A stabilization method of F-barES-FEM-T4 for **dynamic explicit** analysis of nearly incompressible solids

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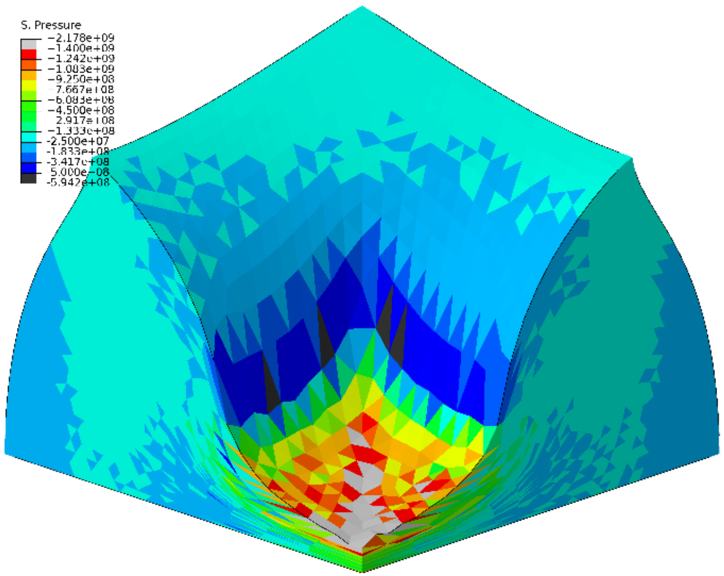
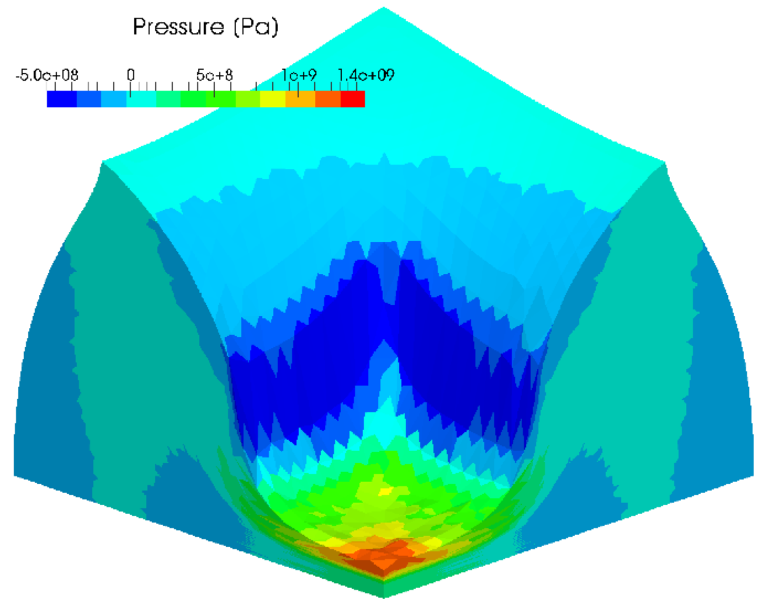
Background

Our group has proposed new FEM formulation named F-bar aided edge-based smoothed finite element method with tetrahedral elements (**F-barES-FEM-T4**).



ABAQUS C3D4H

✗ pressure oscillation



F-barES-FEM-T4(3)

↗ # of cyclic smoothings

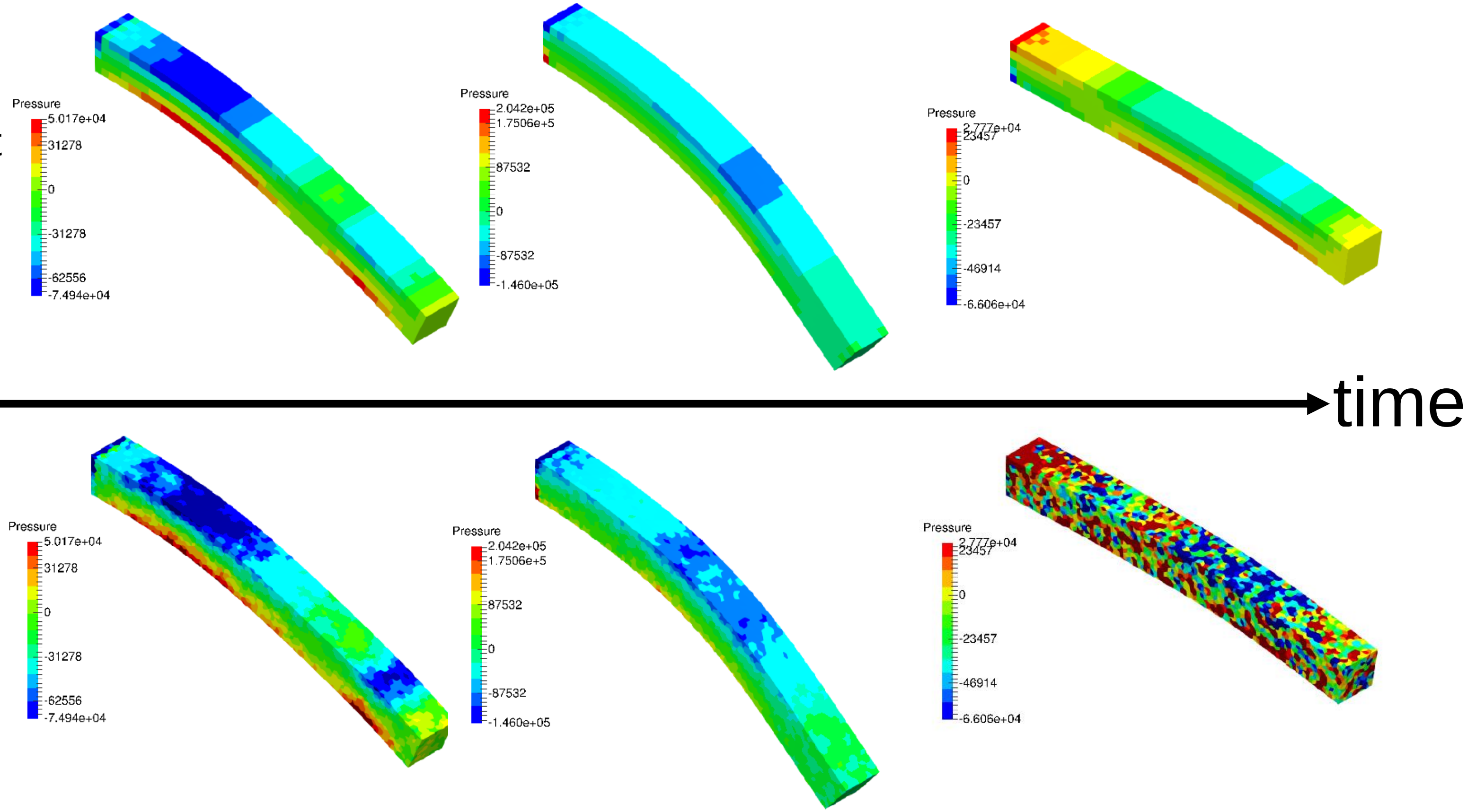
Our method shows excellent accuracy in ***static*** problems!

Drawbacks in explicit dynamics

Cantilever bending analysis

ABAQUS/Explicit
C3D8
Reference

F-barES-FEM
-T4(2)



- Highly accurate results are restricted to short-term analysis.
- F-barES-FEM-T4 causes energy divergence.

Cause of energy divergence

Due to the adoption of F-bar method,
the stiffness matrix $[K]$ becomes asymmetric.

Equation of Motion: $[M]\{\ddot{x}\} + [K]\{x\} = \{f^{\text{ext}}\}$

asymmetric

- Asymmetric stiffness matrix gives rise to imaginary part of natural frequencies and thus causes energy divergence (instability) in dynamic problem.

To realize long-term analysis,
stiffness matrix must be symmetrized.

Objective

Objective

To stabilize **F-barES-FEM-T4** in explicit dynamics for nearly incompressible materials.

Table of Body Contents

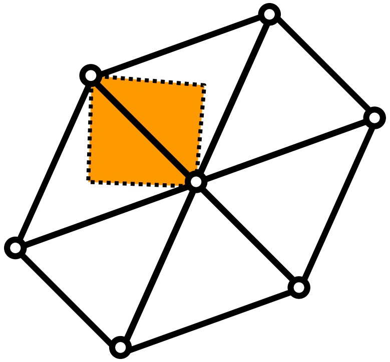
- Methods: Quick introductions of F-barES-FEM-T4 and stabilized method
- Results & Discussion: A few verification analyses
- Summary

Methods

Procedure of F-barES-FEM (1 of 2)

Deformation gradient of each edge, $\overline{\boldsymbol{F}}$ is derived as

$$\overline{\boldsymbol{F}} = \widetilde{\boldsymbol{F}}^{\mathrm{iso}} \cdot \overline{\boldsymbol{F}}^{\mathrm{vol}}$$

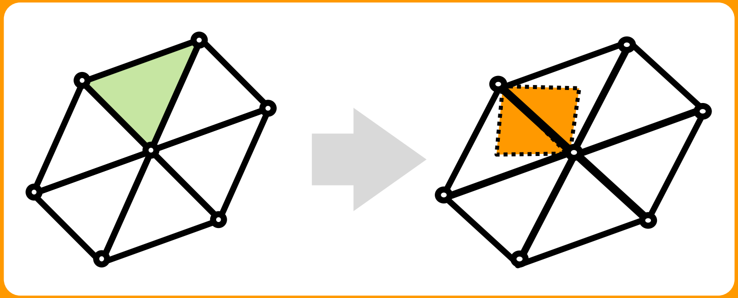

ES−FEM

Procedure of F-barES-FEM (2 of 2)

Each part of $\bar{\boldsymbol{F}}$ is calculated as

$$\bar{\boldsymbol{F}} = \tilde{\boldsymbol{F}}^{\mathrm{iso}} \cdot \bar{\boldsymbol{F}}^{\mathrm{vol}}$$

Isovolumetric part

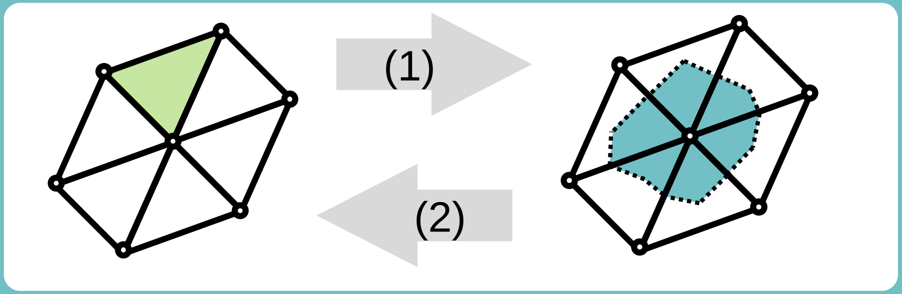


Smoothing the value of adjacent elements.

↓

The same manner as ES-FEM

Volumetric part



(1)Calculating node's value by smoothing the value of adjacent elements

(2)Calculating elements' value by smoothing the value of adjacent nodes

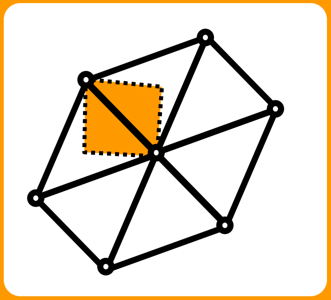
(3)Repeating (1) and (2) a few times

Advantages of F-barES-FEM

This formulation is designed to have 3 advantages.

$$\bar{\boldsymbol{F}} = \tilde{\bar{\boldsymbol{F}}}^{\mathrm{iso}} \cdot \bar{\boldsymbol{F}}^{\mathrm{vol}}$$

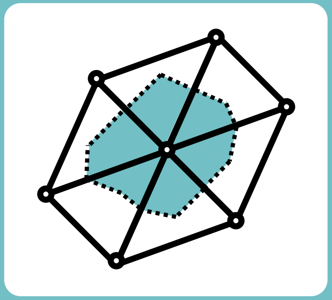
Isovolumetric part



Like a ES-FEM

1. Shear locking free

Volumetric part



Like a NS-FEM

2. Little pressure oscillation

3. Volumetric locking free

with the aid of F-bar method

Construction of internal force vector

Internal force vector of **F-barES-FEM-T4** is calculated as followings:

V of ES-FEM

$$\{f^{\mathrm{int}}\} = \sum [\tilde{B}]\{\bar{T}\}\tilde{V}$$

B-matrix of ES-FEM

Stress derived from $\bar{\boldsymbol{F}}$

Defenition of $\{f^{\mathrm{int}}\}$ in the same fashion as **F-bar method**

■ Combination of $[\tilde{B}]$, $\{\bar{T}\}$ and $\tilde{V}$ causes asymmetric stiffness matrix and thus causes energy divergence.

Proposed method: SymF-barES-FEM-T4

We modify the internal force vector in order to realize stabilization as followings:

F-barES-FEM-T4 $\{f^{\mathrm{int}}\} = \sum [\tilde{B}]\{\bar{T}\}\tilde{V}$

SymF-barES-FEM-T4
Proposed

$$\{f^{\mathrm{int}}\} = \sum [\bar{B}]\{\bar{T}\}\bar{V}$$

- $[\bar{B}]$: B-matrix Derived from $\bar{\boldsymbol{F}}$
- $\{\bar{T}\}$: Stress derived from $\bar{\boldsymbol{F}}$
- $\bar{V}$: volume derived as $\det(\bar{\boldsymbol{F}})V^{\mathrm{ini}}$

We named this formulation SymF-barES-FEM-T4.

Concept of SymF-barES-FEM-T4

SymF-barES-FEM-T4
Proposed

$$\{f^{\mathrm{int}}\} = \sum [\bar{B}]\{\bar{T}\}\bar{V}$$

volume derived as $\det(\bar{\boldsymbol{F}})V^{\mathrm{ini}}$

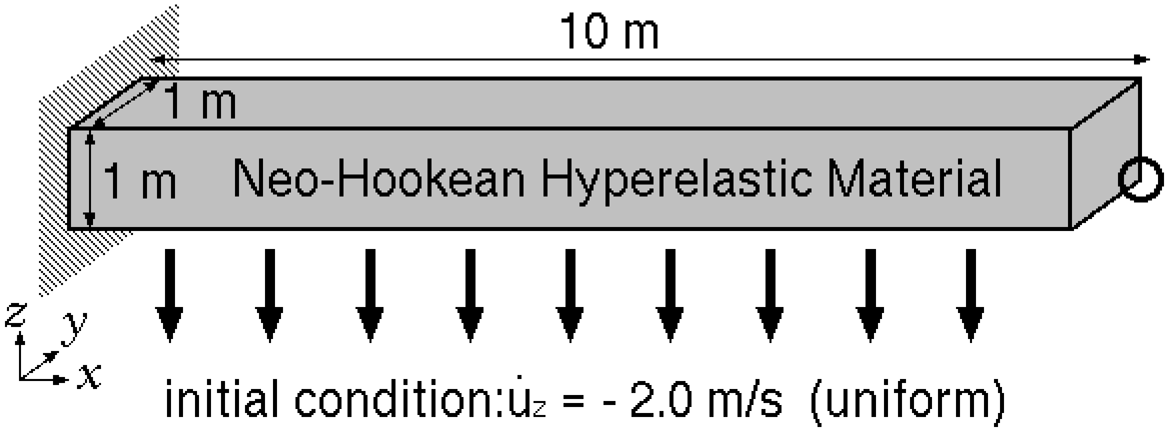
B-matrix Derived from $\bar{\boldsymbol{F}}$

Stress derived from $\bar{\boldsymbol{F}}$

- The replacement $[\tilde{B}]$ to $[\bar{B}]$ means to symmetrize stiffness matrix as standard FEM.
- The replacement $\tilde{V}$ to $\bar{V}$ is natural extension of volume term and introduced empirically.

Result & Discussion

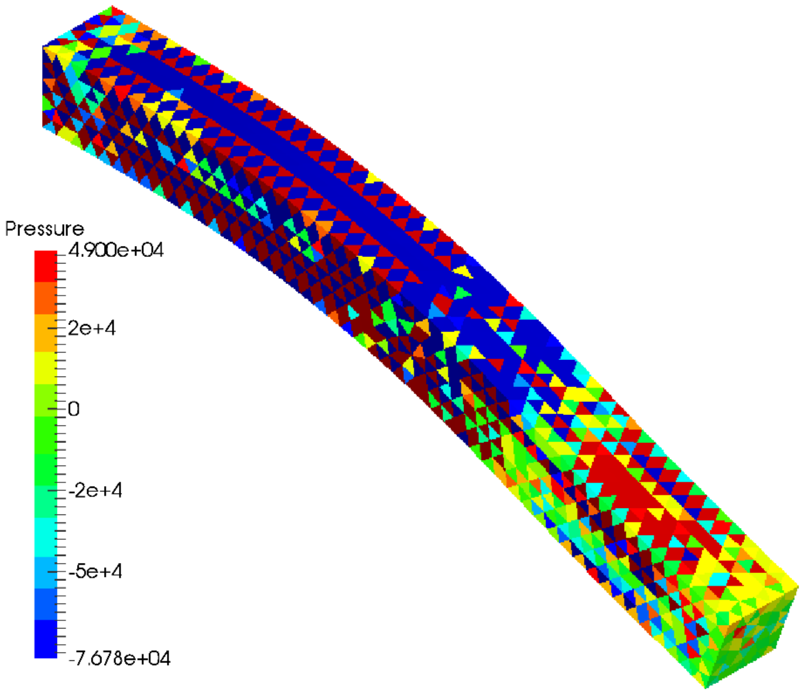
#1 Bending of a cantilever



- Dynamic explicit analysis.
- Neo-Hookean material
  - Initial Young's modulus: 6.0 MPa,
  - Initial Poisson's ratio: 0.49,
  - Density: 920 kg/m^3.
- Compare the results of SymF-barES-FEM-T4, F-barES-FEM-T4 and Selective H8 (ABAQUS/Explicit C3D8) elements.

Inability of standard T4 element

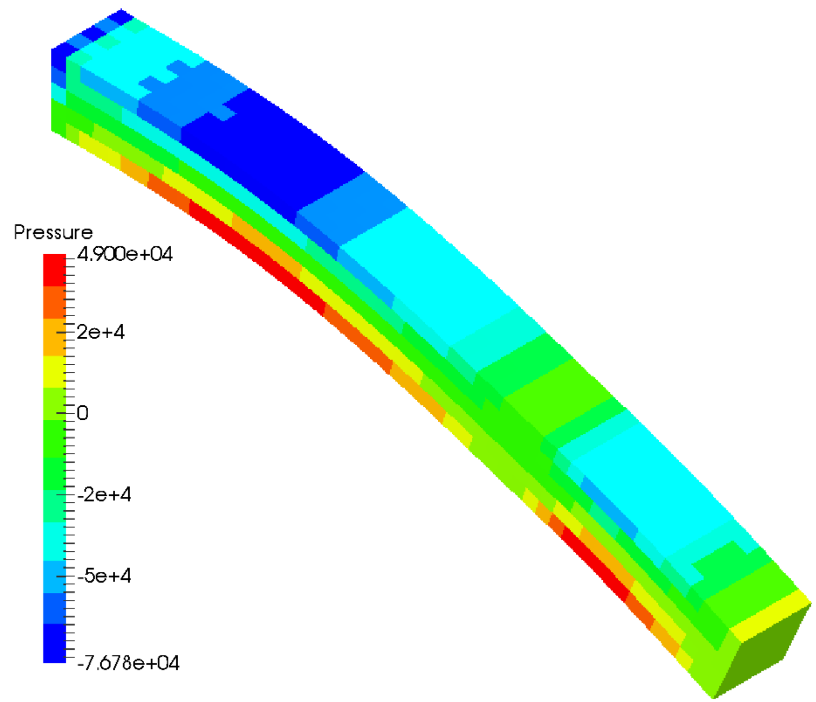
<u>at $t = 0.75$ s</u> (pressure distribution)



ABAQUS/Explicit C3D4

Standard T4 element

✗ Pressure oscillation and locking



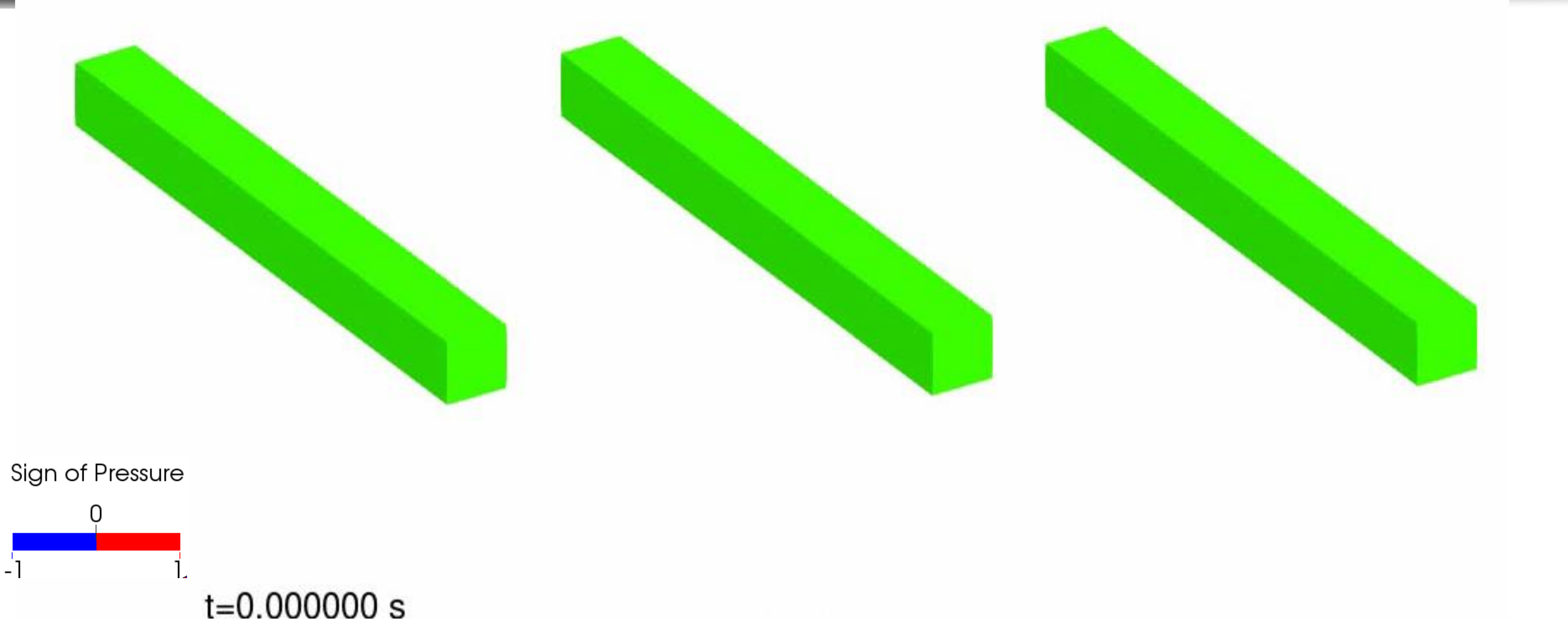
ABAQUS/Explicit C3D8

(Selective H8 element)

Reference

Standard T4 element is useless!

Time history of deformed shapes

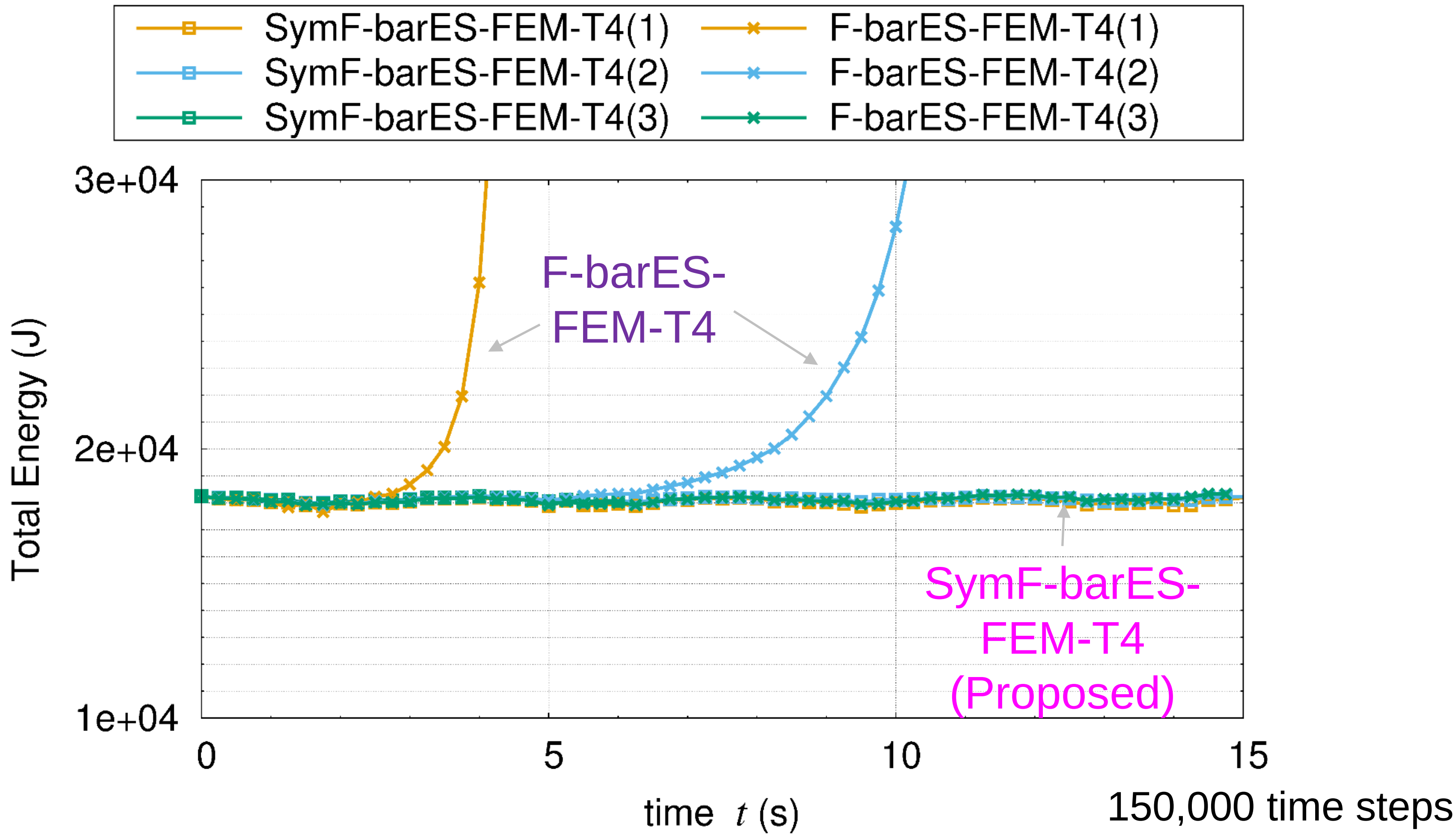


ABAQUS/Explicit C3D8
(Selective H8 element)
Reference

F-barES-FEM-T4(2)
- ✔ No pressure oscillation
- ✔ No locking
- ✗ Energy divergence

SymF-barES-FEM-T4(2)
(Proposed method)
- ✔ Less pressure oscillation
- ✔ No locking
- ✔ No energy divergence

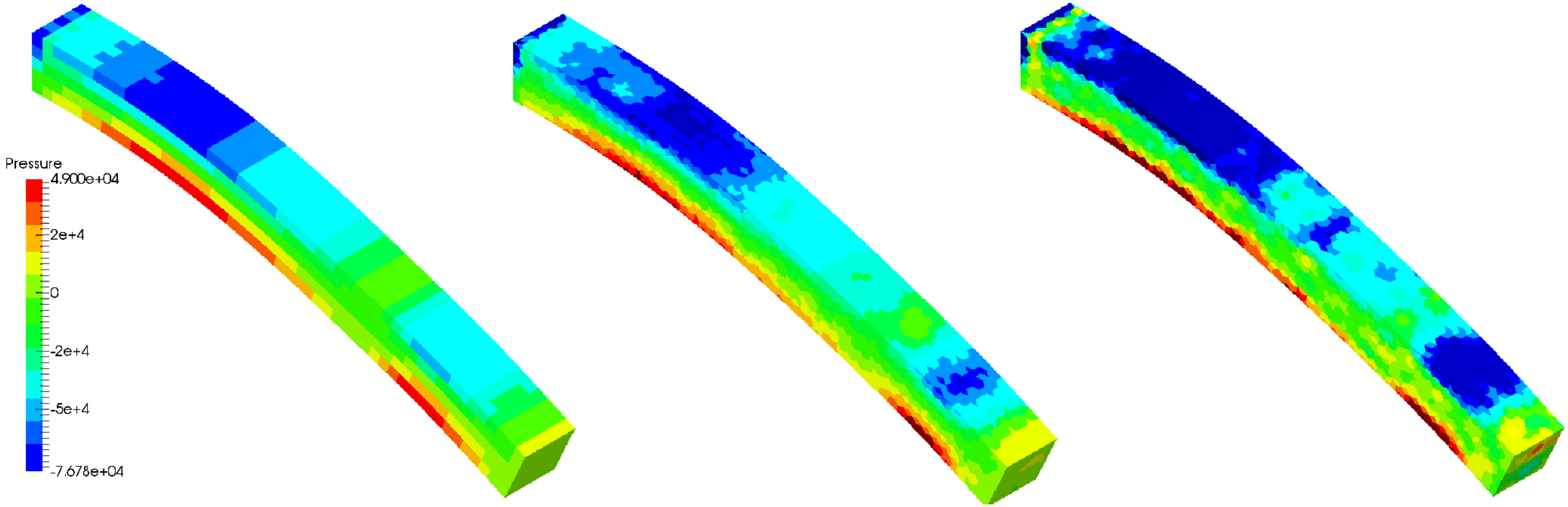
Time history of total energy



SymF-barES-FEM-T4 can suppress energy divergence!

Deformed shapes and pressure distributions

at $t = 0.75$ s



ABAQUS/Explicit C3D8
(Selective H8 element)
Reference

F-barES-FEM-T4(2)

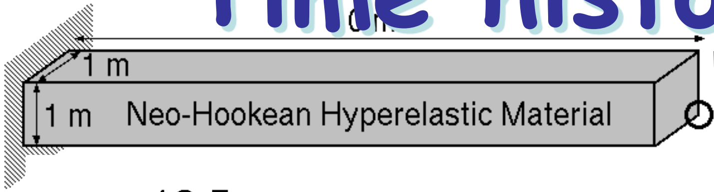
- ✔ No pressure oscillation
- ✔ No locking
- ✗ Energy divergence

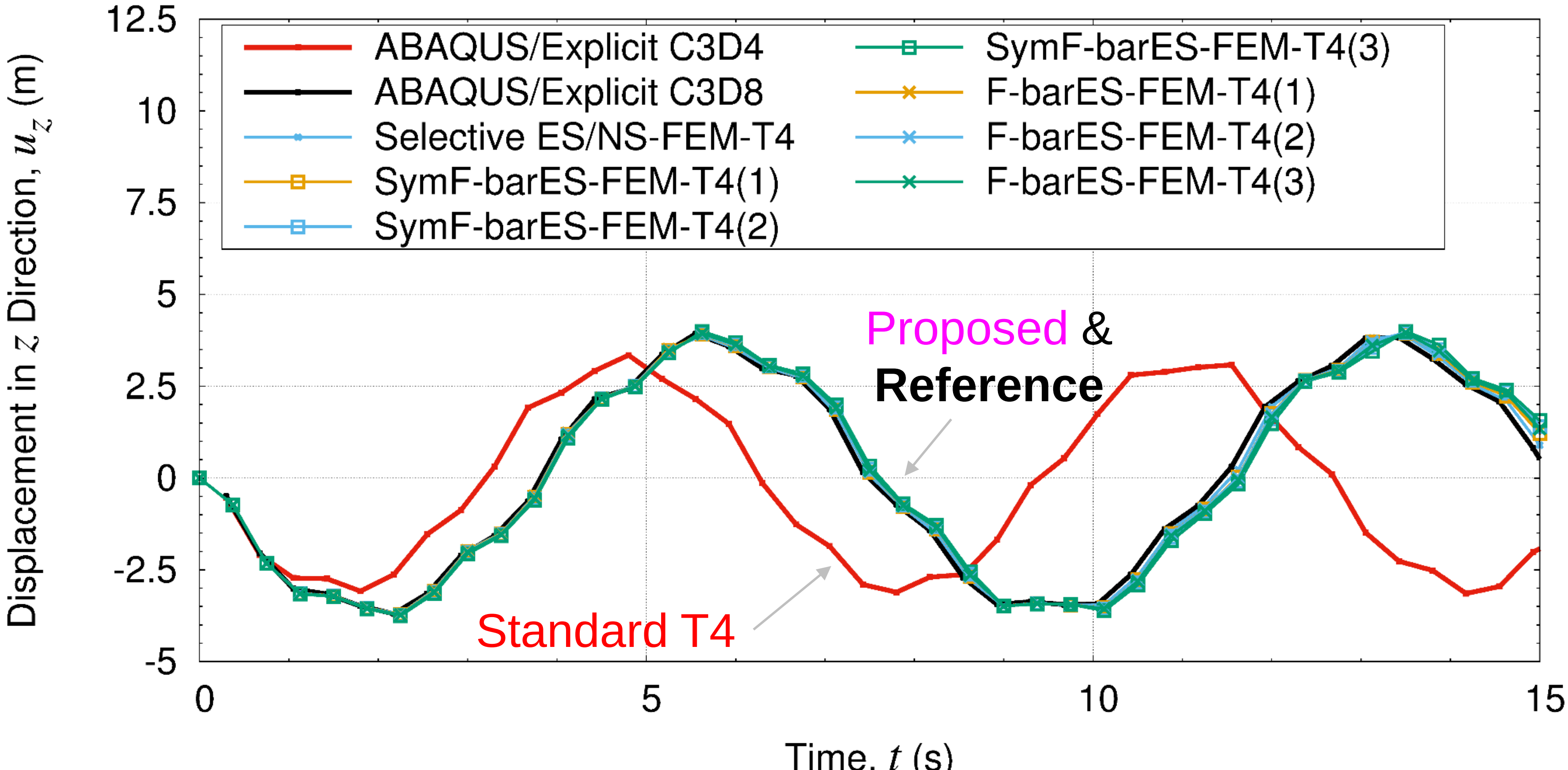
SymF-barES-FEM-T4(2)
(Proposed method)

- ✔ Less pressure oscillation
- ✔ No locking
- ✔ No energy divergence

Proposed method can give acceptable pressure distribution.

Time history of displacement





- Proposed method shows good result without locking.
- The accuracy of displacement does not depend on the number of cyclic smoothings as well as F-baES-FEM-T4.

#2 Cantilever twisting analysis

- Dynamic explicit analysis.
- Twisting initial velocity fields:

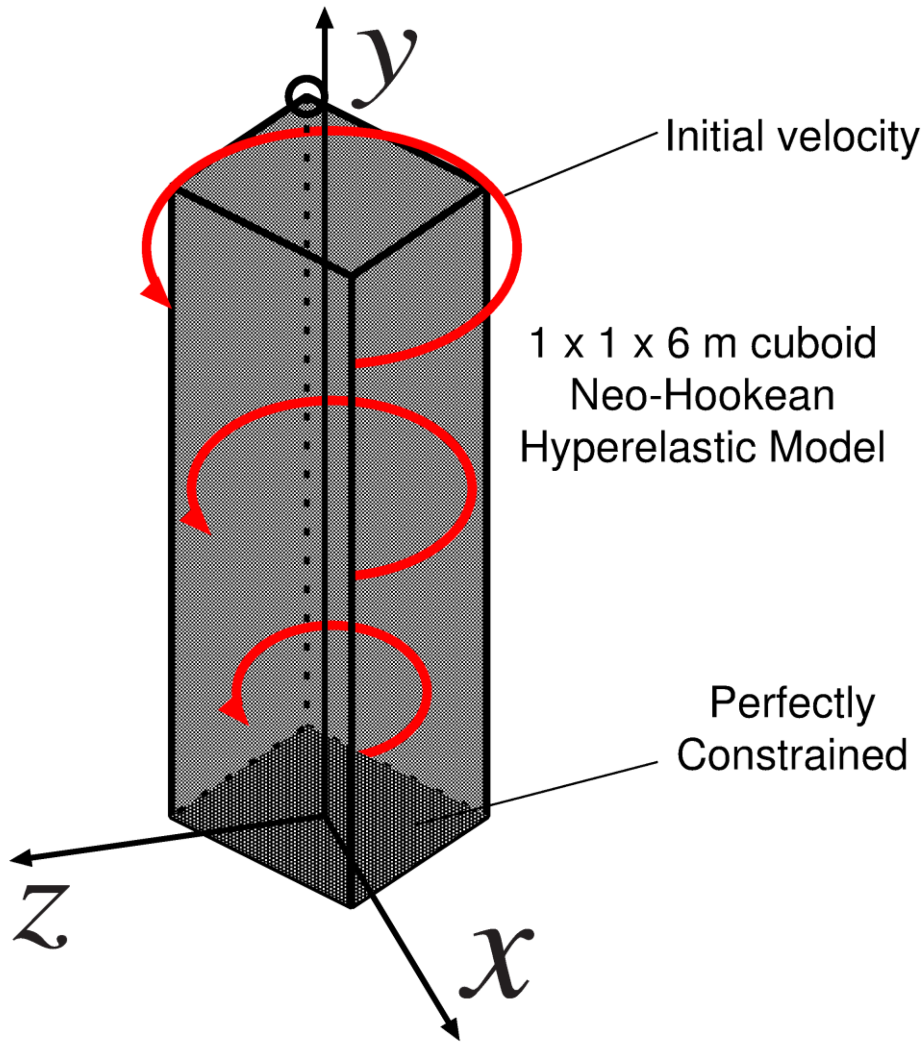
$$\boldsymbol{v}_0(x,y,z) = 100\sin\left(\frac{y\pi}{12}\right)\{z,0,-x\}^T.$$

- Neo-Hookean material

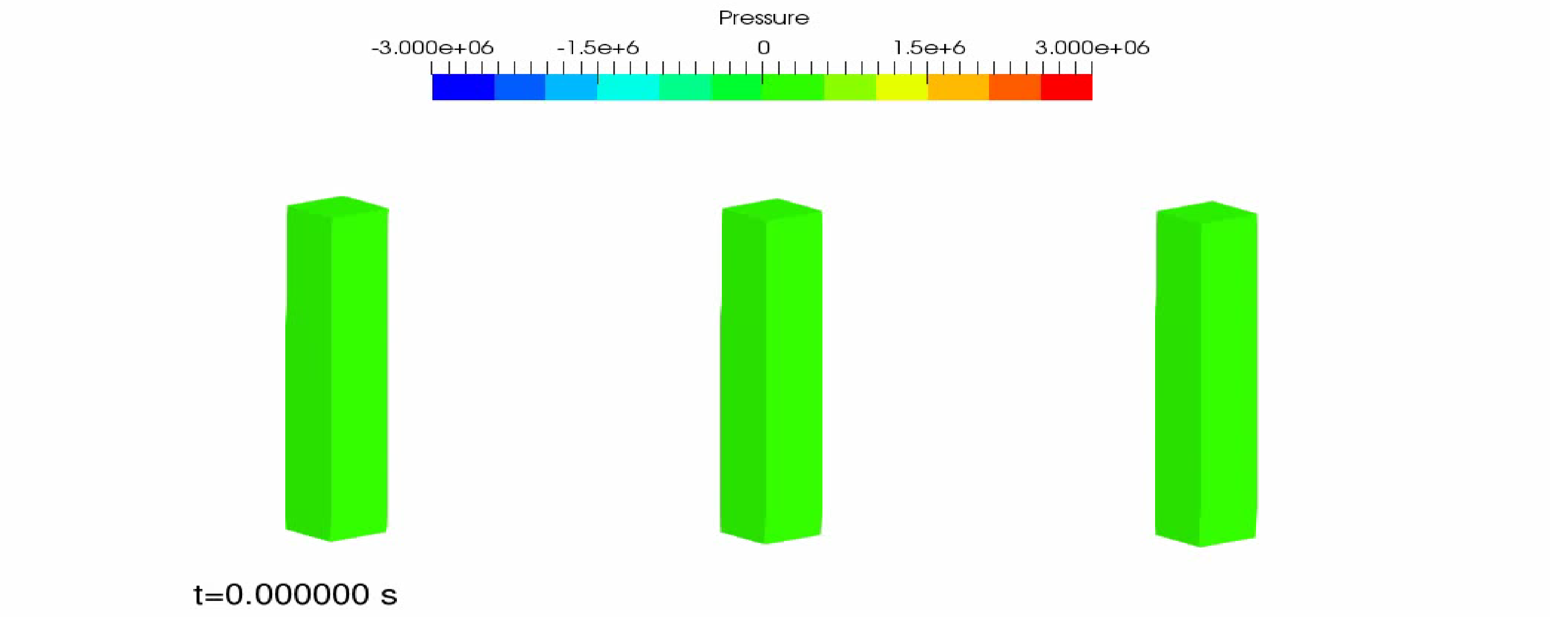
Initial Young's modulus: 17.0 MPa,
Initial Poisson's ratio: 0.49,
Density: 1100 kg/m^3.

- Compare the results of

SymF-barES-FEM-T4,
F-barES-FEM-T4,
and Selective H8



Time history of deformed shapes



ABAQUS/Explicit C3D8
(Selective H8 element)
Reference

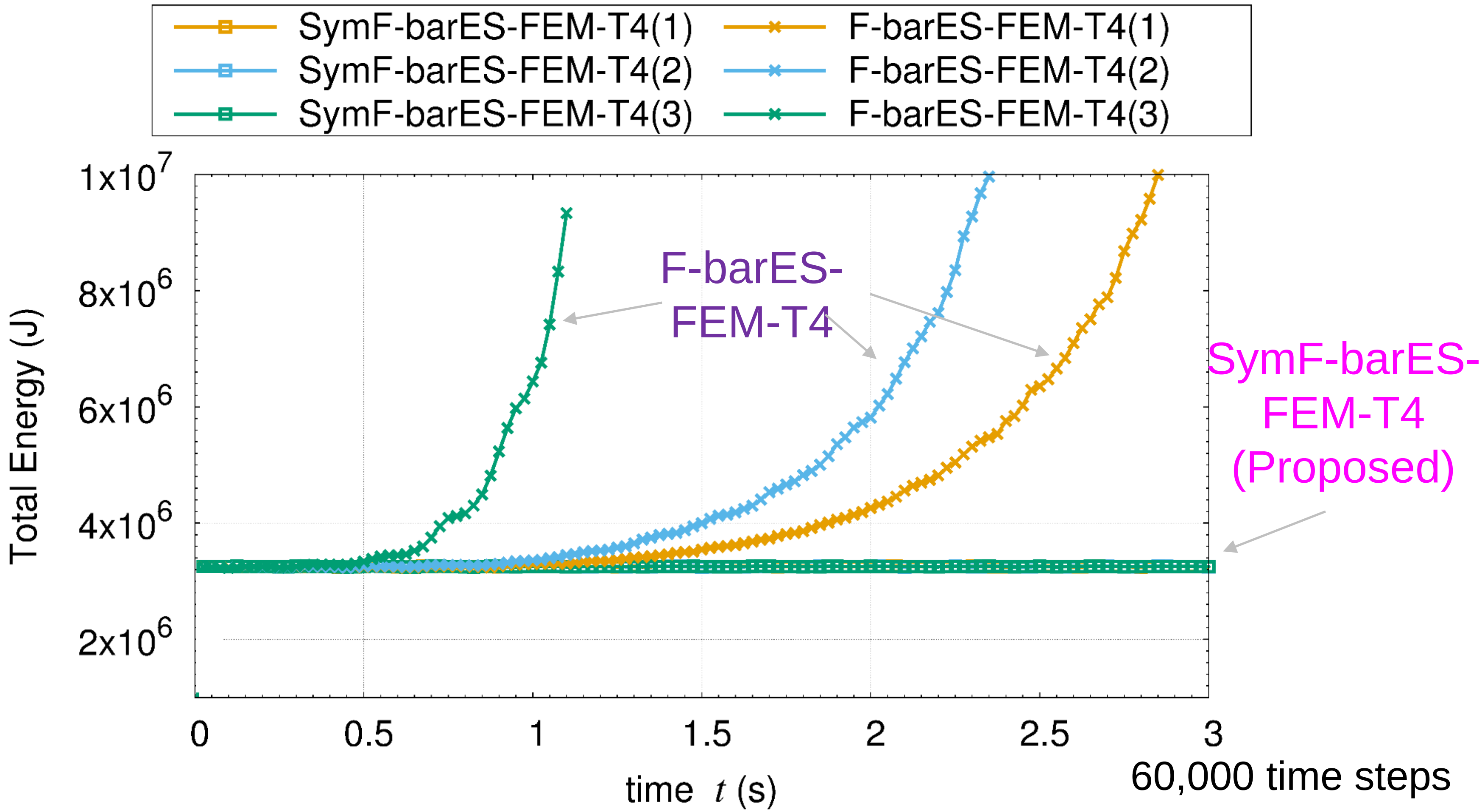
F-barES-FEM-T4(2)

- ✔ No pressure oscillation
- ✔ No locking
- ✗ Energy divergence

SymF-barES-FEM-T4(2)
(Proposed method)

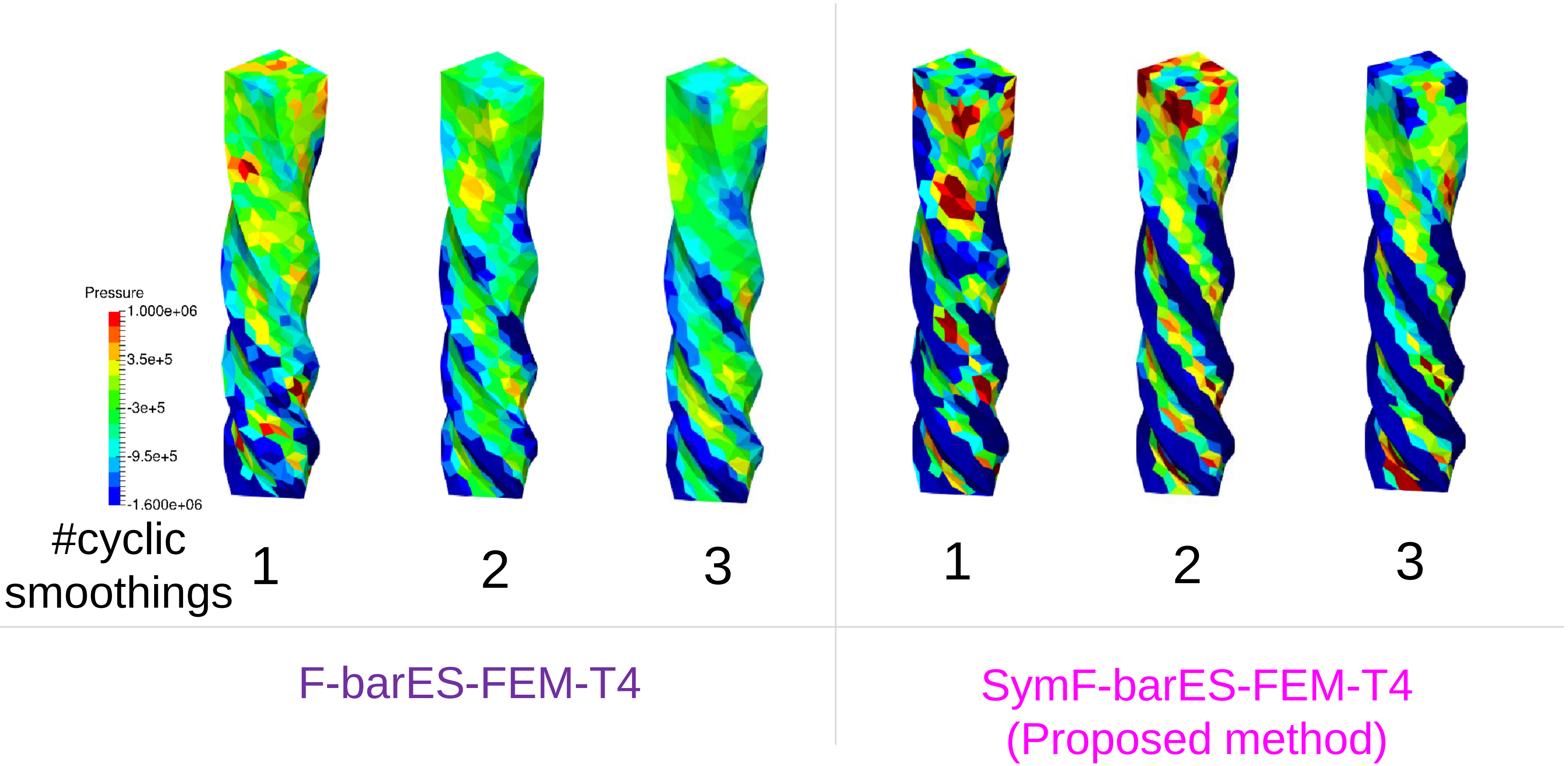
- ✔ Less pressure oscillation
- ✔ No locking
- ✔ No energy divergence

Time history of total energy



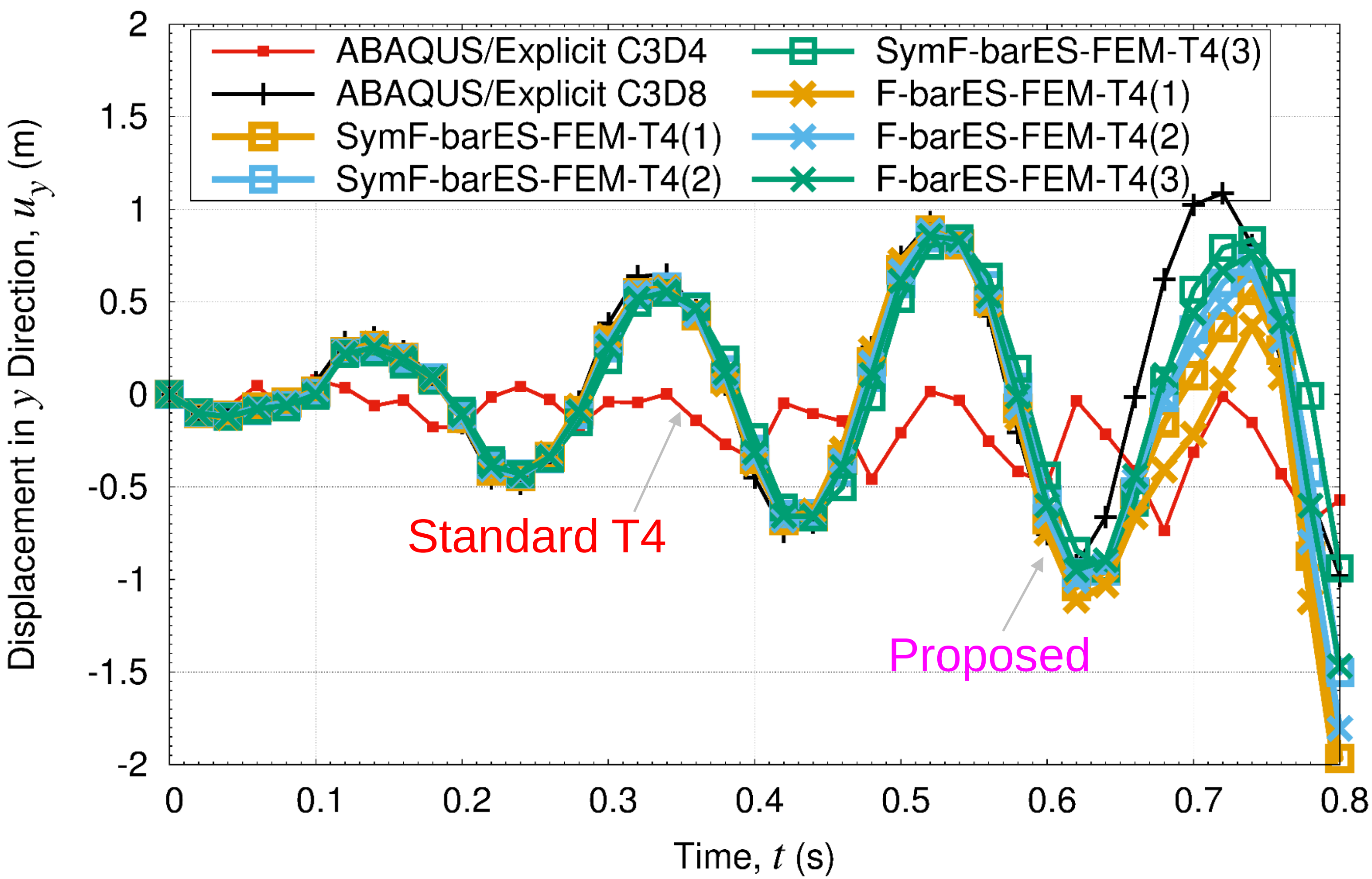
SymF-barES-FEM-T4 can suppress energy divergence!

Effect of cyclic smoothings



The increase in cyclic smoothings no longer improves distribution unlike F-barES-FEM-T4

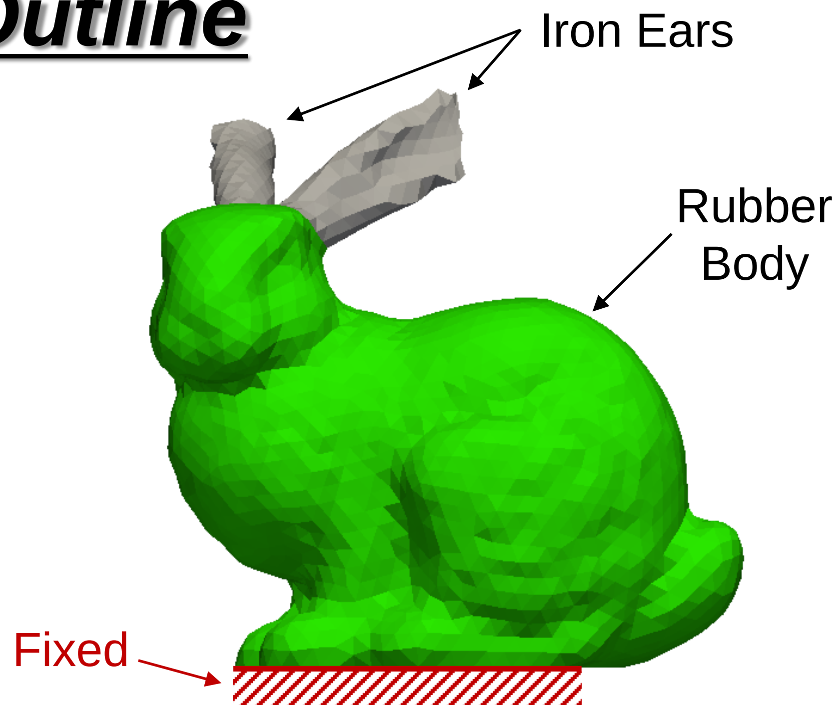
Time history of displacement

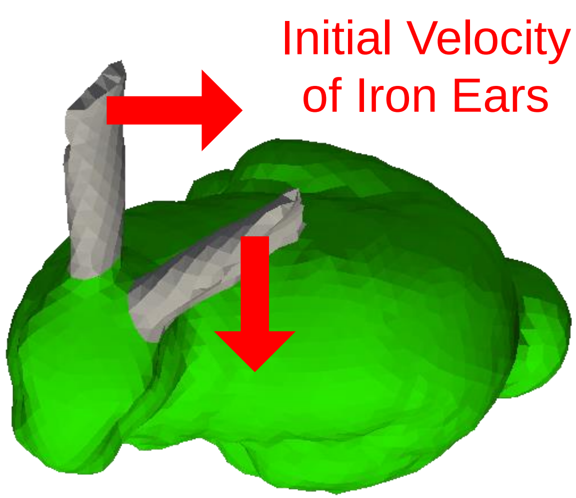


Proposed methods can show as good result as F-barES-FEM-T4

#3 Swinging of Bunny Ears

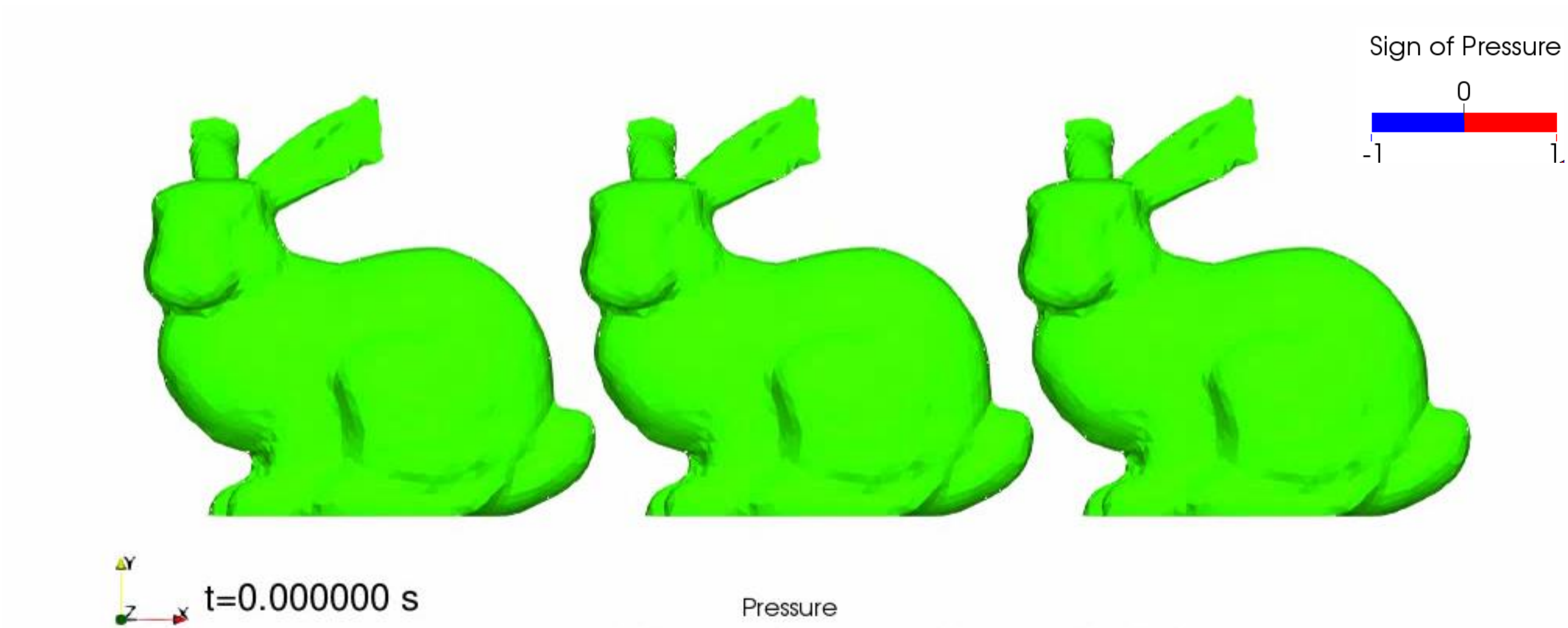
Outline





- Iron ears: $E_{\text{ini}} = 200\ \text{GPa},\ \nu_{\text{ini}} = \mathbf{0.3},\ \rho = 7800\ \text{kg/m}^3$, Neo-Hookean, **No cyclic smoothing.**
- Rubber body: $E_{\text{ini}} = 6\ \text{MPa},\ \nu_{\text{ini}} = \mathbf{0.49},\ \rho = 920\ \text{kg/m}^3$, Neo-Hookean, **1 cycle of smoothing.**
- Compared to ABAQUS/Explicit C3D4. No Hex mesh available!

Time histories of deformed shapes



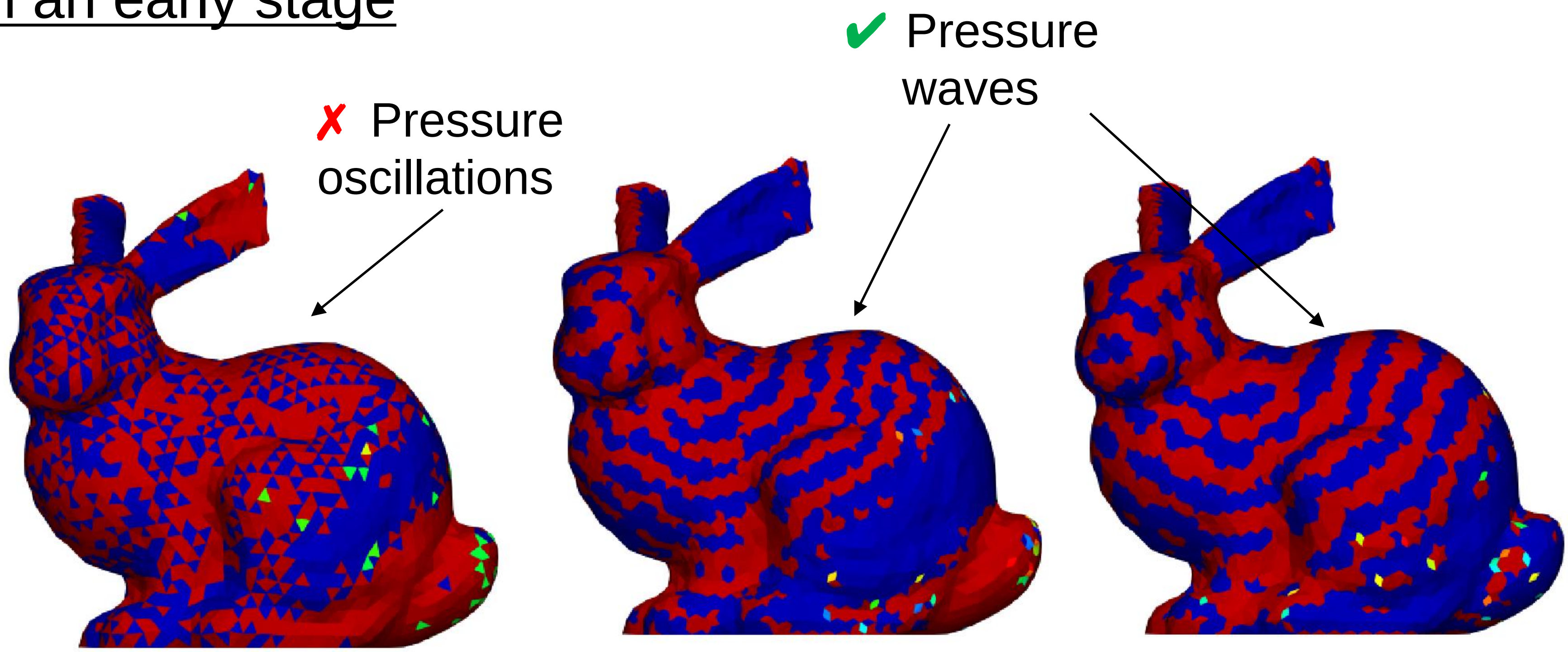
ABAQUS/Explicit C3D4 (Standard T4 element)
- ✗ Pressure oscillation
- ✗ Locking
- ✔ No energy divergence

F-barES-FEM-T4(1)
- ✔ No pressure oscillation
- ✔ No locking
- ✗ Energy divergence

SymF-barES-FEM-T4(1) (Proposed method)
- ✔ Less pressure oscillation
- ✔ No locking
- ✔ No energy divergence

Deformed shapes and sign of pressure

In an early stage



ABAQUS/Explicit C3D4
(Standard T4 element)

F-barES-FEM-T4(1)

SymF-barES-FEM-T4(1)
(Proposed method)

Our method represents pressure waves appropriately!

Summary

Summary

- SymF-barES-FEM-T4 was proposed in order to realize accurate and stable dynamic explicit analysis.

- Proposed method realizes
  - ✔ Less pressure oscillation
  - ✔ No locking
  - ✔ No energy divergence

- Further improvement for perfect suppression of pressure oscillation is our future work.

Thank you for your kind attention.