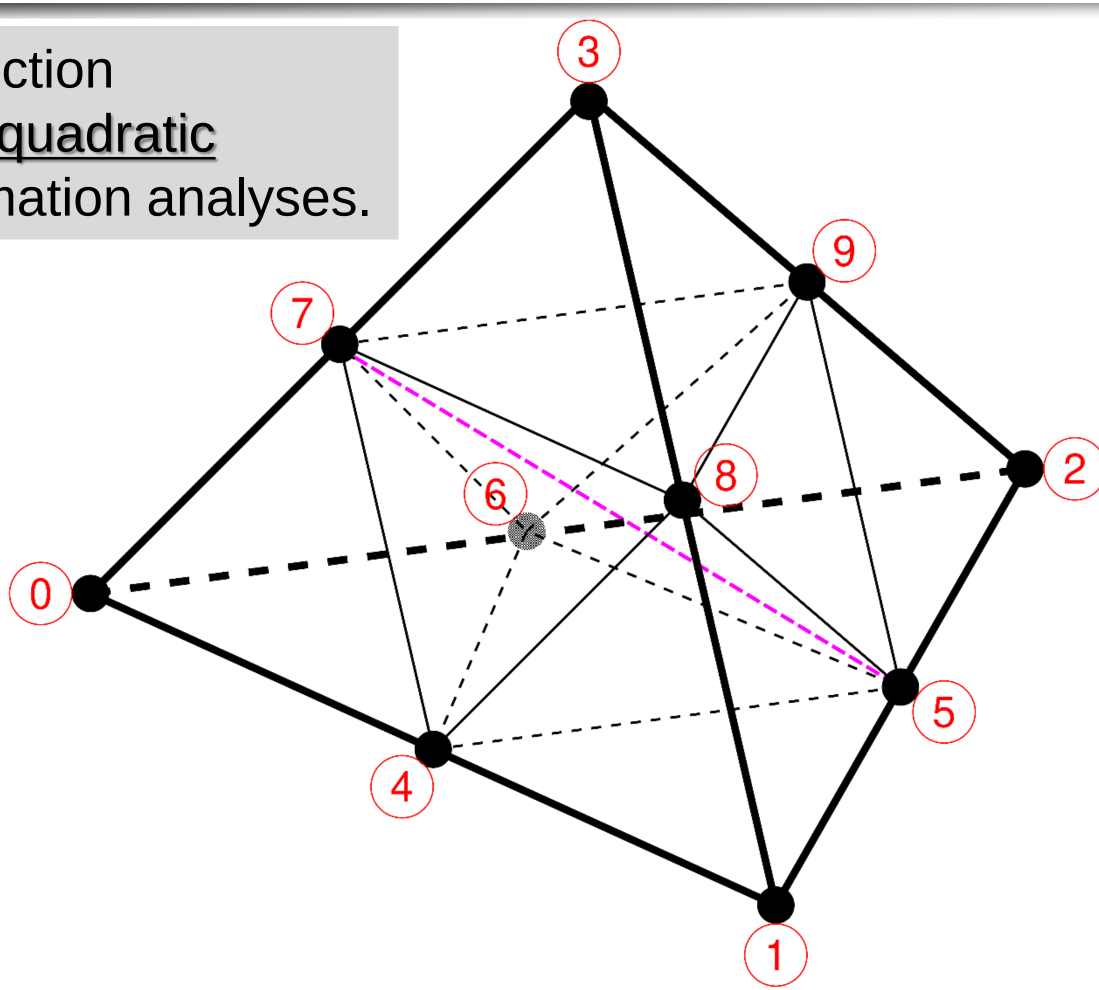
Flowchart of New SelectiveCS-FEM-T10

Explanation in 2D (6-node triangular element) for simplicity



(1) Subdivision into T4 Sub-elements

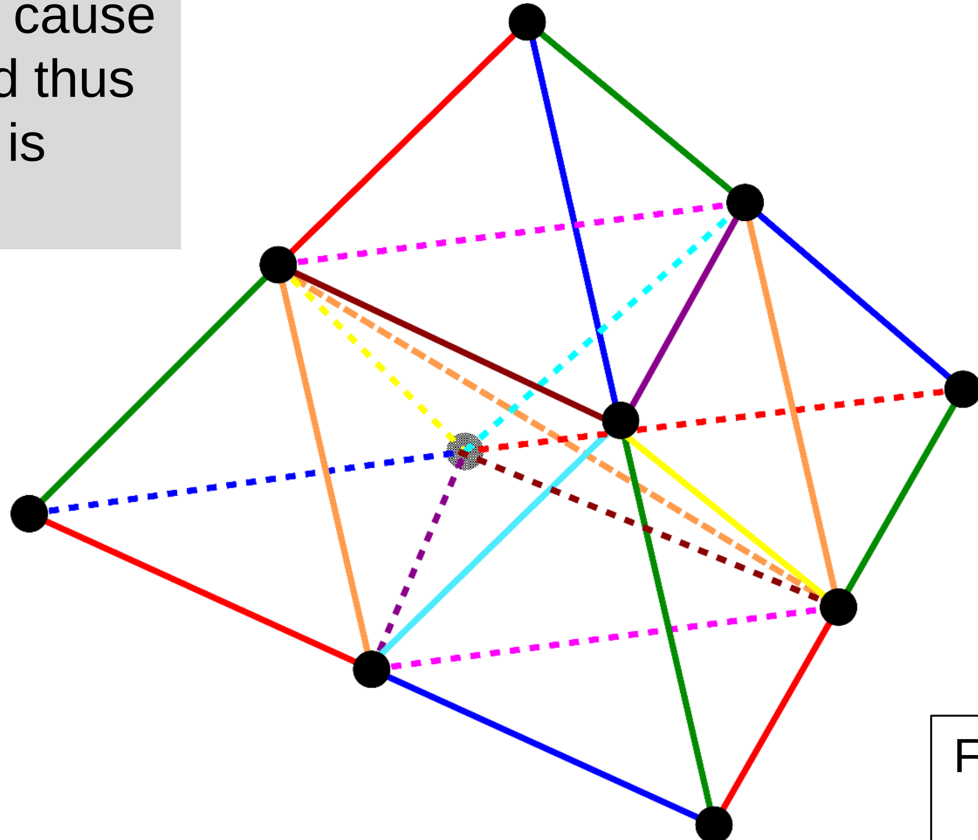
The shape function
<u>should not be quadratic</u>
in large deformation analyses.



- Introduce no dummy node (i.e., asymmetric element).
- Subdivide a T10 element into <u>eight T4 sub-elements</u> and calculate their *B*-matrices and strains.

(2) Deviatoric Strain Smoothing

T4 sub-elements cause shear locking and thus strain smoothing is necessary.

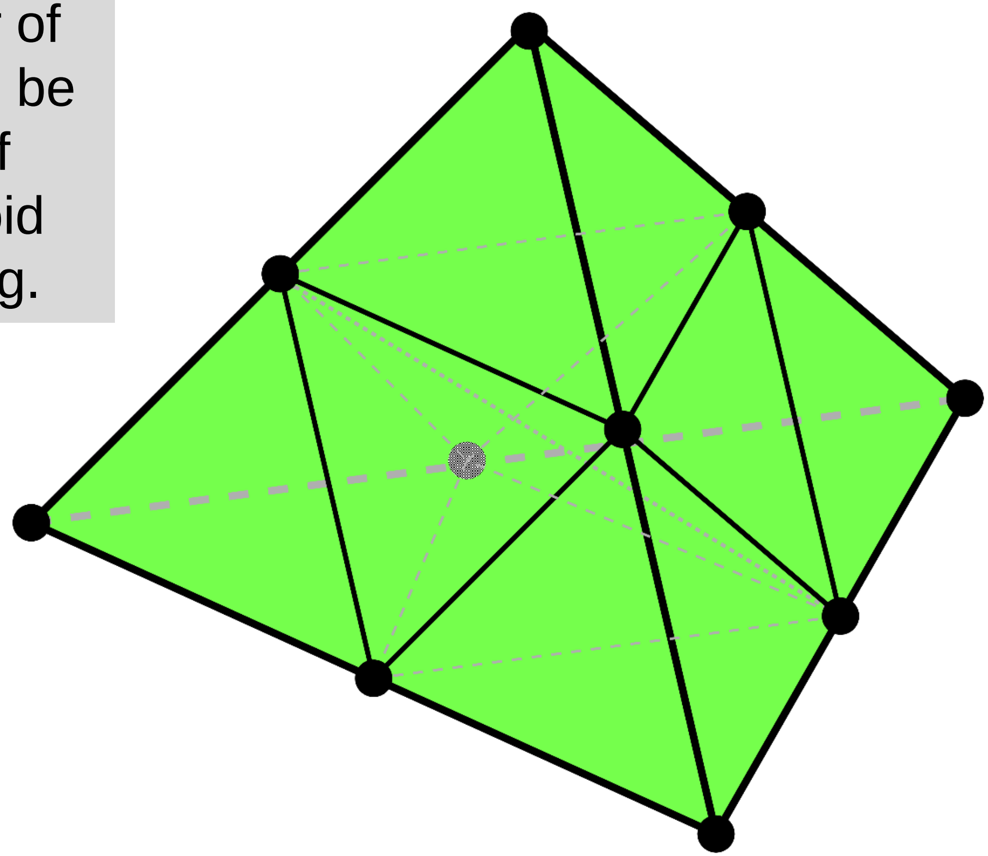


From 8 sub-elements to 25 edges

- Perform strain smoothing in the manner of ES-FEM (i.e., average dev. strains of sub-elements at edges).
- Evaluate deviatoric strain and stress at edges.

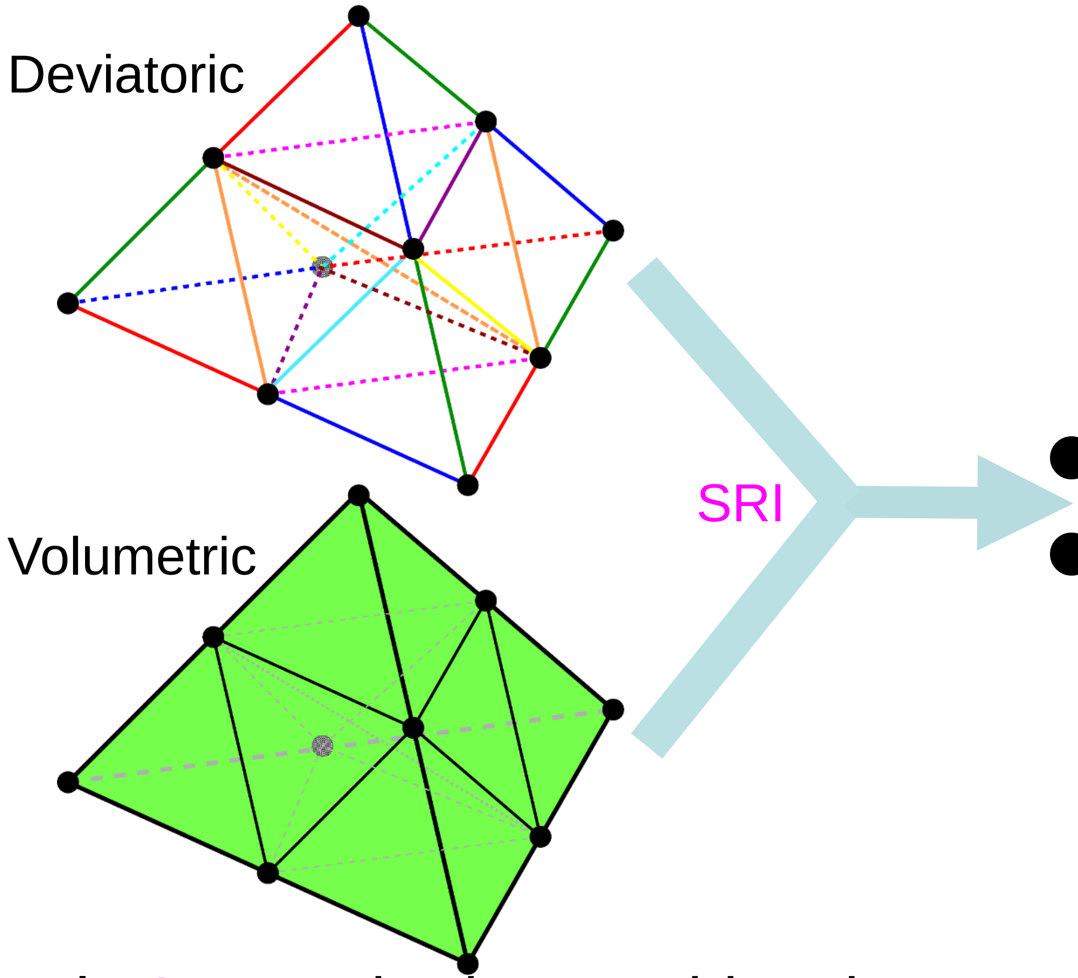
(3) Volumetric Strain Smoothing

The spatial order of vol. strain should be lower than that of dev. strain to avoid volumetric locking.



- Treat the overall mean vol. strain of all sub-elements as the uniform element vol. strain (i.e., same approach as SRI elements).

(4) Combining with SRI Method



- Internal force $\{f^{\text{int}}\}$
- Stiffness $[K]$

- Apply SRI method to combine the Dev. & Vol. parts and obtain $\{f^{\text{int}}\}$ and $[K]$.

Differences between Old and New

1. The new formulation has NO dummy node at the center of an element.
   - Fewer sub-elements and edges.
   - Asymmetric element.

2. The new formulation has No ES-FEM^{-1} after ES-FEM.
   - Strain & stress evaluation at edges.
   - No strain smoothing at frame edges.

Intuitively, the lack of element symmetry and frame edge smoothing is not good for accuracy and stability; however, the new formulation is better in fact.

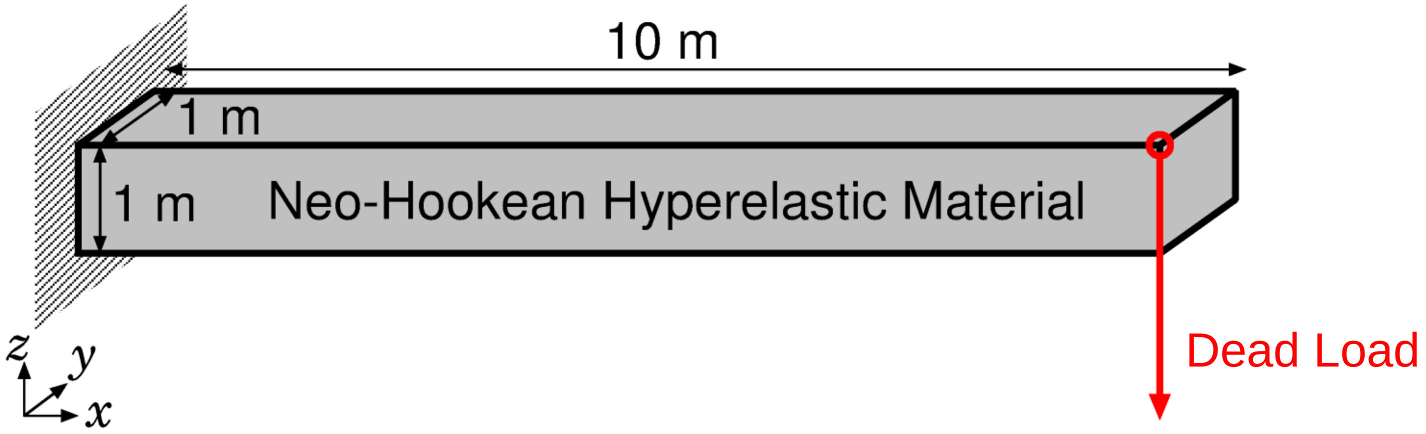
Its reason has not revealed yet.

Demonstration of

New SelectiveCS-FEM-T10

Bending of Hyperelastic Cantilever

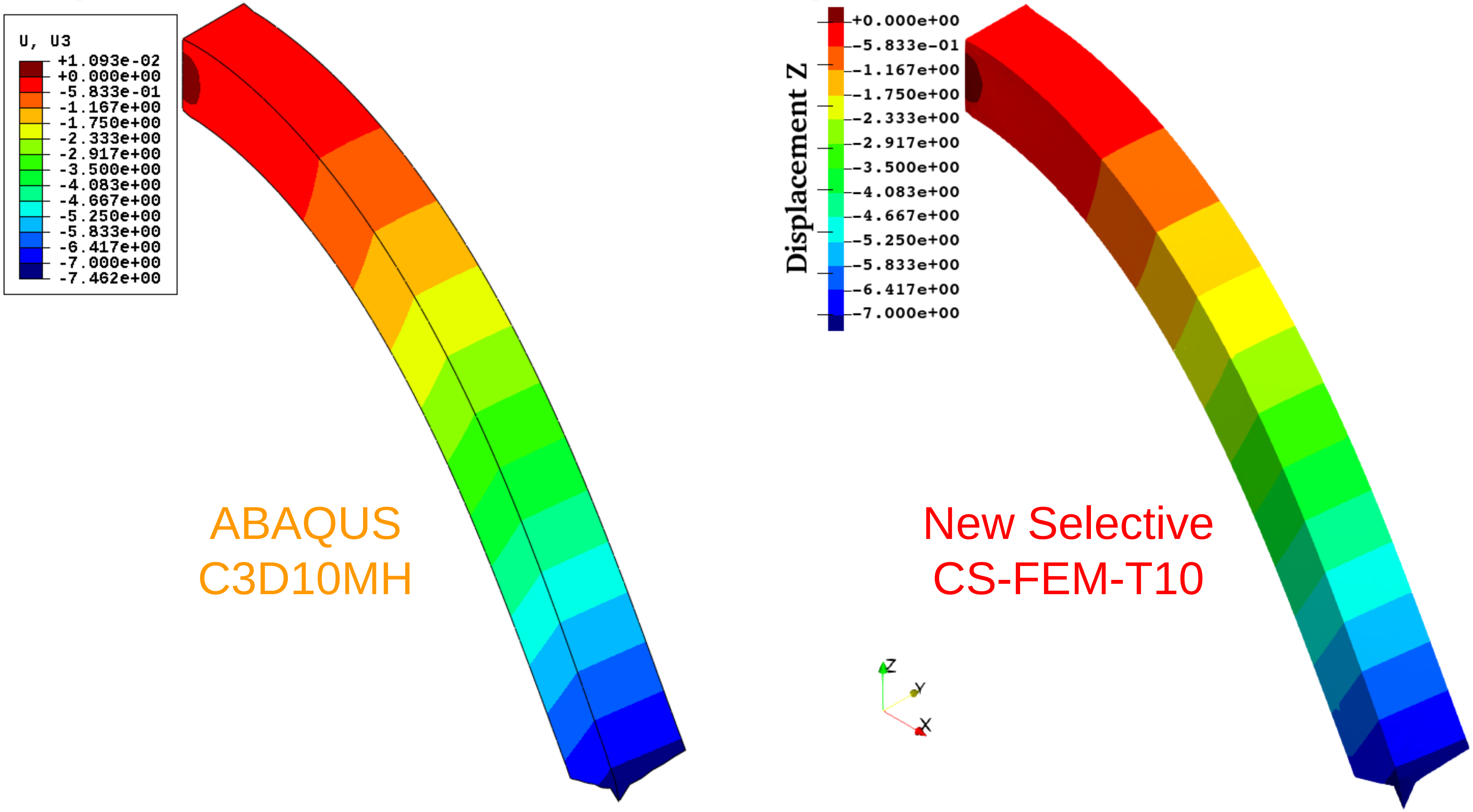
Outline



- Neo-Hookean hyperelastic material
- Initial Poisson's ratio: $\nu_0 = 0.49$
- Compared to ABAQUS C3D10MH (modified hybrid T10 element) with the same mesh.

Bending of Hyperelastic Cantilever

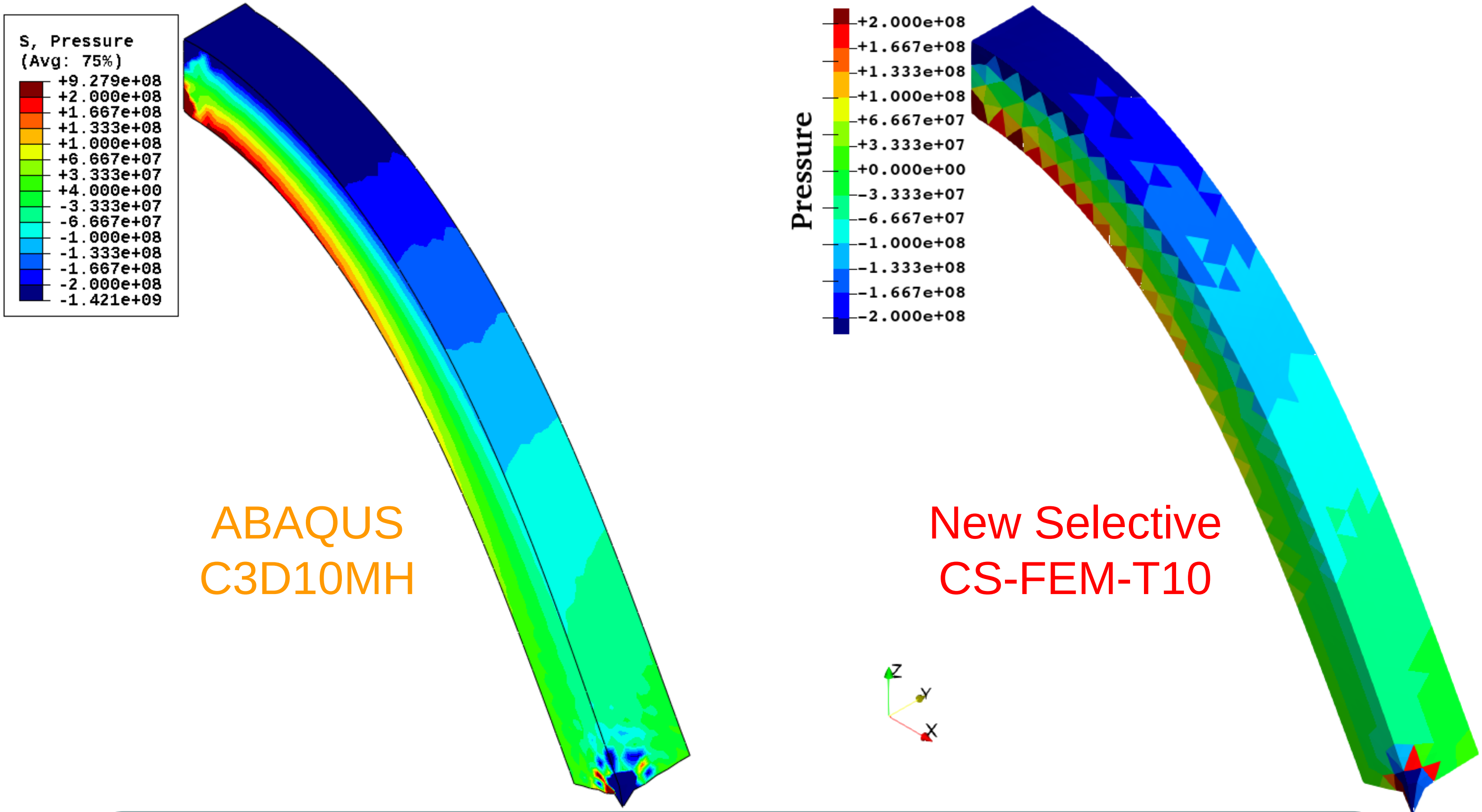
Comparison of the deflection disp. at the final state



No volumetric locking is observed.

Bending of Hyperelastic Cantilever

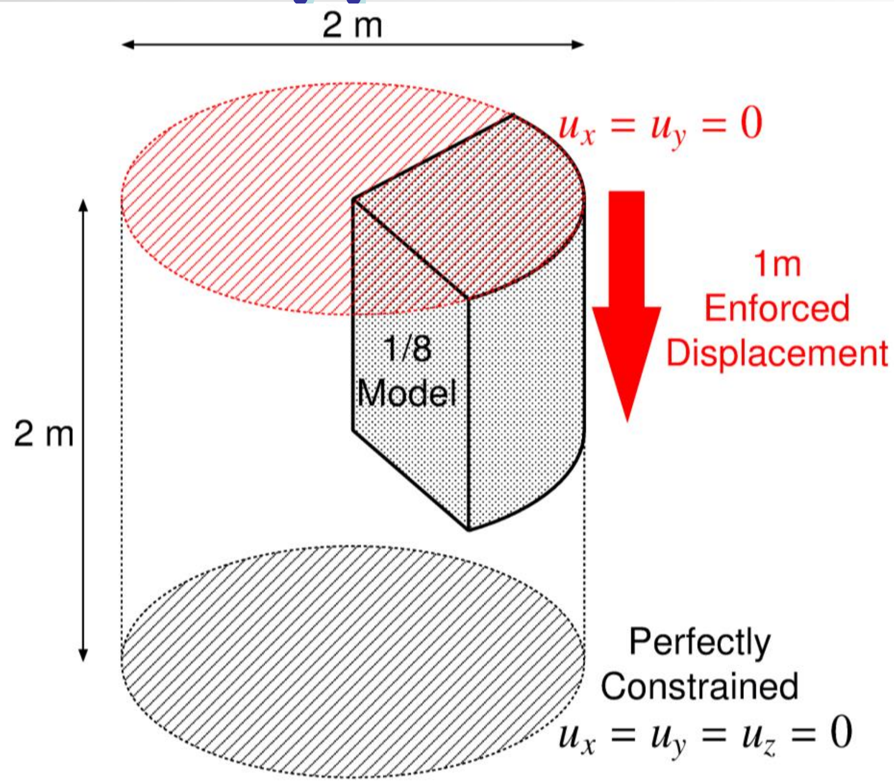
Comparison of the pressure dist. at the final state



Almost the same pressure distributions
with no checkerboarding.

Barreling of Hyperelastic Cylinder

Outline

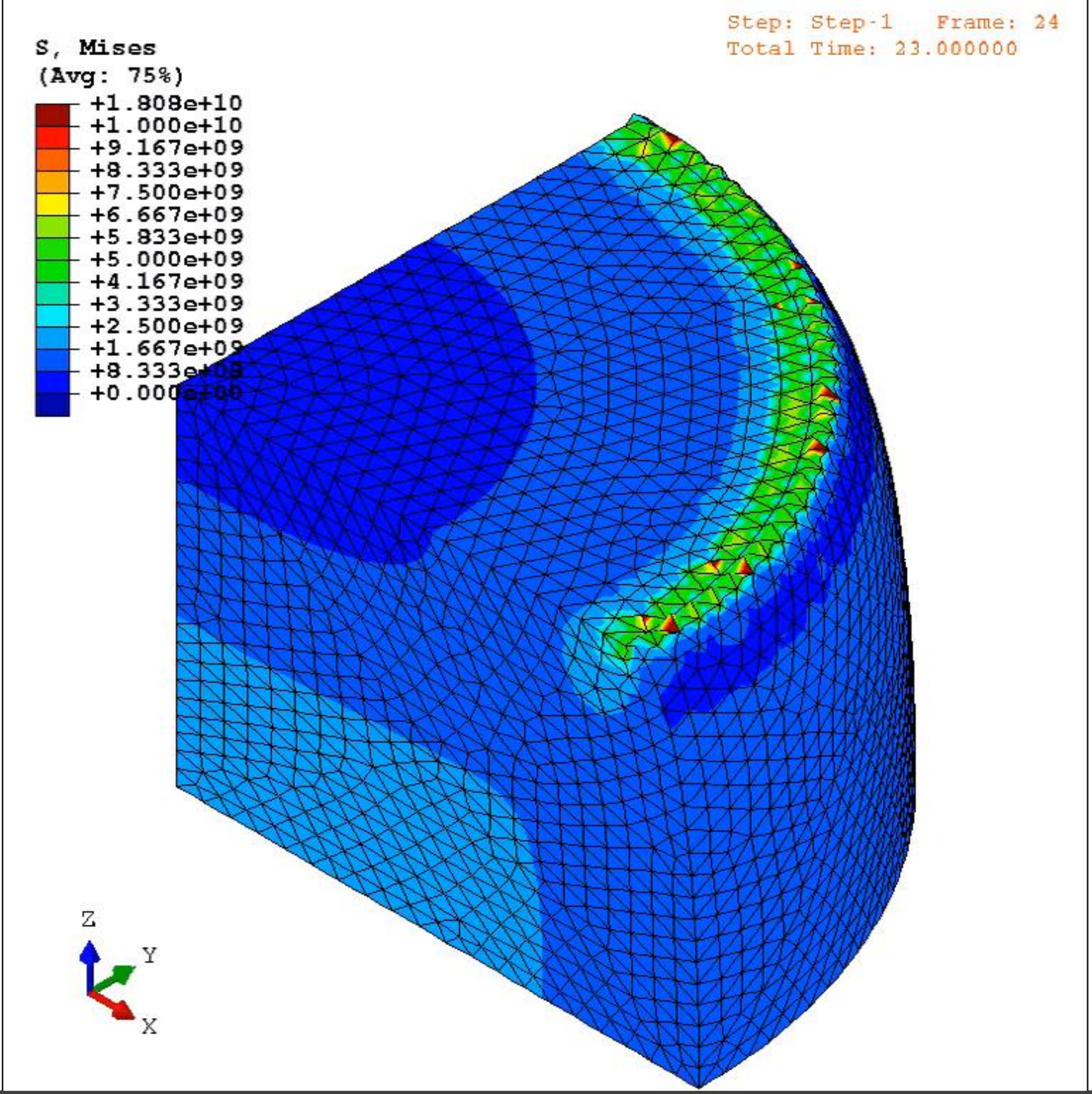


- Enforce axial displacement on the top face.
- Neo-Hookean body with $\nu_{\mathrm{ini}} = 0.49$.
- Compare results with ABAQUS T10 hybrid elements (C3D10H, C3D10MH, C3D10HS) using the same mesh.

Barreling of Hyperelastic Cylinder

Animation of Mises stress (ABAQUS C3D10MH)

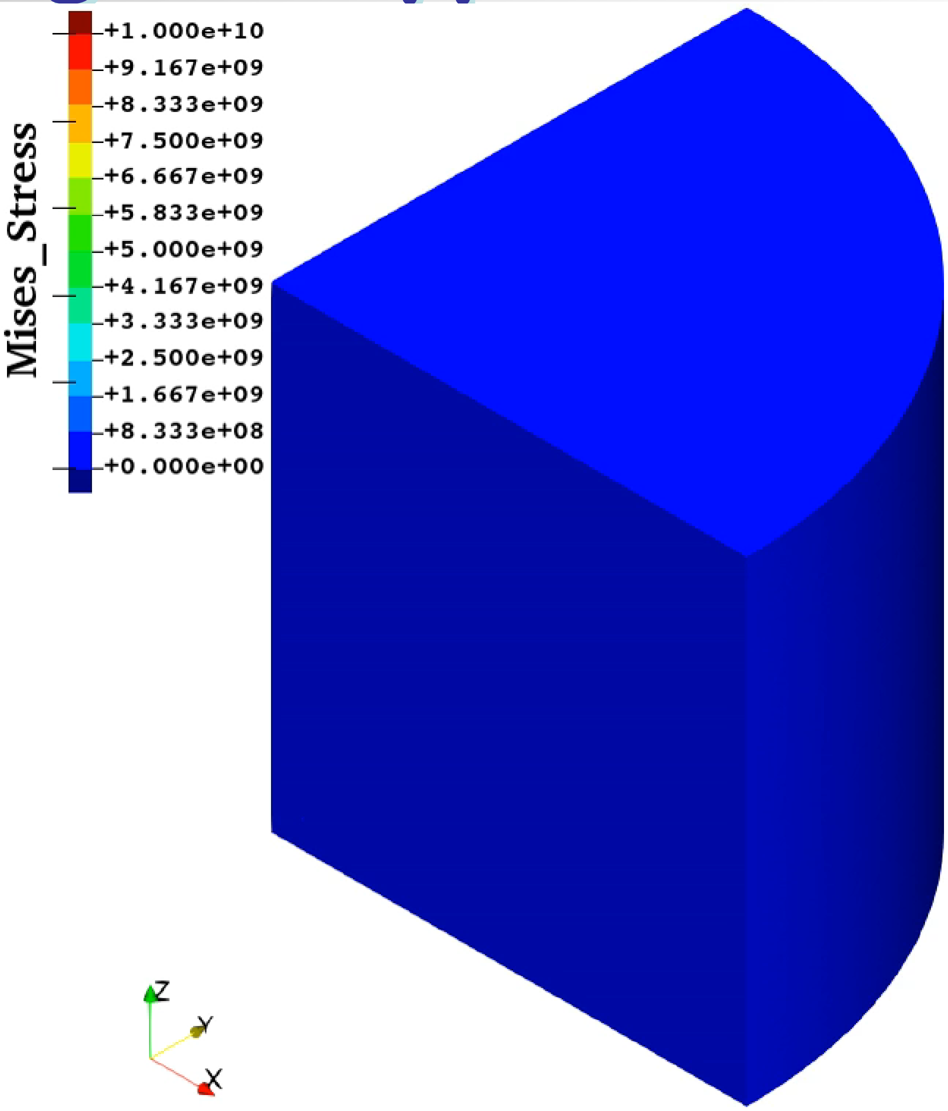
Convergence failure at **24%** compression



Unnaturally oscillating distributions are obtained around the rim.

Barreling of Hyperelastic Cylinder

***Animation of Mises stress* (*New Selective CS-FEM-T10*)**



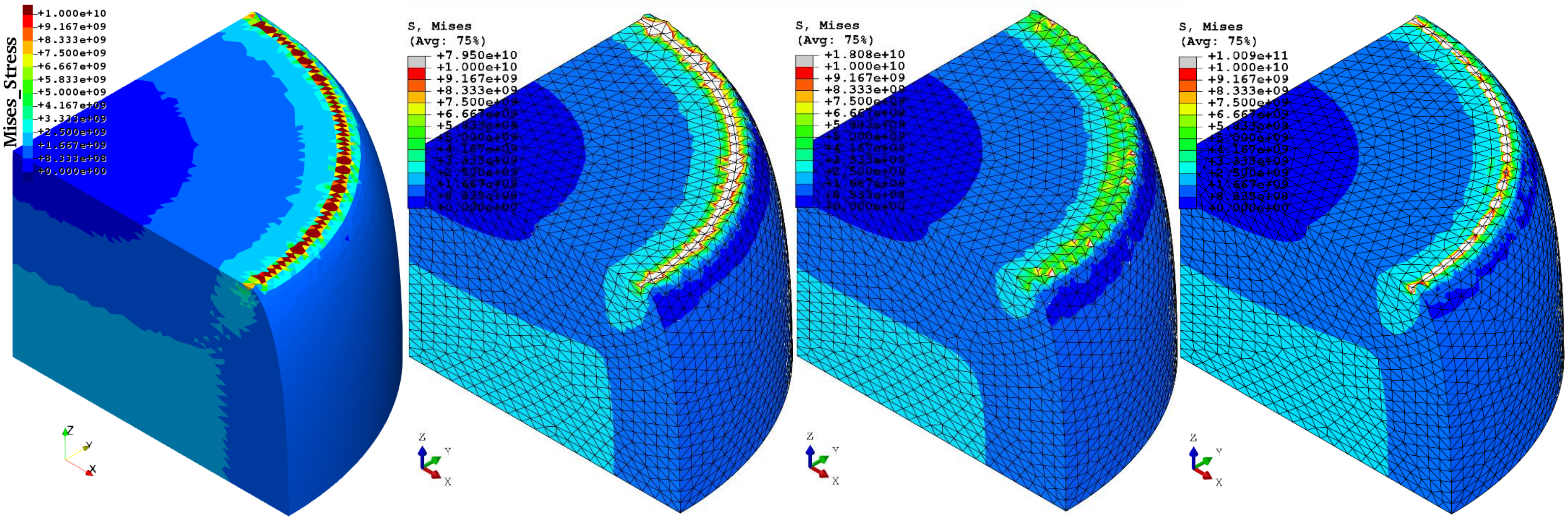
Smooth distributions are obtained except around the rim.

Convergence failure at **43%** compression

The present element is more **robust** than ABAQUS C3D10MH

Barreling of Hyperelastic Cylinder

Comparison of Mises stress at 24% comp.

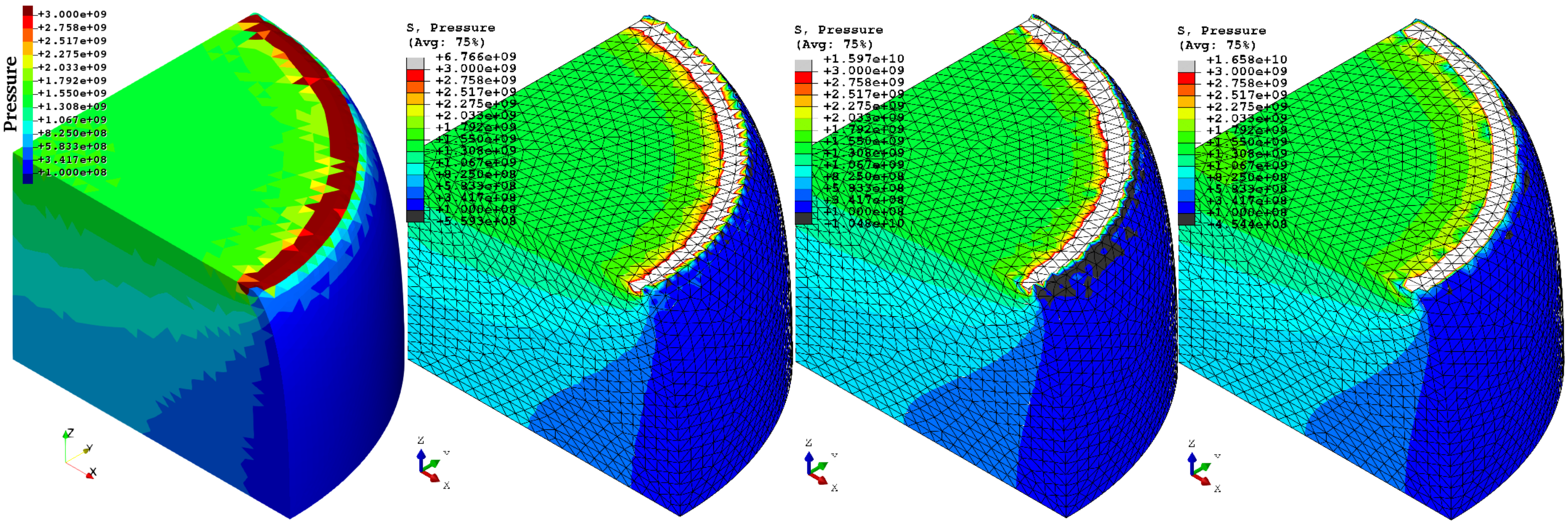


New Selective CS-FEM-T10 | ABAQUS C3D10H | ABAQUS C3D10MH | ABAQUS C3D10HS

All results are similar to each other
except around the rim having stress singularity.

Barreling of Hyperelastic Cylinder

Comparison of pressure at 24% comp.

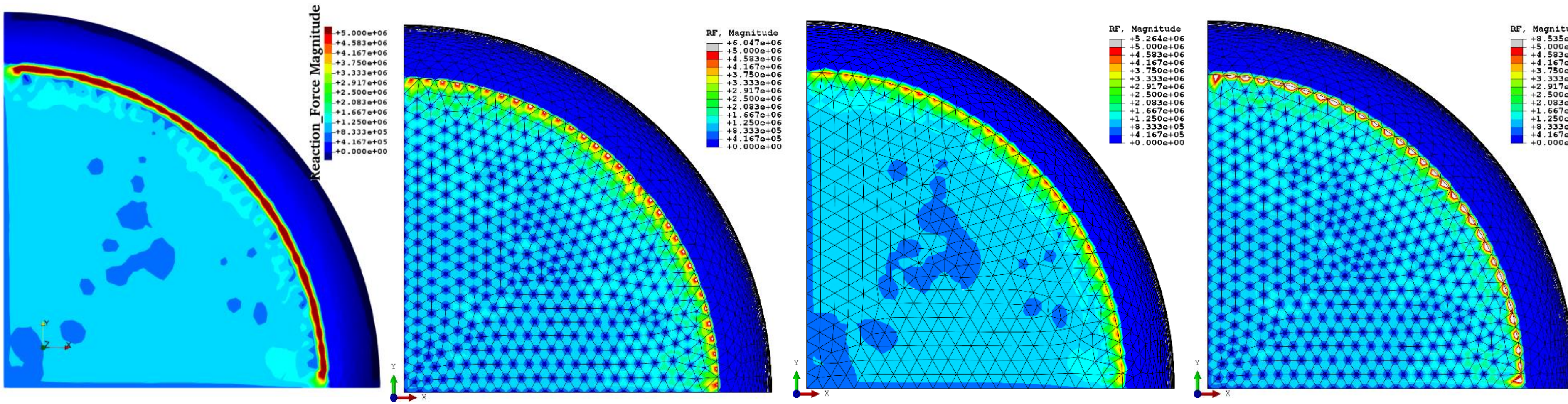


New Selective CS-FEM-T10 | ABAQUS C3D10H | ABAQUS C3D10MH | ABAQUS C3D10HS

All results are similar to each other
except around the rim having stress singularity.

Barreling of Hyperelastic Cylinder

Comparison of nodal reaction force at 24% comp.



New Selective CS-FEM-T10

ABAQUS C3D10H

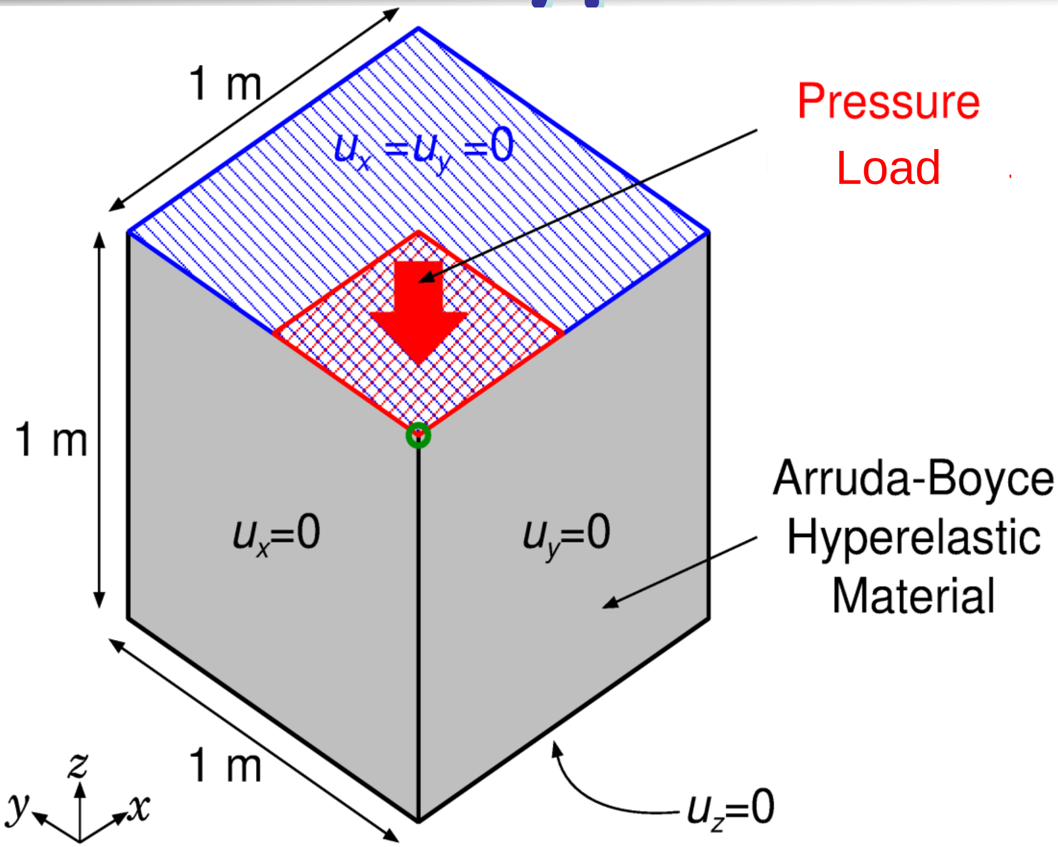
ABAQUS C3D10MH

ABAQUS C3D10HS

ABAQUS C3D10H and C3D10HS suffer from nodal force oscillation.

Compression of Hyperelastic Block

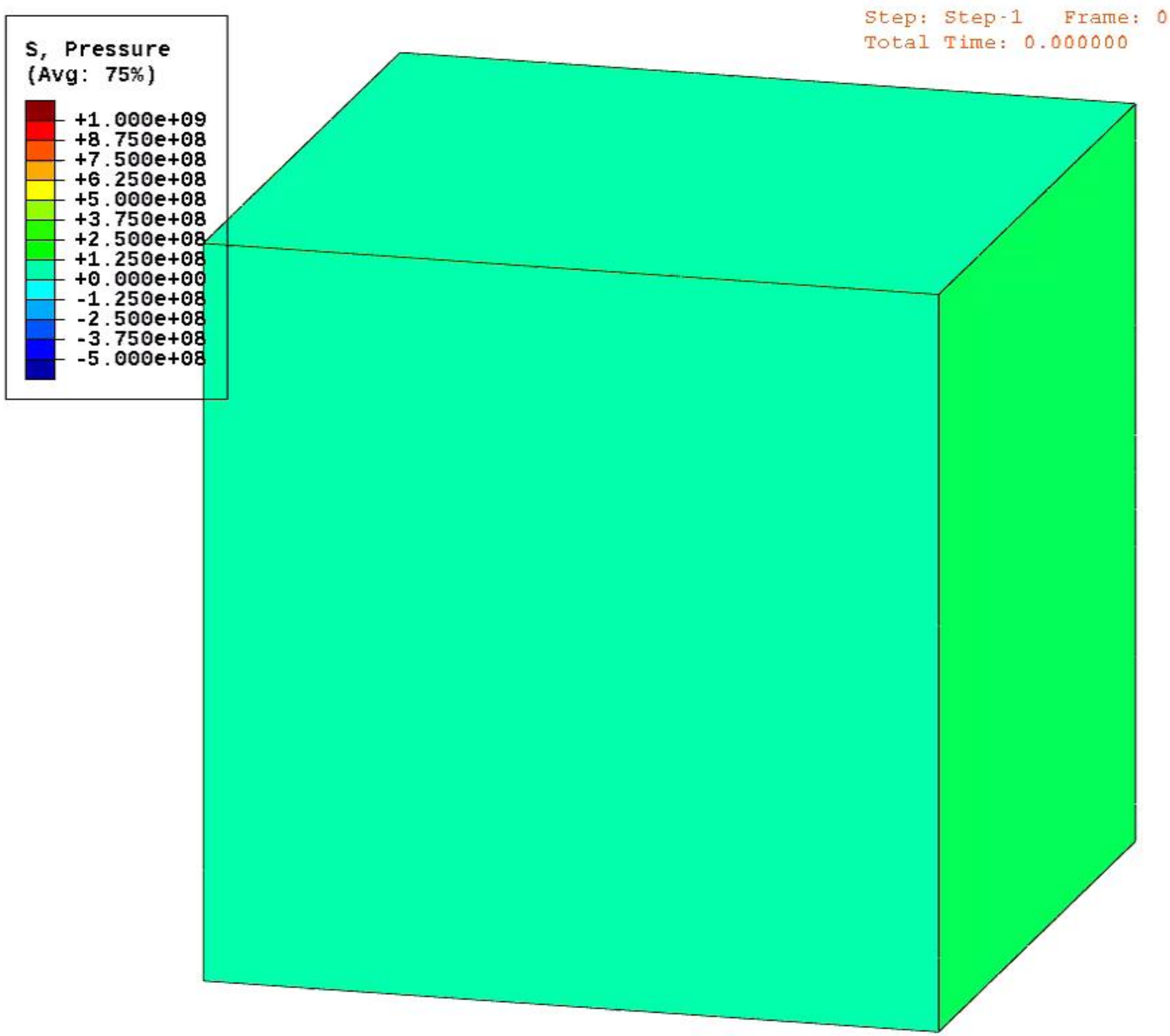
Outline



- Arruda-Boyce hyperelastic material ($\nu_{\mathrm{ini}} = 0.499$).
- Applying pressure on ¼ of the top face.
- Compared to ABAQUS C3D10MH with the same unstructured T10 mesh.

Compression of Hyperelastic Block

Animation of pressure dist. (ABAQUS C3D10MH)

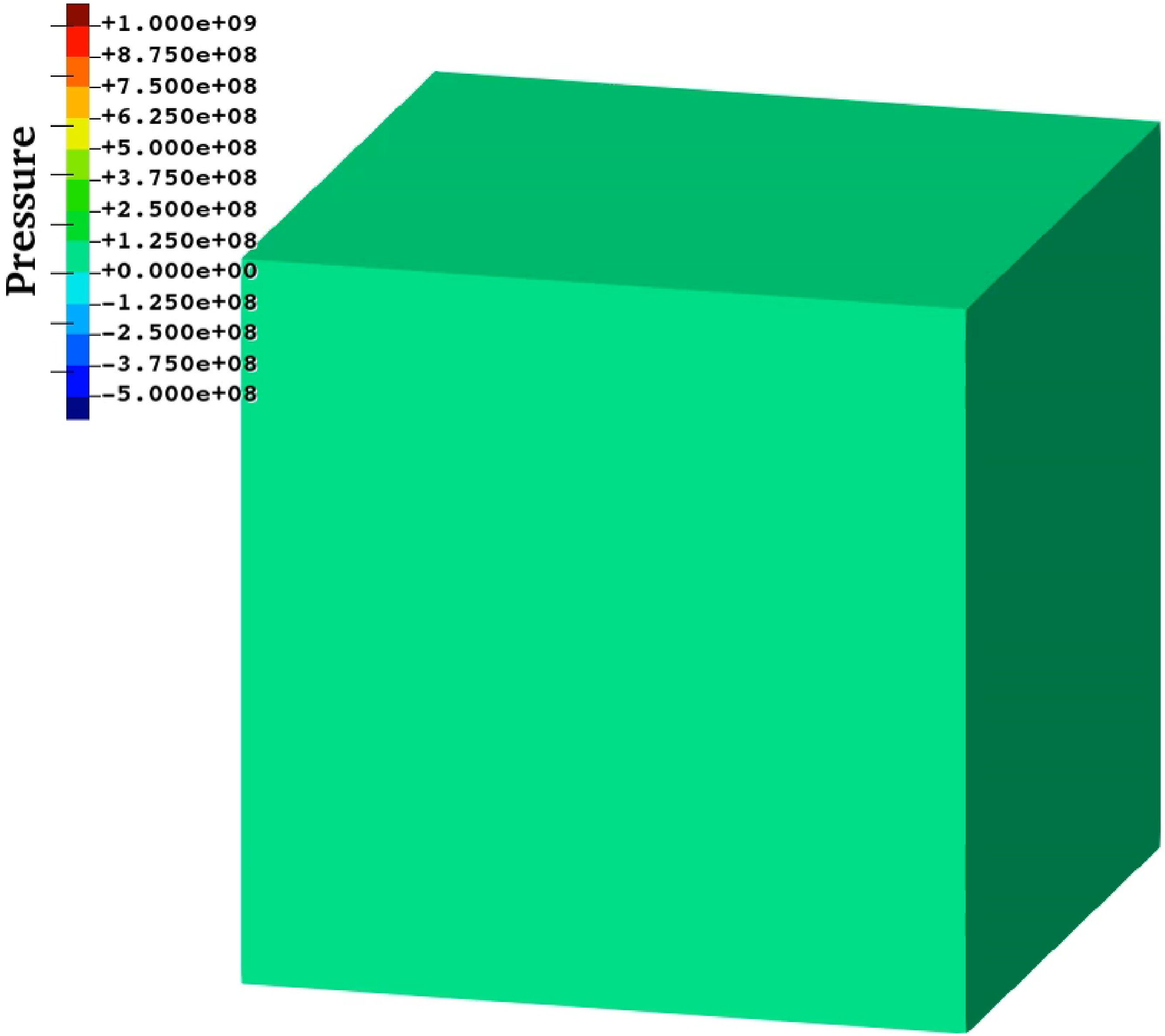


Convergence failure at **0.71 GPa** pressure

Compression of Hyperelastic Block

Animation of pressure dist. (New Selective CS-FEM-T10)

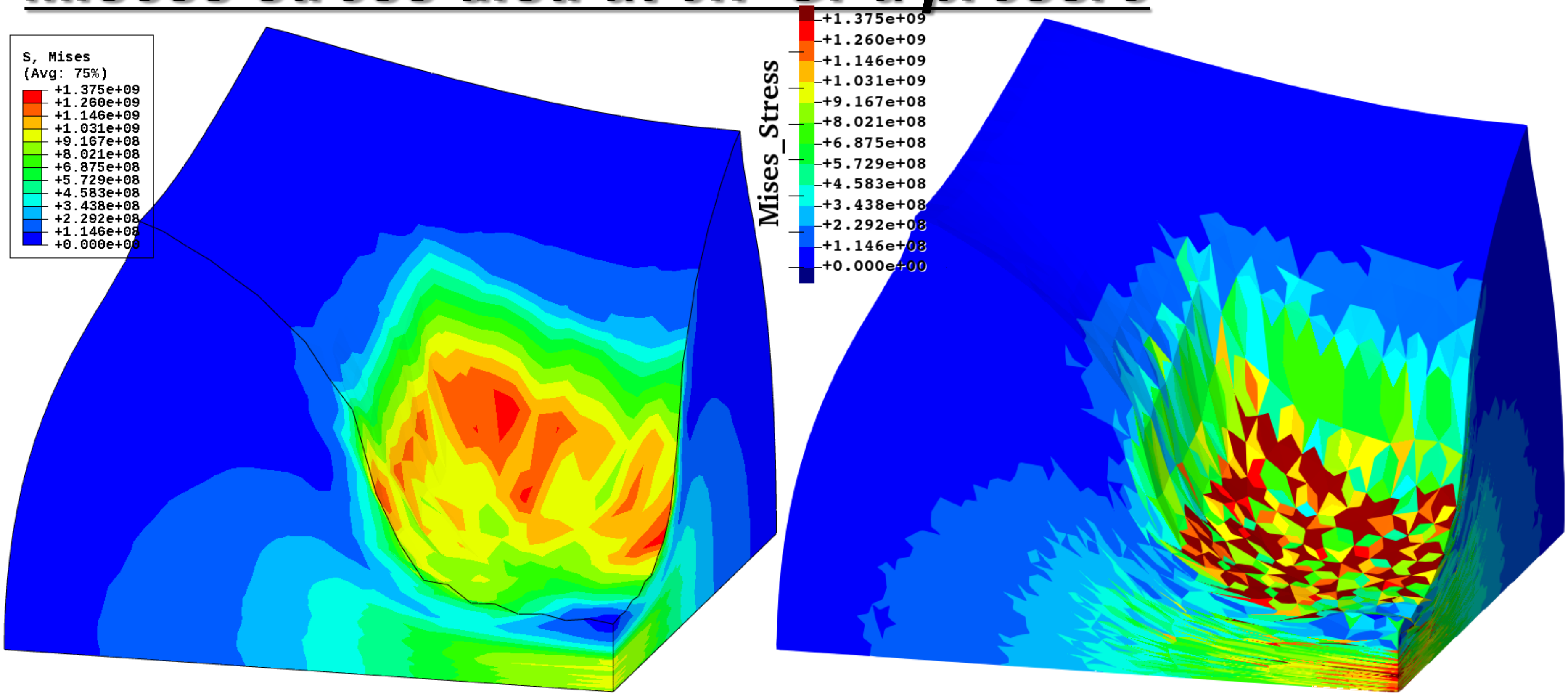
Convergence failure at **1.35 GPa** pressure



The present element is more **robust** than ABAQUS C3D10MH

Compression of Hyperelastic Block

Misess stress dist. at 0.7 GPa pressre



ABAQUS C3D10MH

New SelectiveCS-FEM-T10

Less smoothed Mises stress is observed in New SelectiveCS-FEM-T10.
Further improvement is still required.

Summary

Summary of SelectiveCS-FEM-T10

Benefits

- ✓ Accurate
  (no locking, no checkerboarding, no force oscillation).
- ✓ Robust (long-lasting in large deformation).
- ✓ No increase in DOF (No static condensation).
- ✓ Same memory & CPU costs as the other T10 elements.
- ✓ Implementable to commercial FE codes.

Very close to practical use!!

Drawbacks

- ✗ No longer a T4 formulation.

Take-home message

Please consider implementing SelectiveCS-FEM-T10 to your in-house code. It's supremely useful & easy to code!!

Thank you for your kind attention!