Performance Evaluation of Various Smoothed Finite Element Methods with Tetrahedral Elements in Large Deformation Dynamic Analysis

Ryoya IIDA, Yuki ONISHI, Kenji AMAYA

Tokyo Institute of Technology, Japan

Goal and Requirements

Goal

To analyze large deformation dynamic problems for rubber-like materials

For such analysis, these 3 properties are required.

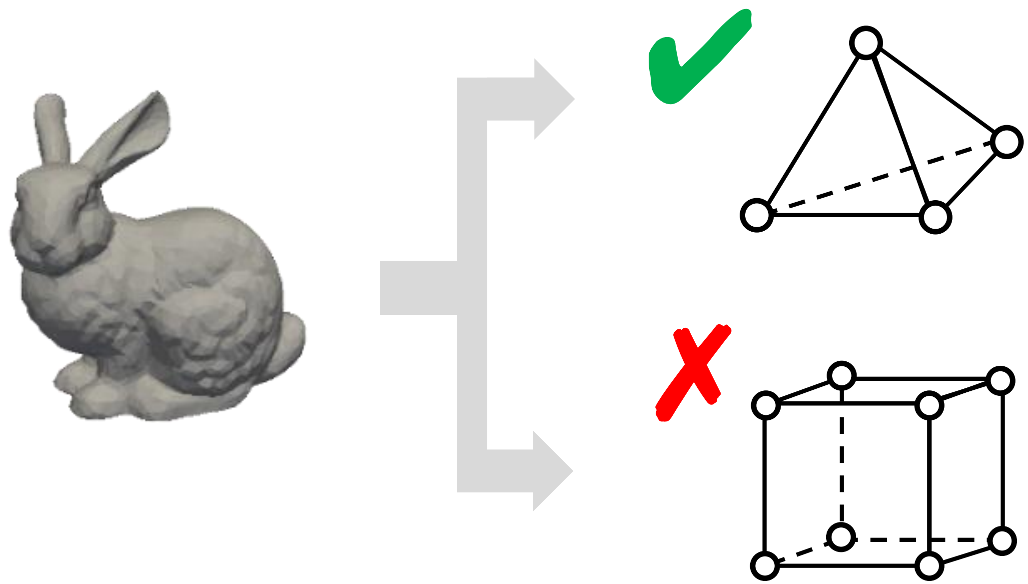
(1)Being applied to 4-noded Tet (T4) elements

(2)Stability even in Nearly incompressible materials

(3)Being directly applied to Explicit Dynamics

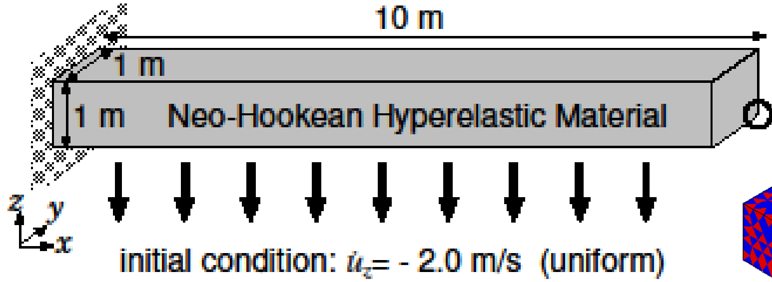
Requirements (1 of 3): T4 elements

Arbitrary shapes cannot be meshed
into good-quality Hex elements automatically.

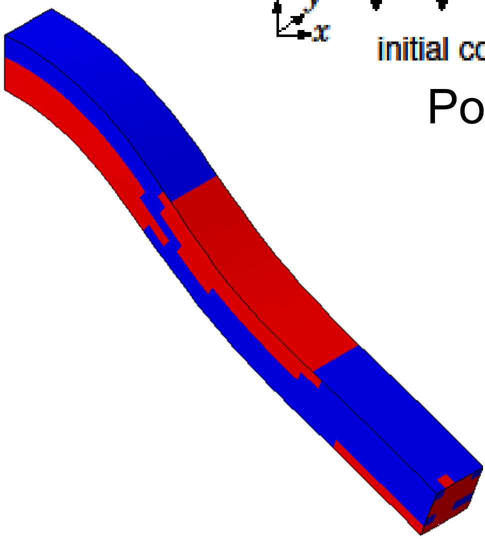


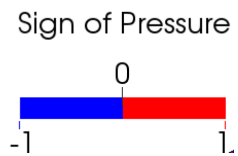
- Intermediate nodes cause bad-accuracy in Large deformation problem.
- First-order Tet elements (T4) are preferred for such analysis.

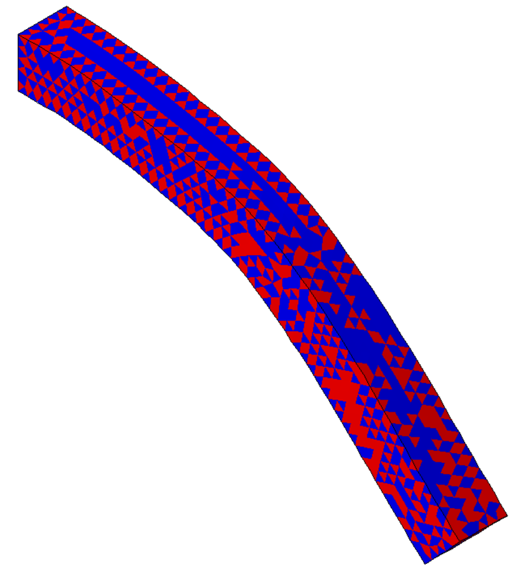
Requirements (2 of 3): Stability for rubber-like materials



Poisson's ratio: 0.499







Reference solution

Standard T4 elements
✗ pressure oscillation
✗ shear & volumetric locking

In FE analysis for rubber-like materials,
Pressure oscillation & Locking easily arise.

Requirements (3 of 3): Explicit dynamics

2 types of time integration scheme

1. Implicit is suitable for long-term problems
2. Explicit is suitable for short-term problems

- u/p hybrid formulations cannot be easily applied to explicit dynamics.
- General methods of explicit dynamics for rubber like materials have not been established !
- The advantage of S-FEM is to be directly applied to explicit dynamics

Objective

Objective

To evaluate the performance of competitive **S-FEMs**

- **Selective ES/NS-FEM**
- **F-barES-FEM**

in explicit dynamics for nearly incompressible materials.

Table of Body Contents

- Methods: Quick introduction of S-FEMs
- Results & Discussion: A few verification analyses
- Summary

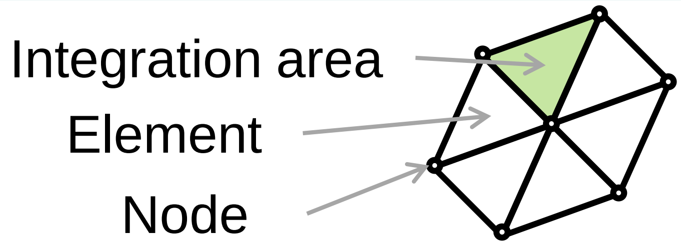
Methods

Selective ES/NS-FEM

F-barES-FEM

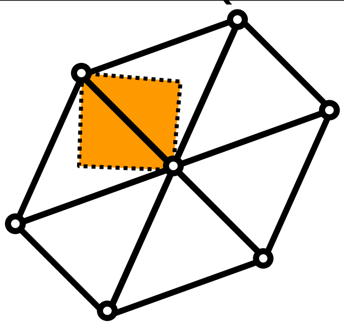
Quick review of S-FEM

Standard FEM

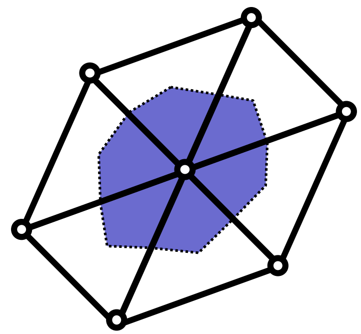


Nodal forces are calculated by summing up of each elements’

Smoothed-FEM (S-FEM)



ES-FEM sums up each *Edge* values.



NS-FEM sums up each *Node* values.

✔ High accuracy in isovolumetric part without shear locking

✔ High accuracy in volumetric part with little pressure oscillation

Selective ES/NS-FEM (1 of 2)

Cauchy stress tensor $\boldsymbol{T}$ is derived as

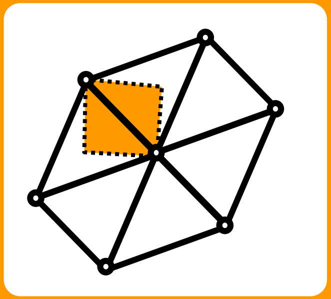
$$\boldsymbol{T} = \boldsymbol{T}^{\mathrm{dev}} + \boldsymbol{T}^{\mathrm{hyd}}$$

Selective ES/NS-FEM (2 of 2)

This formulation is designed to have 3 advantages.

$$\boldsymbol{T} = \boldsymbol{T}^{\mathrm{dev}} + \boldsymbol{T}^{\mathrm{hyd}}$$

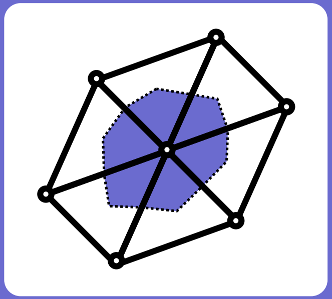
Deviatoric part



Like a ES-FEM

1. Shear locking free

Hydrostatic-pressure part



Like a NS-FEM

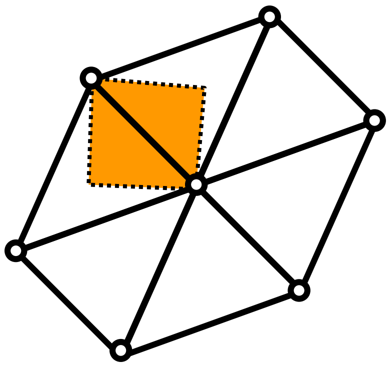
2. Little pressure oscillation

3. Volumetric locking free

F-barES-FEM (1 of 3)

Deformation gradient of each edge, $\bar{\boldsymbol{F}}$ is derived as

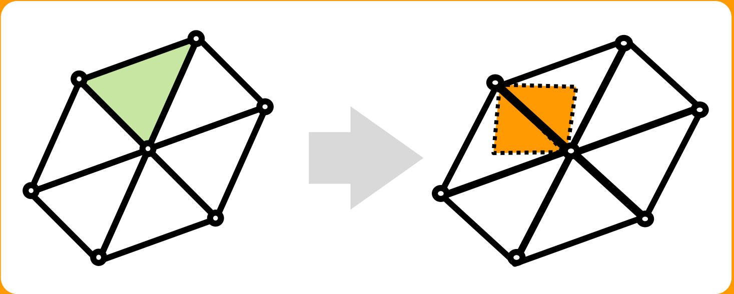
$$\bar{\boldsymbol{F}} = \widetilde{\boldsymbol{F}}^{\mathrm{iso}} \cdot \bar{\boldsymbol{F}}^{\mathrm{vol}}$$



Each part of $\bar{\boldsymbol{F}}$ is calculated as

$$\bar{\boldsymbol{F}} = \tilde{\boldsymbol{F}}^{\mathrm{iso}} \cdot \bar{\boldsymbol{F}}^{\mathrm{vol}}$$

Isovolumetric part

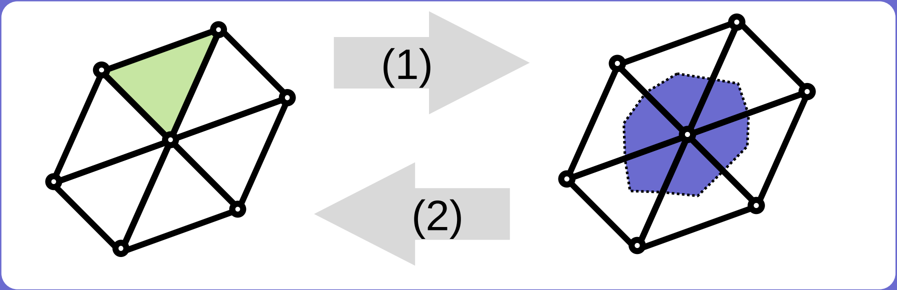


Smoothing the value of adjacent elements.

↓

The same manner as ES-FEM

Volumetric part



(1)Calculating node's value by smoothing the value of adjacent elements

(2)Calculating elements' value by smoothing the value of adjacent nodes

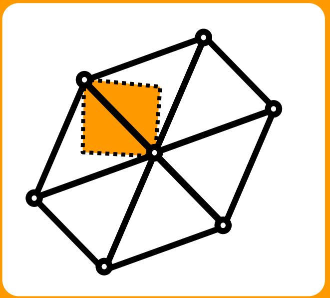
(3)Repeating (1) and (2) a few times

F-barES-FEM (3 of 3)

This formulation is designed to have 3 advantages.

$$\bar{\boldsymbol{F}} = \tilde{\boldsymbol{F}}^{\mathrm{iso}} \cdot \bar{\boldsymbol{F}}^{\mathrm{vol}}$$

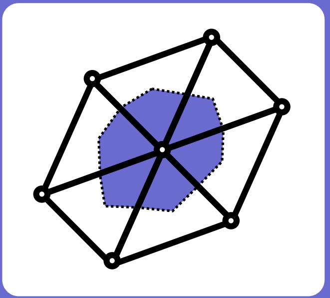
Isovolumetric part



Like a ES-FEM

1. Shear locking free

Volumetric part



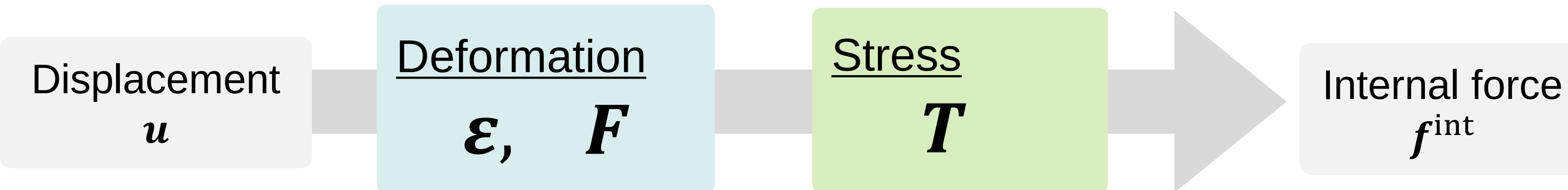
Like a NS-FEM

2. Little pressure oscillation

3. Volumetric locking free
with the aid of F-bar method

Characteristics of S-FEMs

Selective ES/NS-FEM decomposes this part
✗ Constitutive models are restricted



F-barES-FEM decomposes this part.
✗ Stiffiness matrix becomes asymmetric

Both of S-FEMs have disadvantages…

Equation to solve

Equation of Motion

$$[M]\{\ddot{u}\} = \{f^{\text{ext}}\} - \{f^{\text{int}}\},$$

Internal force vector is calculated as

Selective ES/NS-FEM

$$\{f^{\text{int}}\} = \sum_{\text{Edge}} [\tilde{B}^{\text{Edge}}]\{\tilde{T}^{\text{dev}}\}V + \sum_{\text{Node}} [\tilde{B}^{\text{Node}}]\{\tilde{T}^{\text{hyd}}\}V$$

Dev. parts are derived from **ES-FEM**

Hyd. parts are derived from **NS-FEM**

F-barES-FEM

$$\{f^{\text{int}}\} = \sum_{\text{Edge}} [\tilde{B}^{\text{Edge}}]\{\bar{T}\}V$$

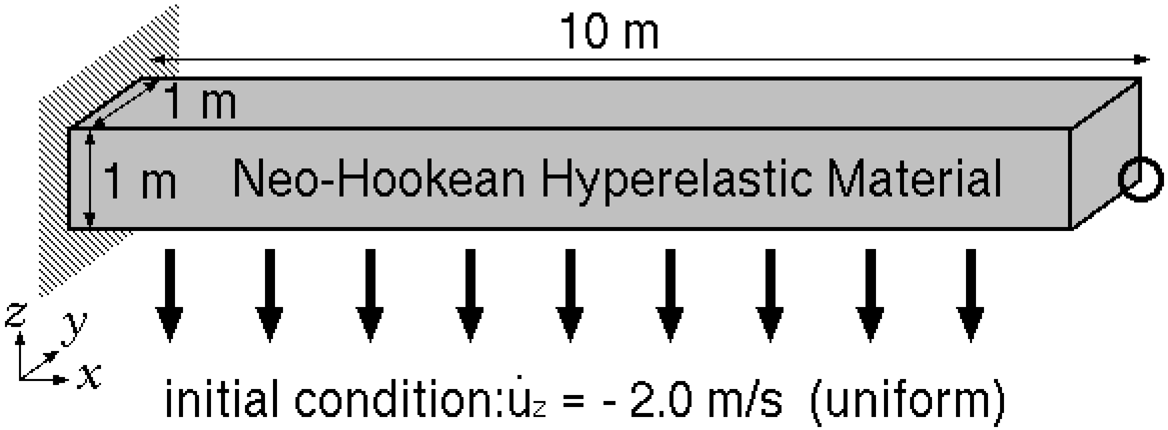
Defenition of $\{f^{\text{int}}\}$ in the same fashion as **F-bar method**

B-matrix of **ES-FEM**

Stress derived from $\bar{\boldsymbol{F}}$

Result & Discussion

Bending of a cantilever



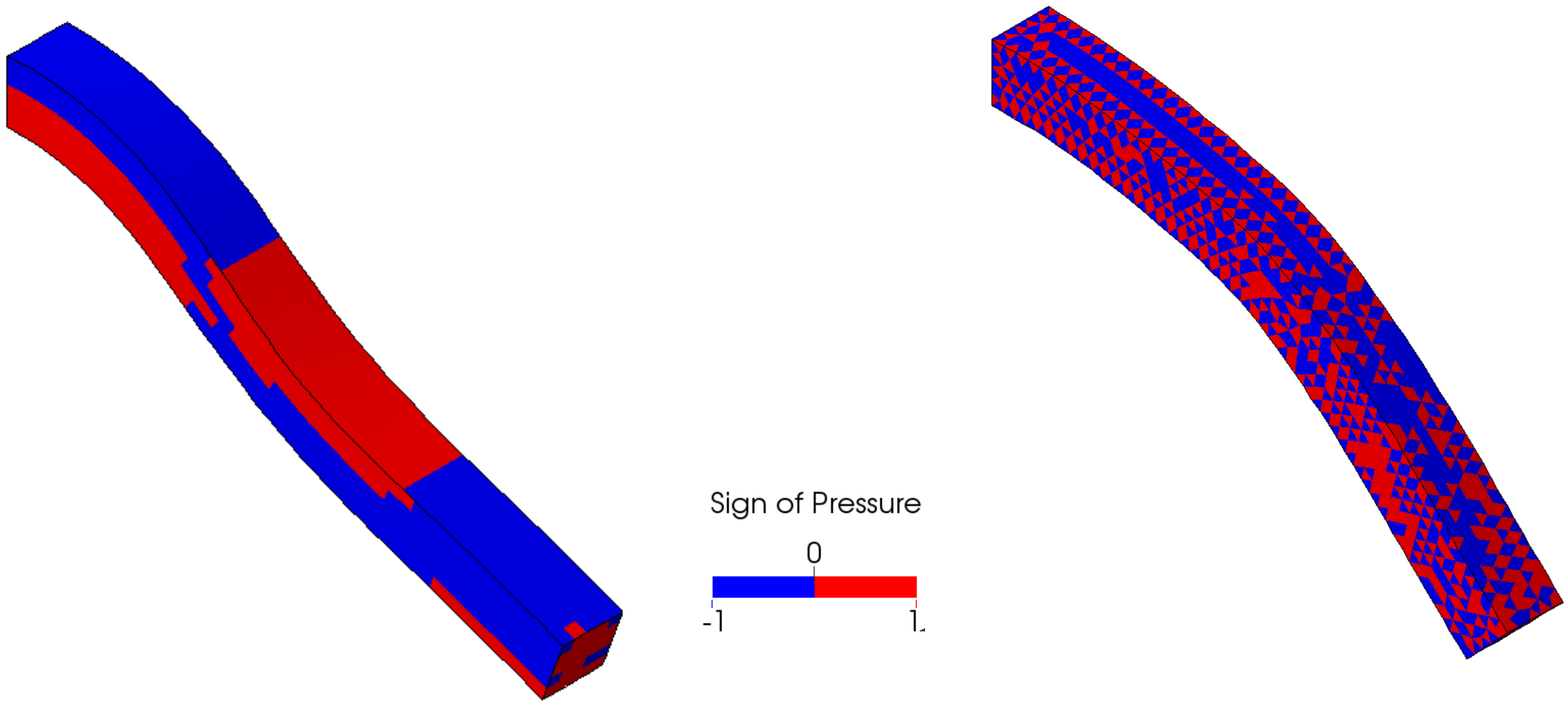
- Dynamic explicit analysis.
- Neo-Hookean material

Initial Young's modulus:	6.0 MPa,
Initial Poisson's ratio:	0.499,
Density:	10000 kg/m^3.

- Compare the results of S-FEMs with Selective H8 (ABAQUS/Explicit C3D8) elements.

Result of Standard T4 elements

<u>at $t = 1.5$ s</u>



ABAQUS/Explicit
C3D8
(**Reference**)

Standard T4 element
× pressure oscillation
× locking

The result of standard T4 elements is useless…

Time history of deformed shapes

ABAQUS/Explicit C3D8 **Reference**

Selective ES/NS-FEM

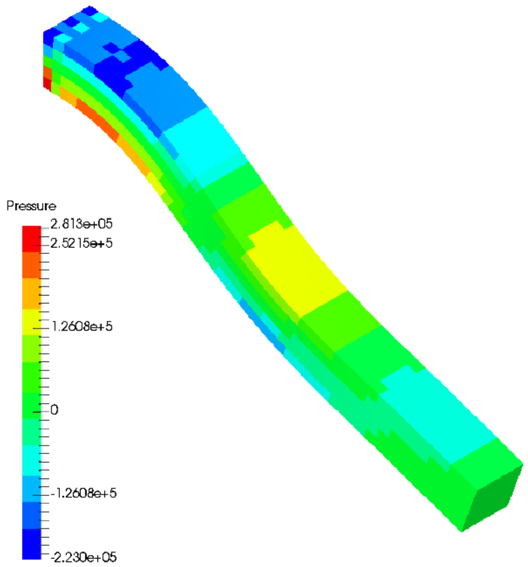
F-barES-FEM (2)

NS-FEM

- All of S-FEMs are locking free.
- **F-barES-FEM** shows good result in earlier stage but gets worse in later stage.
- **Selective ES/NS-FEM** and NS-FEM cannot suppress pressure oscillation…

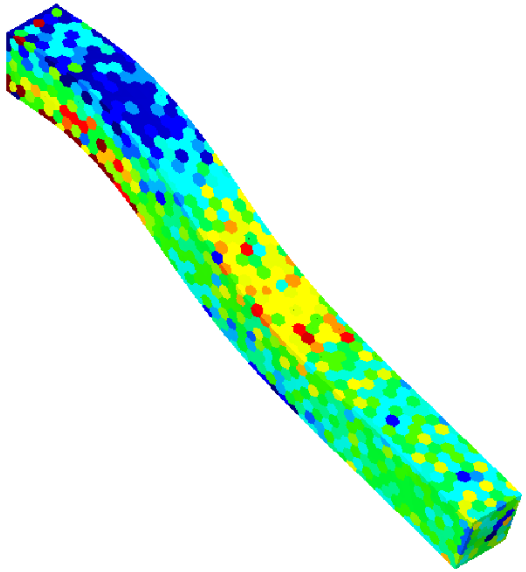
Deformed shapes and pressure distributions

at $t = 1.5$ s

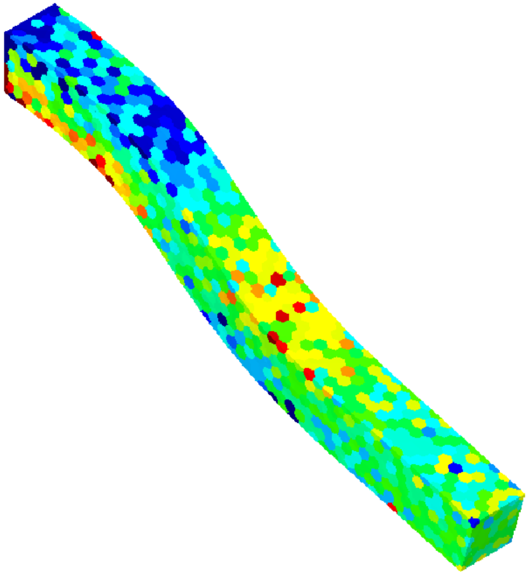


ABAQUS/Explicit
C3D8
(Reference)

Selective ES/NS-FEM



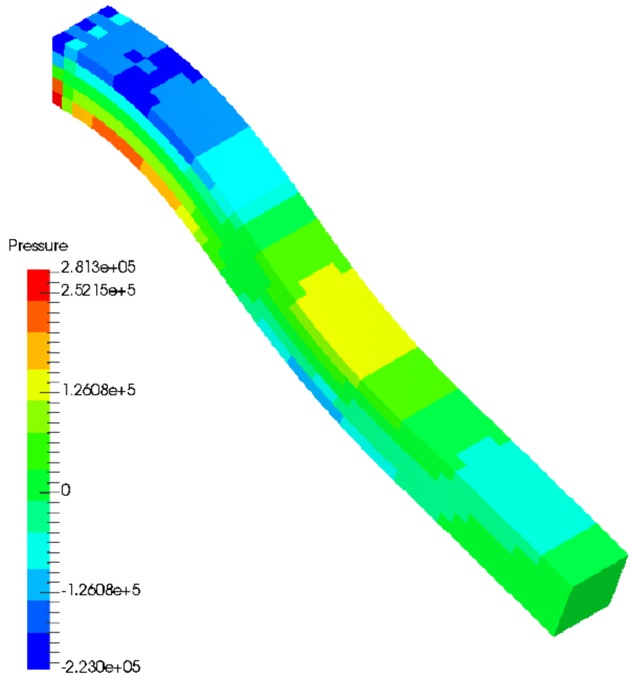
NS-FEM



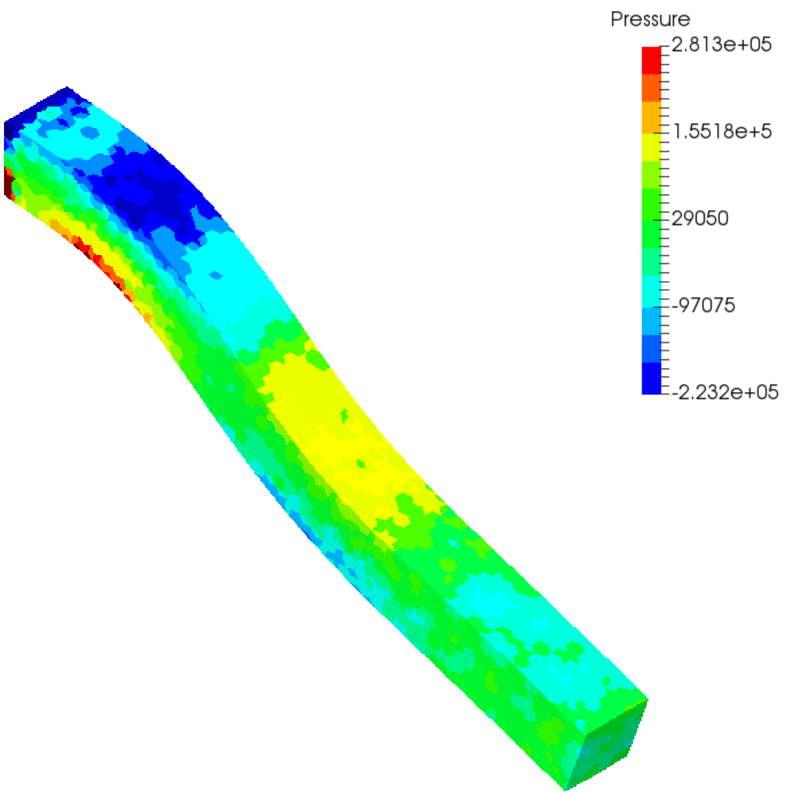
Selective ES/NS-FEM and **NS-FEM** cannot suppress pressure oscillation due to the insufficient smoothing.

Deformed shapes and pressure distributions

at $t = 1.5$ s



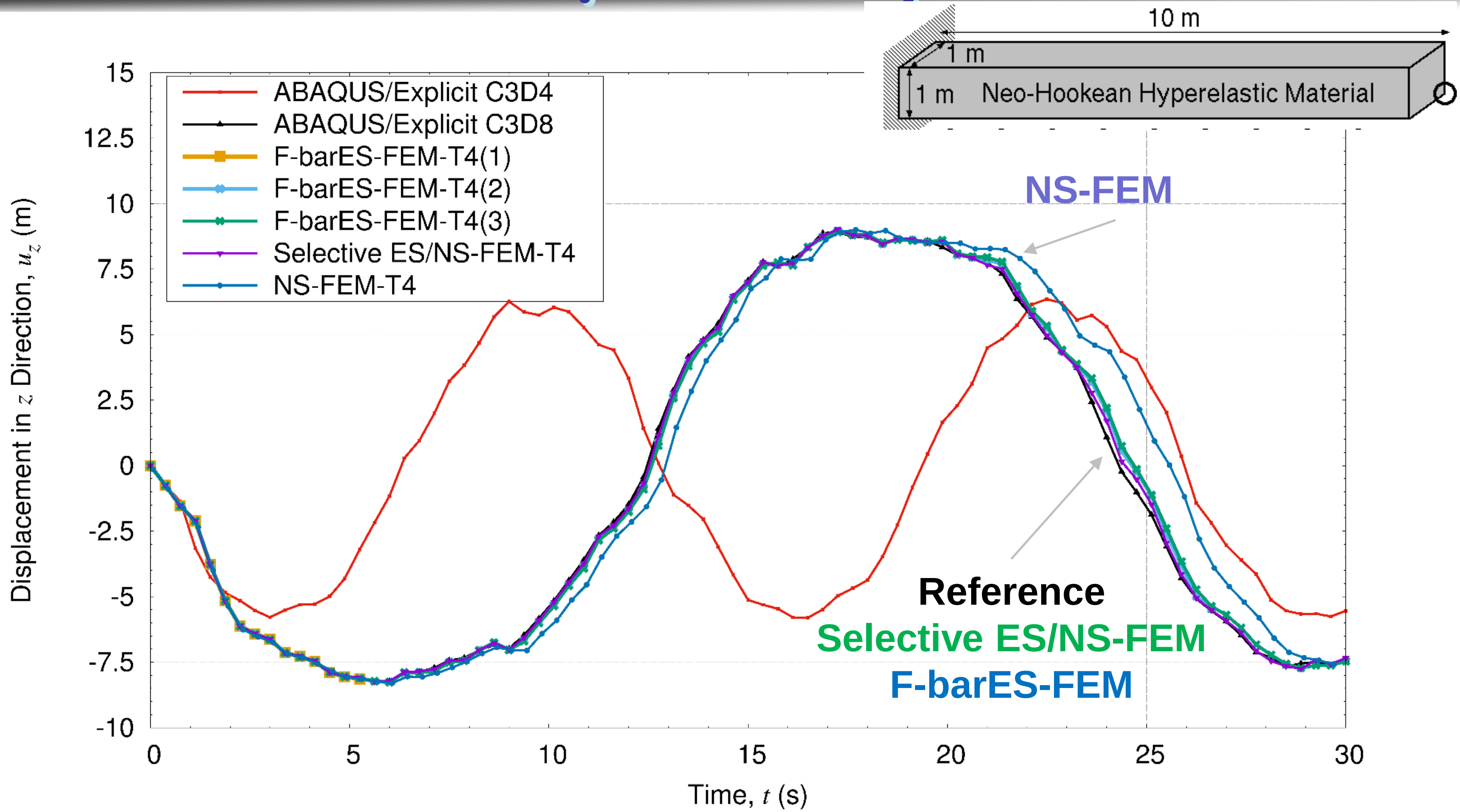
ABAQUS/Explicit C3D8
(Selective H8 element)
Reference



F-barES-FEM(2)

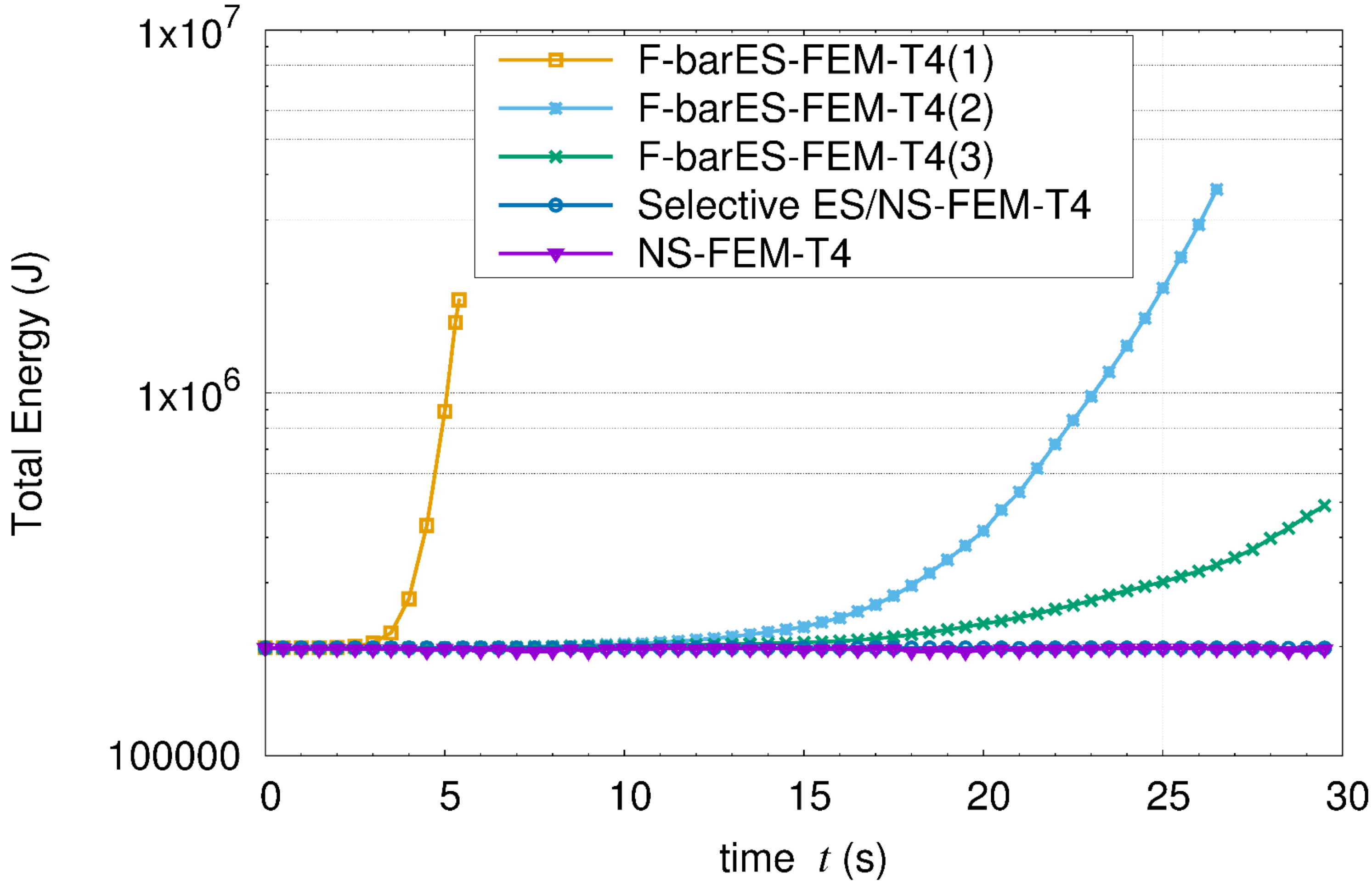
F-barES-FEM(2) is comparable to Selective H8 element!

Time history of displacement



- **NS-FEM** shows slightly soft result.
- **Selective ES/NS-FEM** and **F-barES-FEM** agree with the reference.

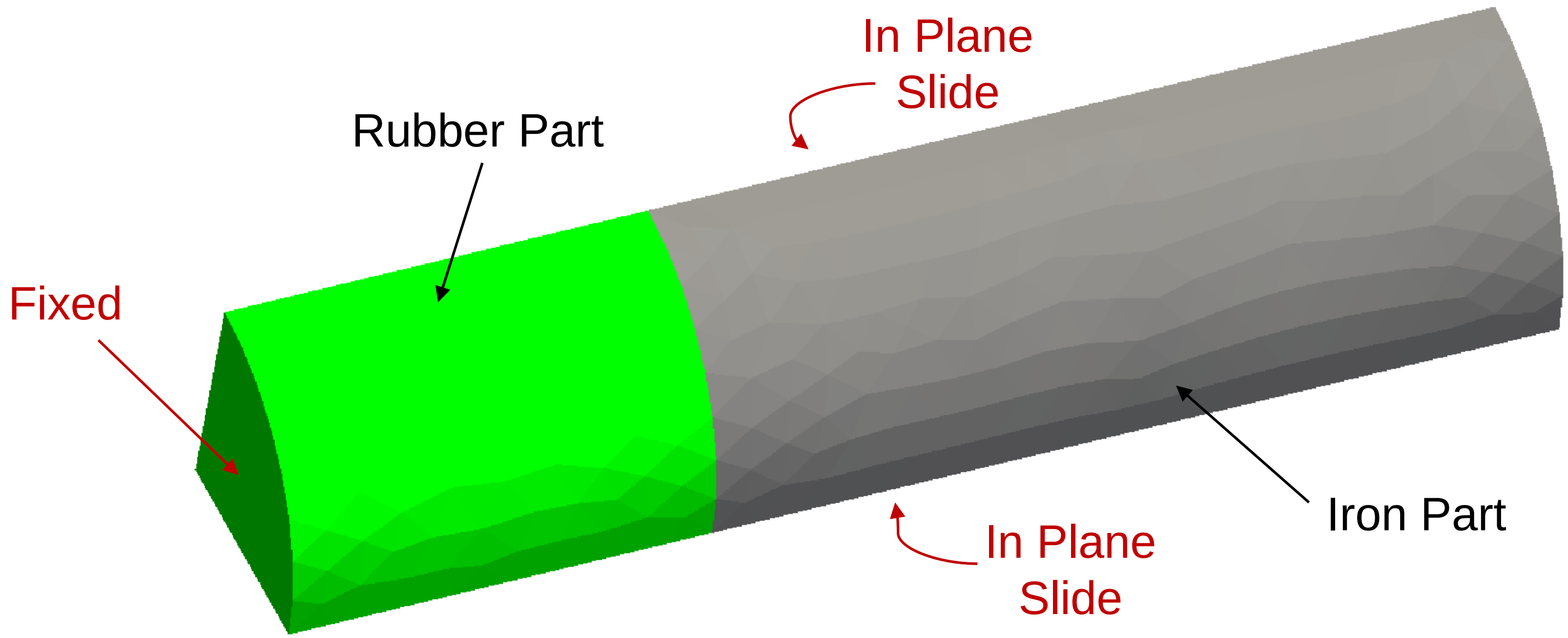
Time history of total energy



- **F-barES-FEM** causes energy divergences in earlier stage…
- Increasing the number of smoothings suppresses the speed of divergence.

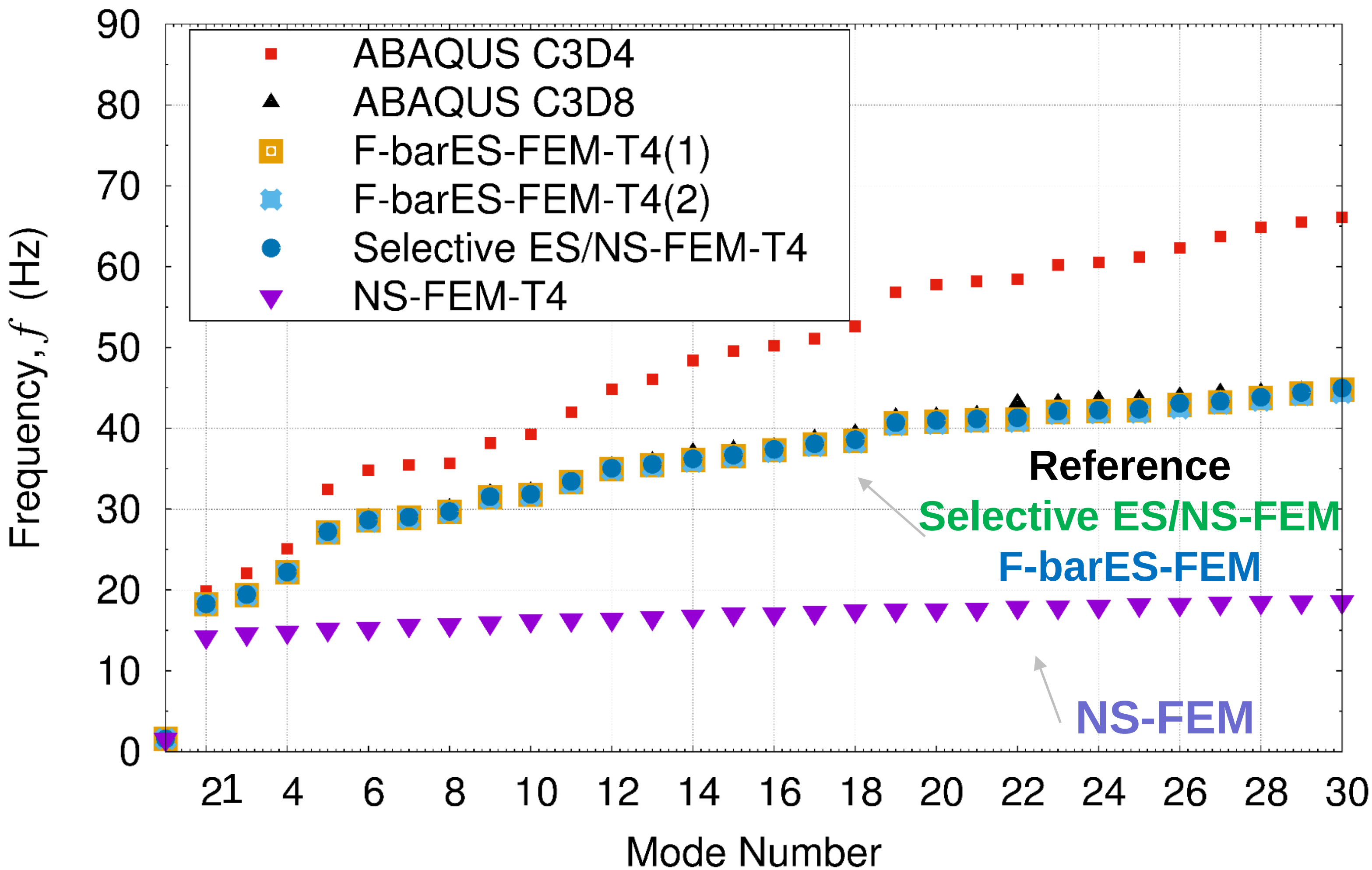
Natural Modes of ¼ Cylinder

Outline



- Iron part: $E_{\text{ini}} = 200\ \text{GPa},\ \nu_{ini} = \mathbf{0.3},\ \rho = 7800\ \text{kg/m}^3$, Elastic, **No cyclic smoothing.**
- Rubber part: $E_{\text{ini}} = 6\ \text{MPa},\ \nu_{ini} = \mathbf{0.499},\ \rho = 920\ \text{kg/m}^3$, Elastic, **1 or 2 cycles of smoothing.**
- Compare S-FEMs with ABAQUS C3D4 and C3D8.

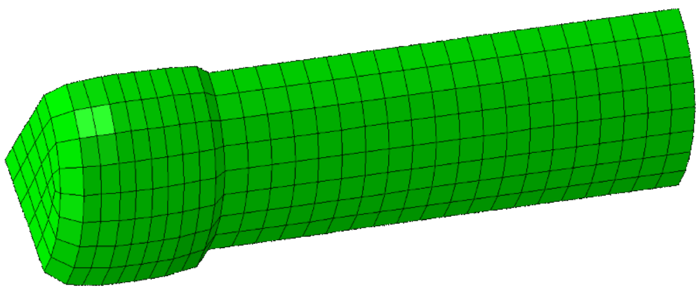
Natural frequencies of each mode



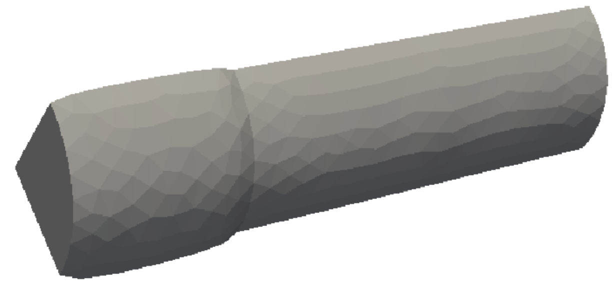
NS-FEM shows bad results due to spurious low-energy modes.

Selective ES/NS-FEM and **F-barES-FEM** agree with the reference.

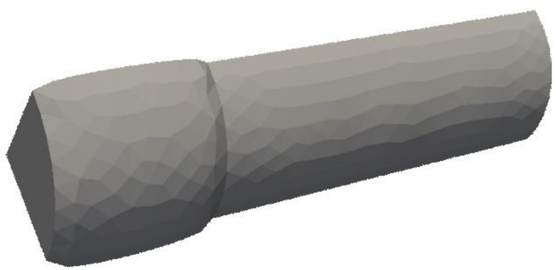
1^{st} mode shapes



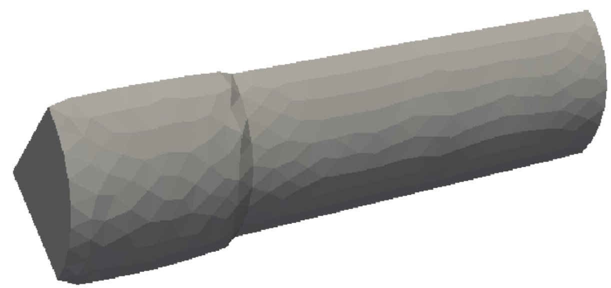
ABAQUS C3D8
(**Reference**)



Selective ES/NS-FEM



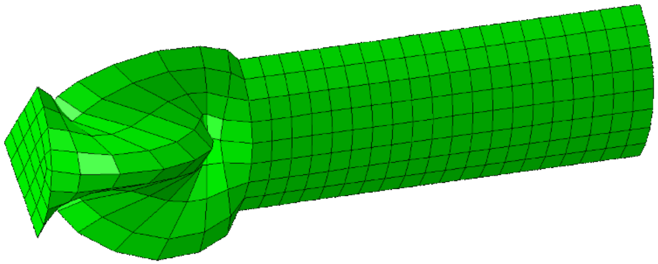
F-barES-FEM(2)



NS-FEM

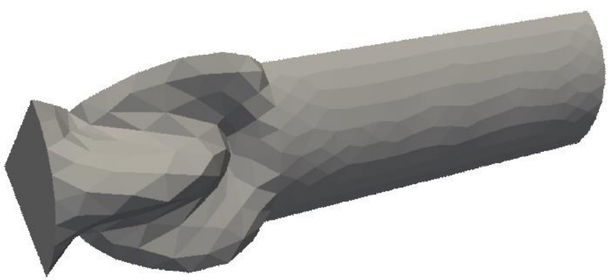
1^{st} mode shapes also agree with the reference solutions.

11^{th} mode shapes



ABAQUS C3D8
(**Reference**)

Selective ES/NS-FEM



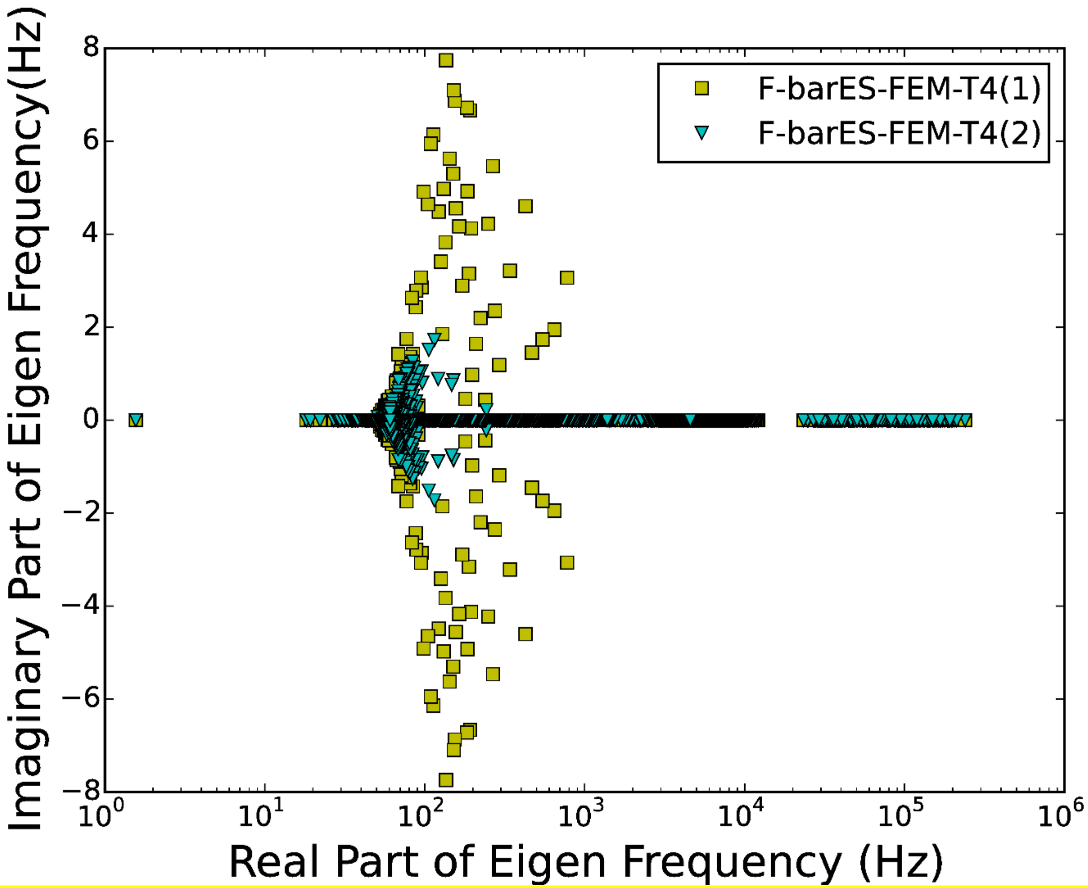
F-barES-FEM(2)

NS-FEM
spurious low-energy mode

NS-FEM shows spurious low-energy mode!

Distributions of natural frequencies of F-barES-FEM

Imag.
Part



Some natural frequencies have small imaginary part…

Increasing the number of smoothings makes the frequencies close to real numbers.

Cause of energy divergence

Due to the adoption of F-bar method,
the stiffness matrix $[K]$ becomes asymmetric.

Equation of Motion: $[M]\{\ddot{x}\} + [K]\{x\} = \{f^{\text{ext}}\}$

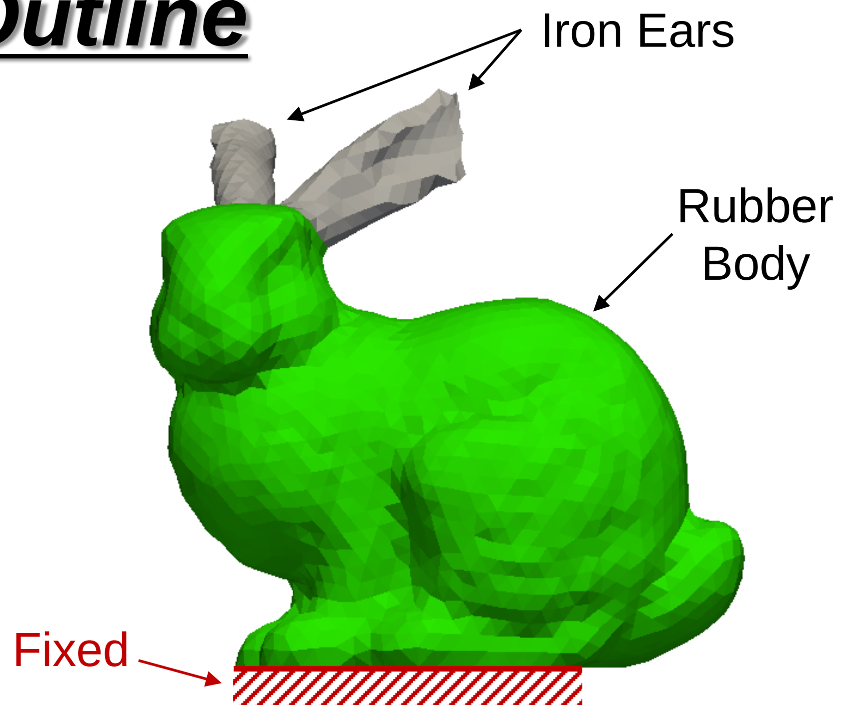
asymmetric

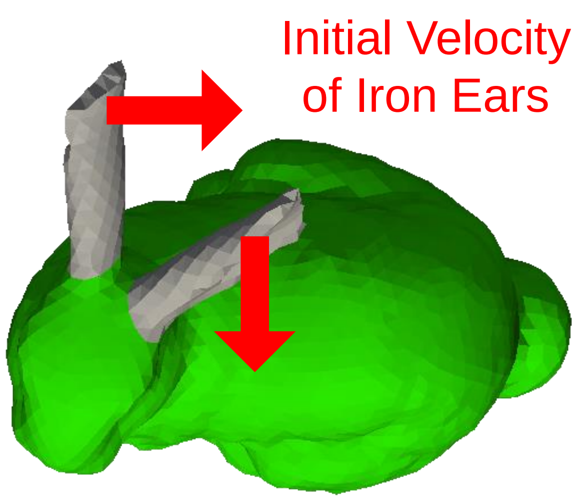
- Asymmetric stiffness matrix gives rise to imaginary part of natural frequencies and instability in dynamic problem.
- As shown before, increasing the number of smoothings suppress the energy divergence speed.

F-barES-FEM is restricted to
short-term analysis (such as impact analysis)
with a sufficient number of smoothings.

Swinging of Bunny Ears

Outline

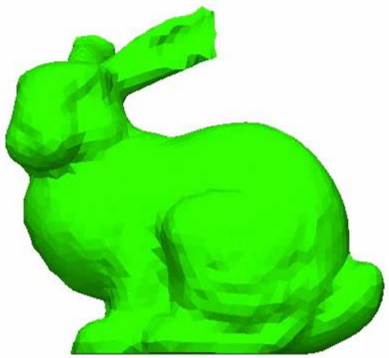




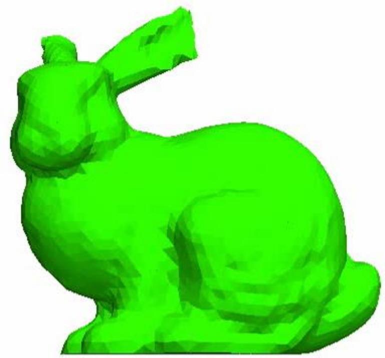
- Iron ears: $E_{\mathrm{ini}} = 200\ \mathrm{GPa},\ \nu_{\mathrm{ini}} = \mathbf{0.3},\ \rho = 7800\ \mathrm{kg/m^3}$, Neo-Hookean, **No cyclic smoothing.**
- Rubber body: $E_{\mathrm{ini}} = 6\ \mathrm{MPa},\ \nu_{\mathrm{ini}} = \mathbf{0.49},\ \rho = 920\ \mathrm{kg/m^3}$, Neo-Hookean, **1 cycle of smoothing.**
- Compared to ABAQUS/Explicit C3D4. No Hex mesh available!

Time histories of deformed shapes

✗ Pressure oscillation

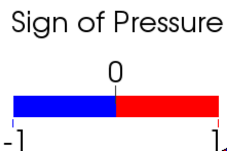


ABAQUS/Explicit C3D4

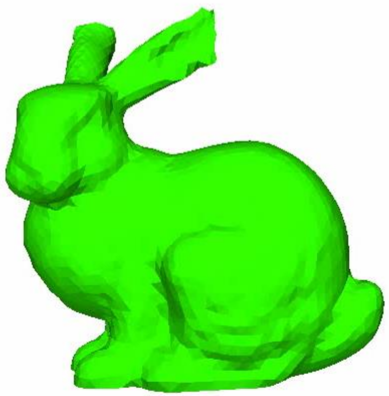


Selective ES/NS-FEM

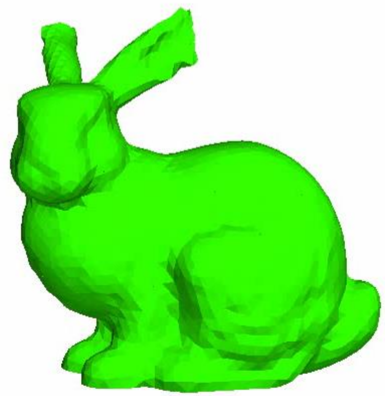
✔ Pressure waves



✗ Pressure oscillation



F-barES-FEM



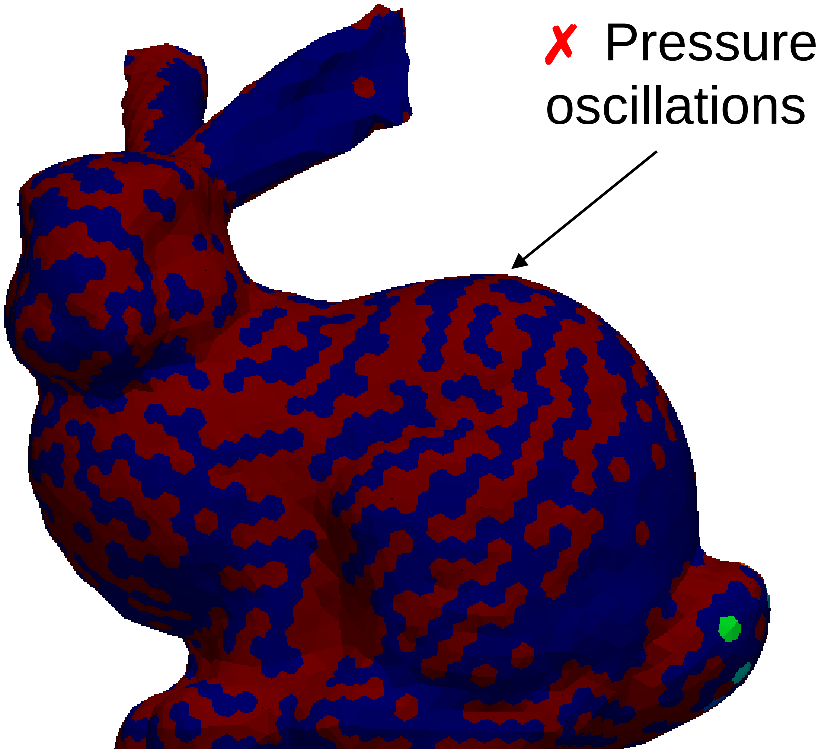
NS-FEM

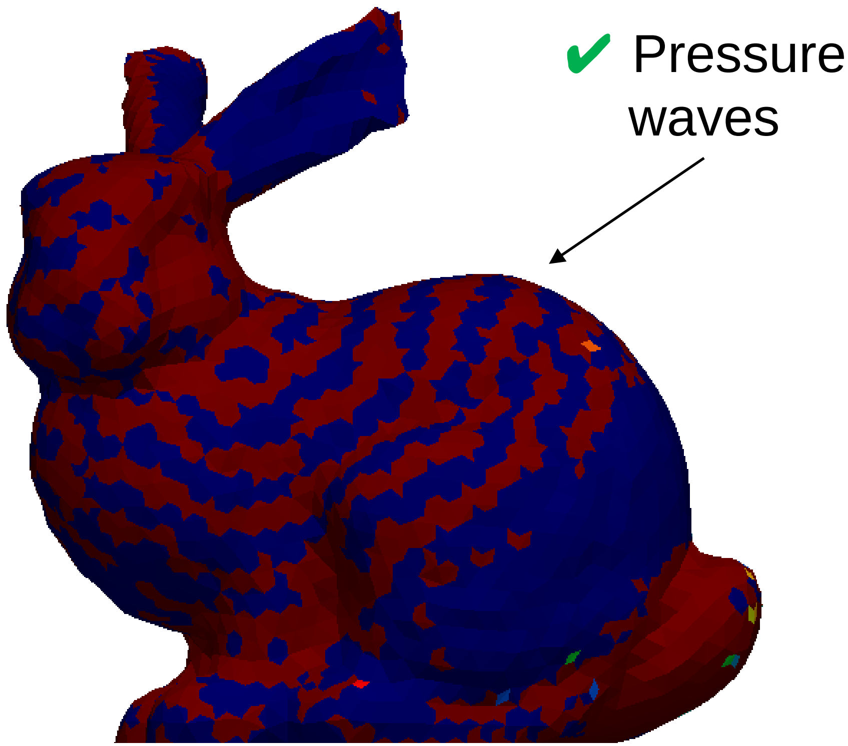
✗ Pressure oscillation
✗ suspension in an early stage

Only F-barES-FEM seems to be representing not pressure oscillations but pressure waves.

Deformed shapes and sign of pressure

In an early stage

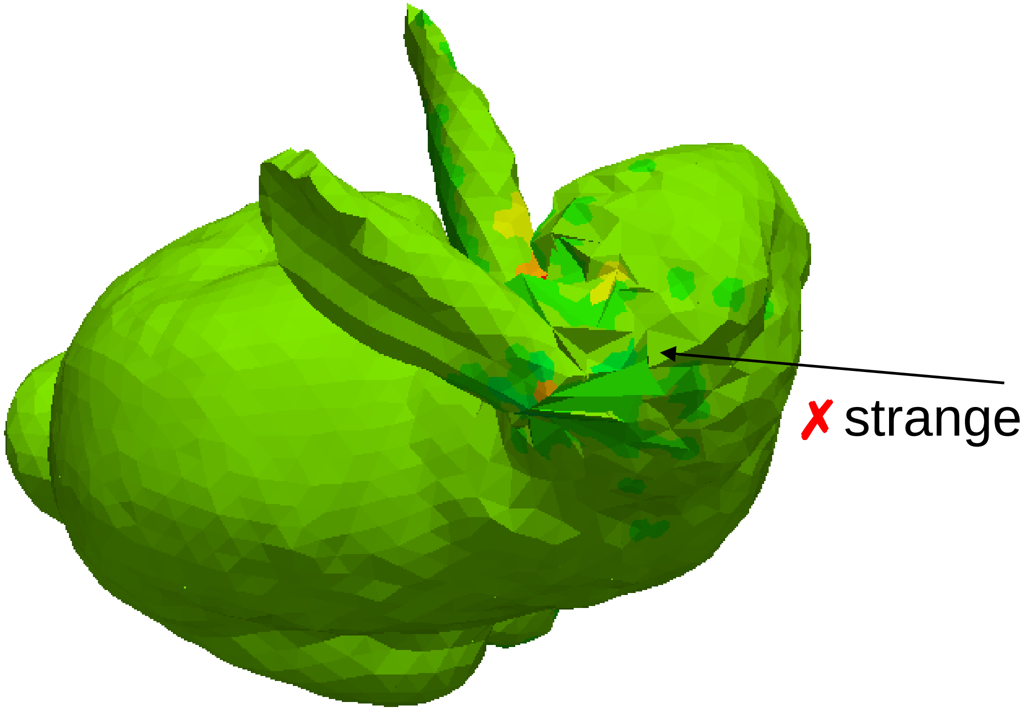




Selective ES/NS-FEM **F-barES-FEM**

F-barES-FEM represents pressure waves correctly!

Deformed shapes and pressure distributions



NS-FEM

NS-FEM shows strange shapes in large deformed part

Summary

Summary

Characteristics of S-FEMs are summarized like them.

Selective ES/NS-FEM

- Dynamic: with little pressure oscillation, temporary stable.
- Modal : high accuracy
- Constitutive equations are restricted.

F-barES-FEM

- Dynamic: with no pressure oscillation, temporary unstable.
- Modal : high accuracy, imaginary part of natural frequencies.
- It is restricted to short-term analysis.

Thank you for your kind attention.

Appendix

B-barES-FEM

F-barES-FEM

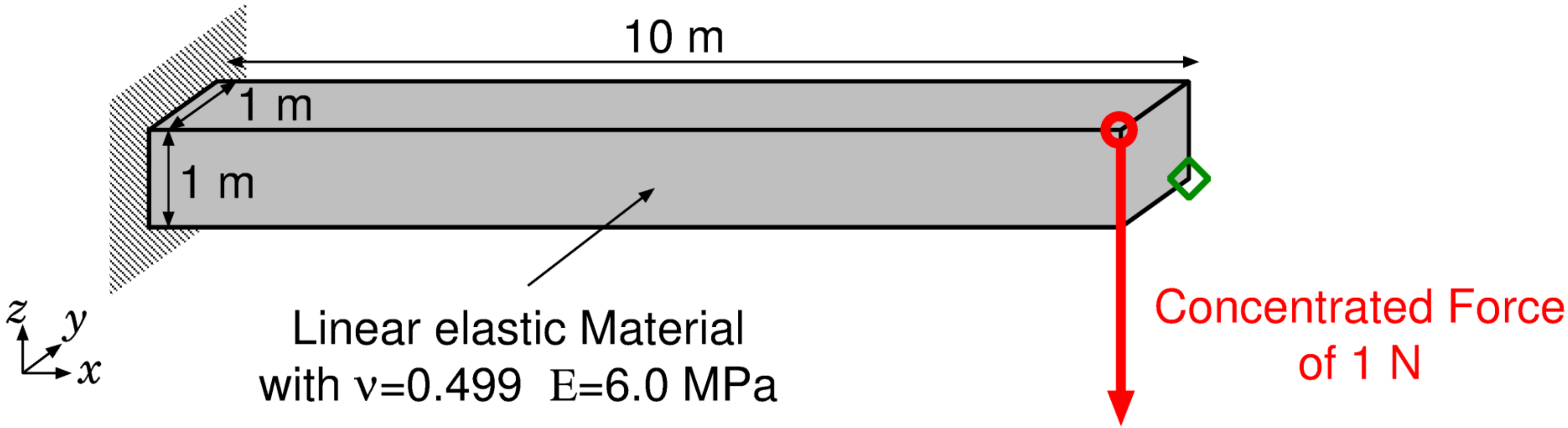
$$\{f^{\text{int}}\} = \sum_{\text{Edge}} [\tilde{B}^{\text{Edge}}]\{\bar{T}\}V$$

derived from **ES-FEM** derived from $\bar{\boldsymbol{F}}$

B-barES-FEM

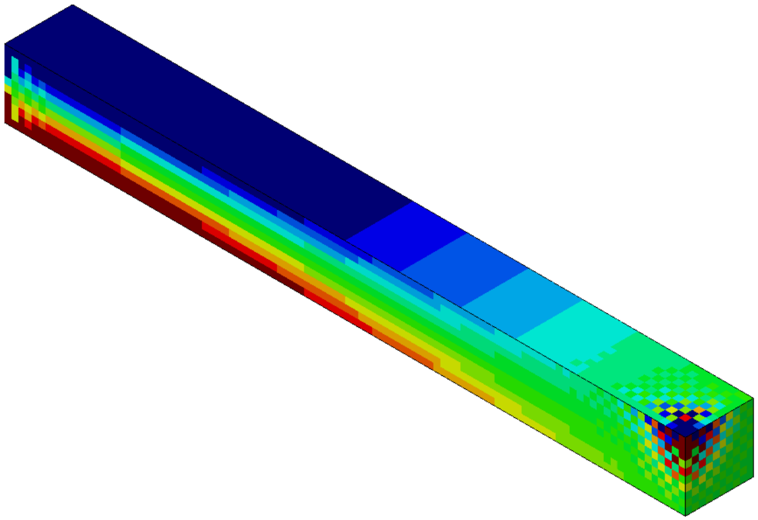
$$\{f^{\text{int}}\} = \sum_{\text{Edge}} [\bar{B}^{\text{Edge}}]\{\bar{T}\}V$$

Cantilever Bending Analysis

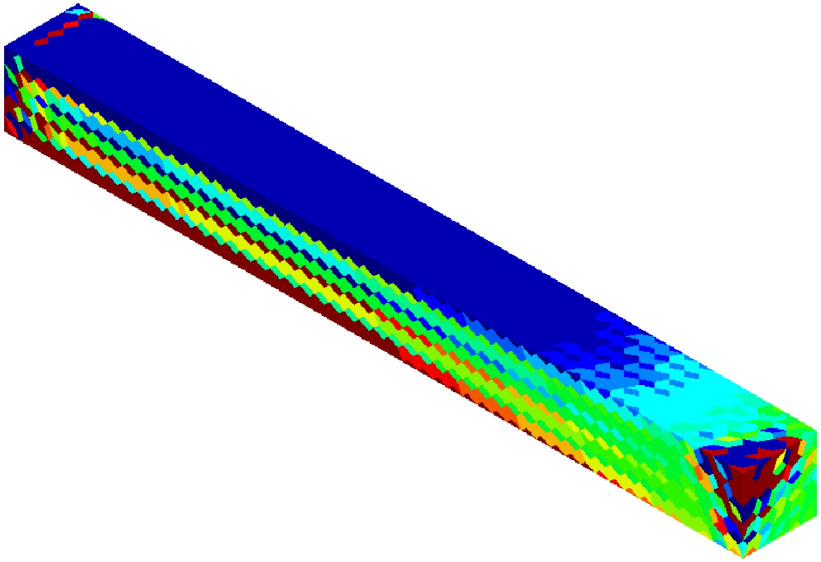


- Small deformation static analysis
- Compare B-barES-FEM with ABAQUS C3D20H

Pressure distributions

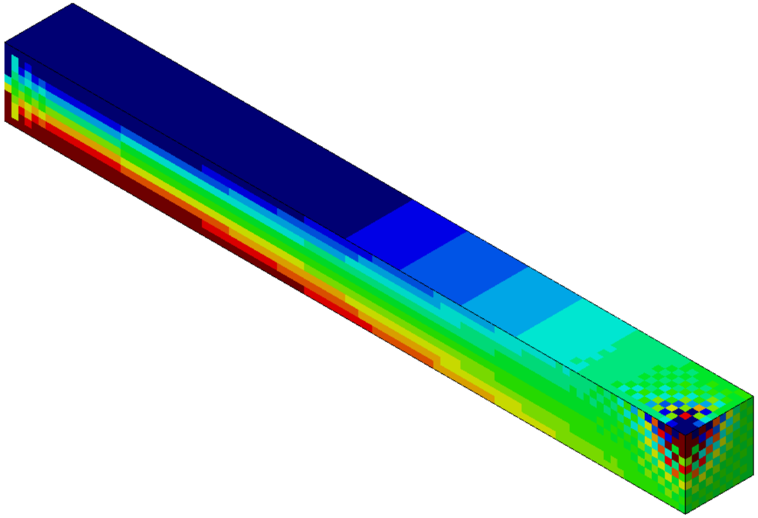


ABAQUS C3D20H
Reference

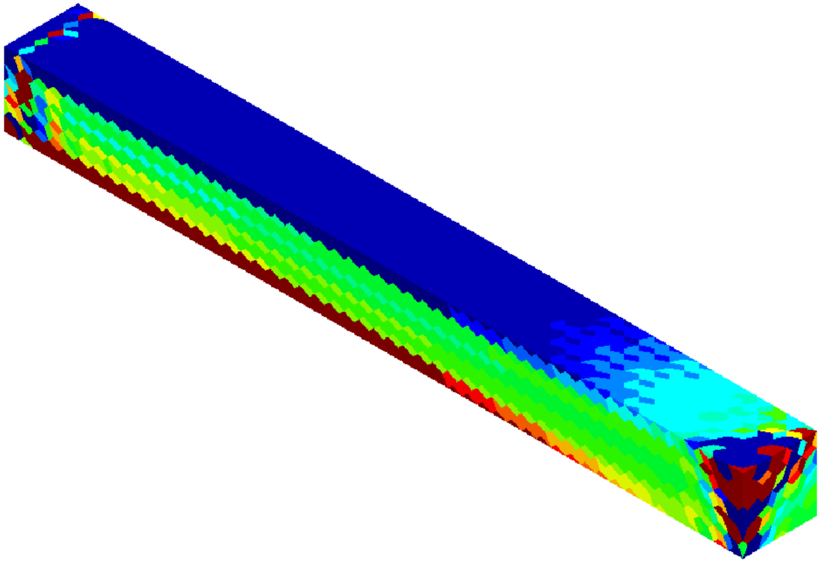


B-barES-FEM(**1**)
Reference

Pressure distributions

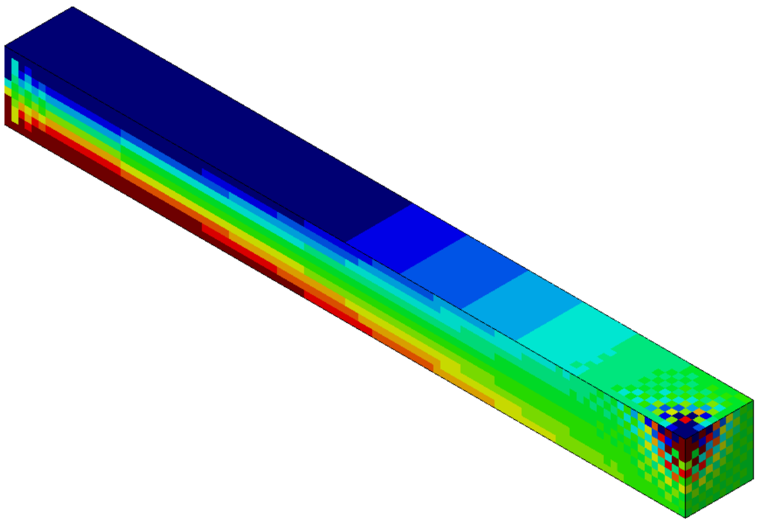


ABAQUS C3D20H
Reference

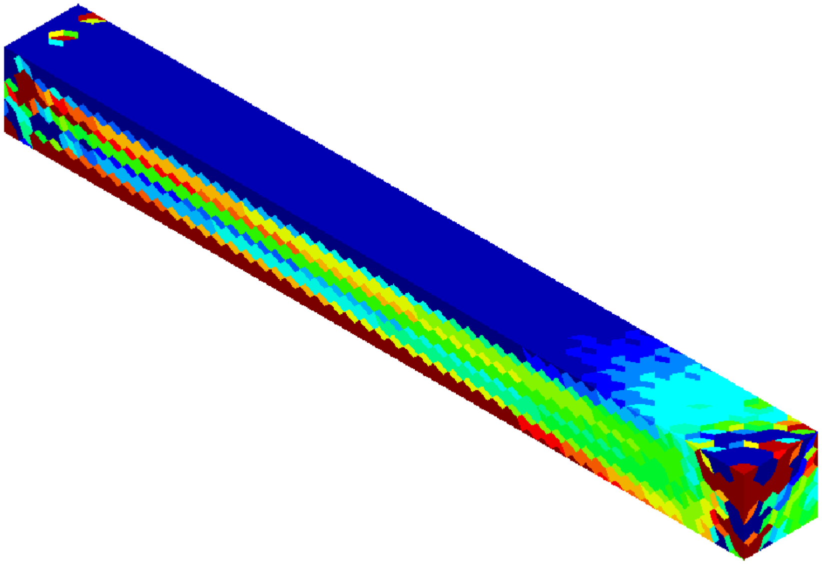


B-barES-FEM(**2**)
Reference

Pressure distributions

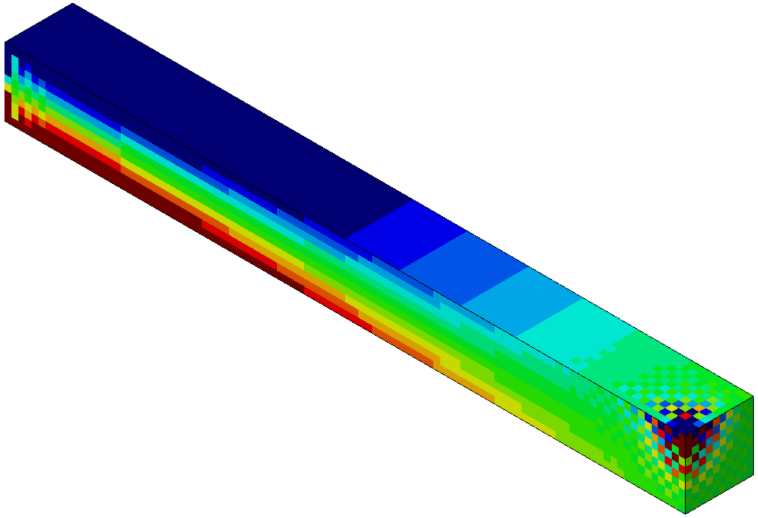


ABAQUS C3D20H
Reference

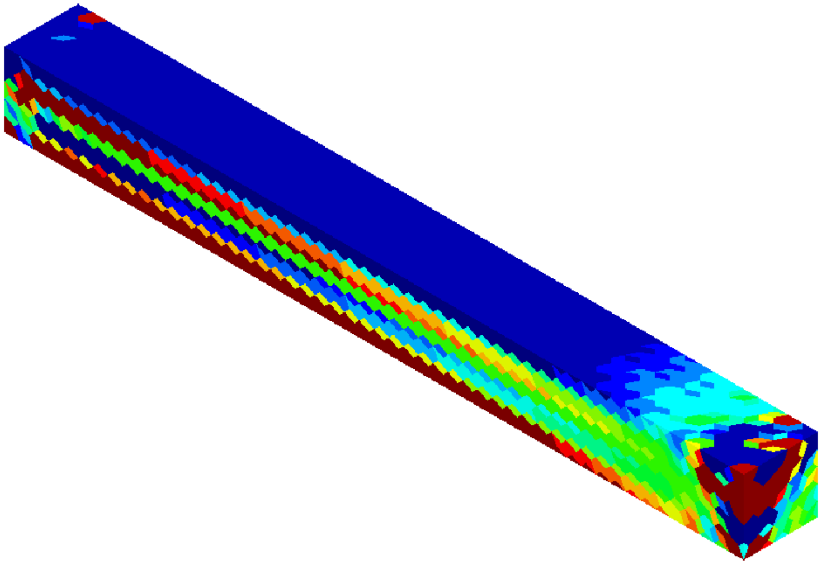


B-barES-FEM(**3**)
Reference

Pressure distributions



ABAQUS C3D20H
Reference

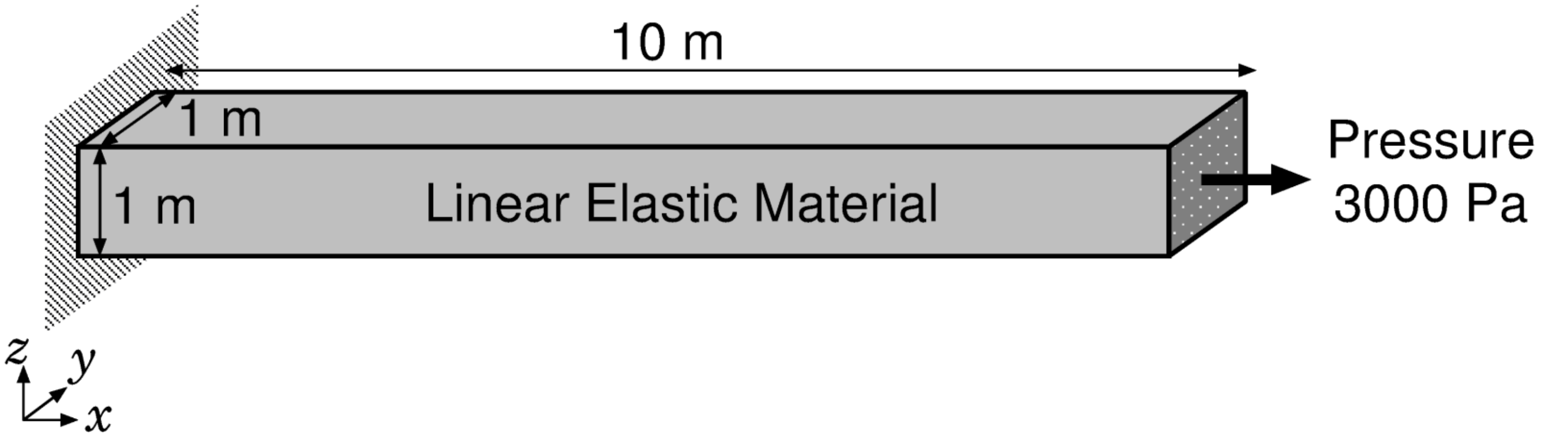


B-barES-FEM(**4**)
Reference

東京工業大学
Tokyo Institute of Technology
TOKYO TECH
Pursuing Excellence

Propagation of 1D pressure wave

Outline



- Small deformation analysis.
- Linear elastic material,

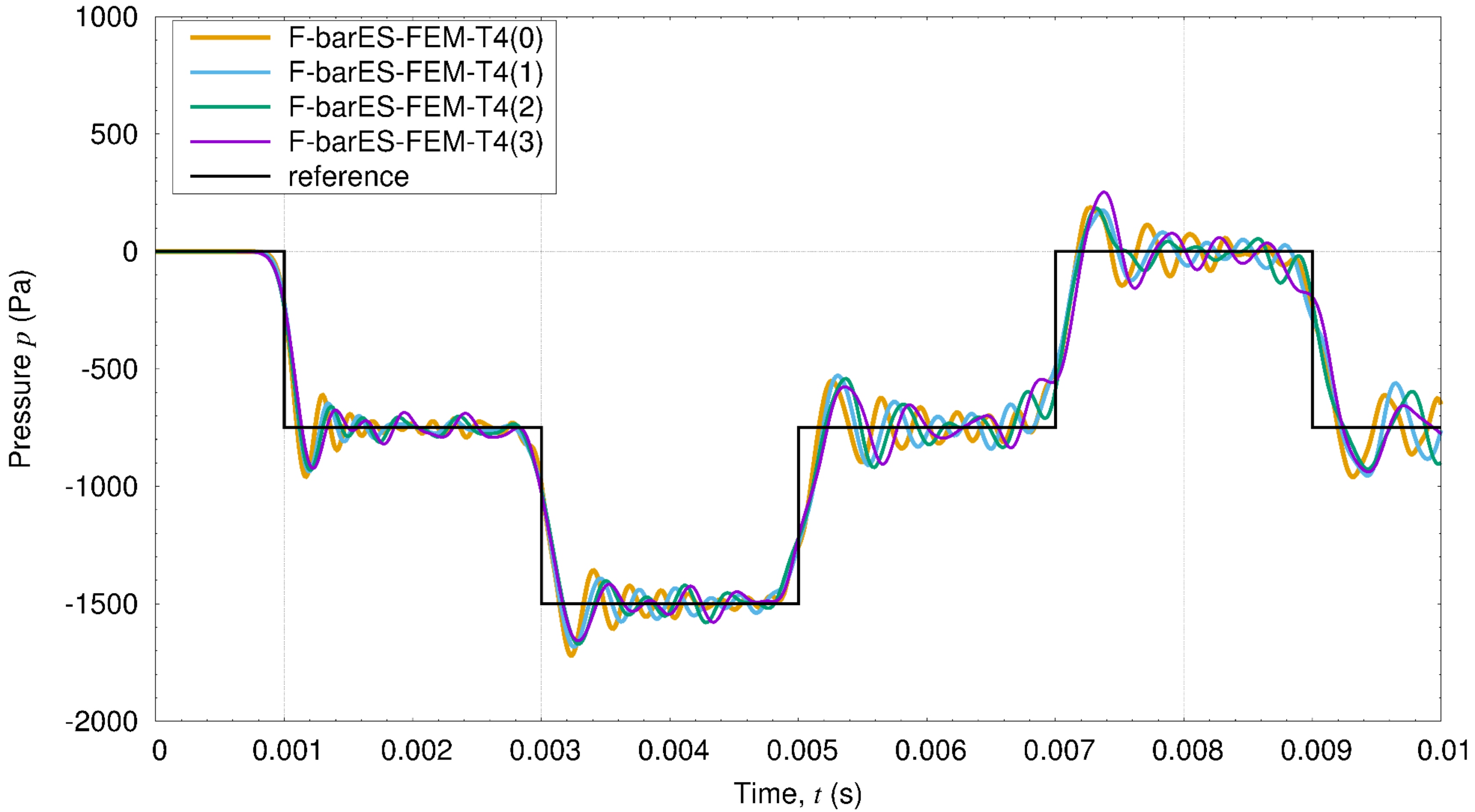
  Young's modulus: 200 GPa,

  Poisson's ratio: 0.0,

  Density: 8000 kg/m^3.
- Results of F-barES-FEM(0), (1), (2), and (3) are compared to the analysical solution.

Propagation of 1D pressure wave

Results



Velocity Verlet Method

Algorithm

1. Calculate the next displacement $\{u_{n+1}\}$ as

$$\{u_{n+1}\} = \{u_n\} + \{\dot{u}_n\}\Delta \mathrm{t} + \frac{1}{2}\{\ddot{u}_n\}\Delta \mathrm{t}^2.$$

2. Calculate the next acceleration $\{\ddot{u}_{n+1}\}$ as

$$\{\ddot{u}_{n+1}\} = [M^{-1}](\{f^{\mathrm{ext}}\} - \{f^{\mathrm{int}}(u_{n+1})\}).$$

3. Calculate the next velocity $\{\dot{u}_{n+1}\}$ as

$$\{\dot{u}_{n+1}\} = \{\dot{u}_n\} + \{\ddot{u}_{n+1}\}\Delta t$$

Characteristics

- 2nd order symplectic scheme in time.
- Less energy divergence.

Cause of energy divergence

Due to the adoption of F-bar method, the stiffness matrix $[K]$ becomes asymmetric and thus the dynamic system turns to unstable.

Equation of natural vibration, $[M]\{\ddot{u}\} + [K]\{u\} = \{0\}$, derives an eigen equation, $([M]^{-1}[K])\{u\} = \omega^2\{u\}$, which has asymmetric left-hand side matrix.

⇒Some of eigen frequencies could be complex numbers.

⇒When an angular frequency $\omega_k = a + \mathrm{i}b$ $(b > 0)$, the time variation of the kth mode is

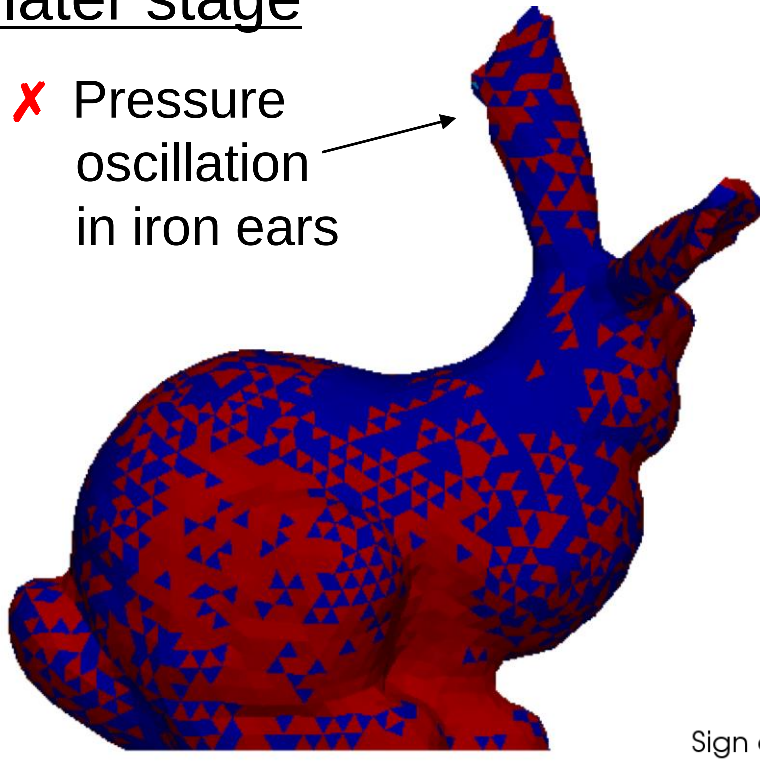
$$\{u(t)\} = \mathrm{Re}[\{u_k\}\exp(-\mathrm{i}\omega_k t)]$$
$$= \mathrm{Re}[\{u_k\}\exp(-\mathrm{i}at)\exp(bt)]$$

Divergent term!

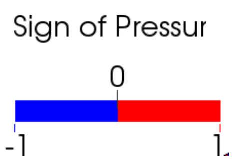
Deformed shapes and sign of pressure

In a later stage

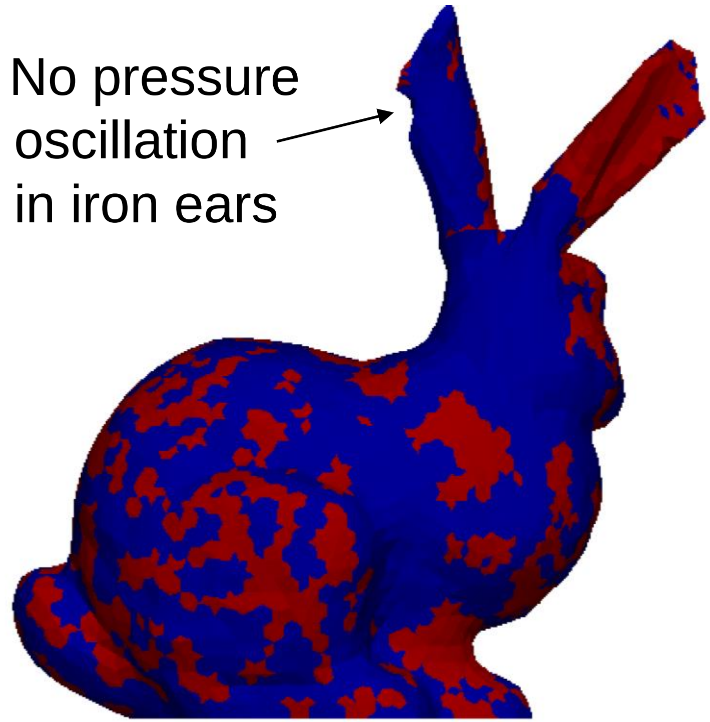
✗ Pressure oscillation in iron ears



ABAQUS/Explicit C3D4
(Standard T4 element)



✔ No pressure oscillation in iron ears



F-barES-FEM

It should be noted that a presence of rubber spoils over all accuracy of the analysis with Standard T4 elements.

A rubber parts is a “bad apple” when Standard T4 elements are used.