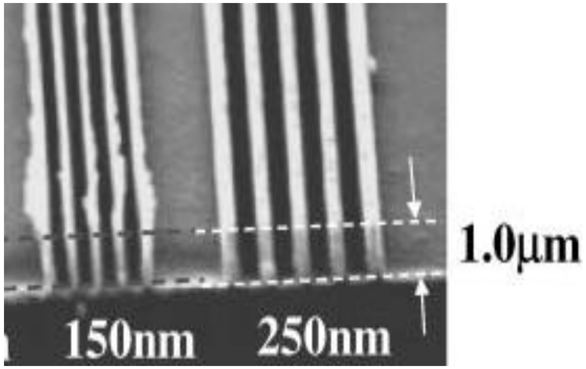
# Performance Evaluation of the Selective Smoothed Finite Element Method with Deviatoric/Hydrostatic Split

**Yuki ONISHI**, **Kenji AMAYA**
**Tokyo Institute of Technology (Japan)**

# Motivation & Background

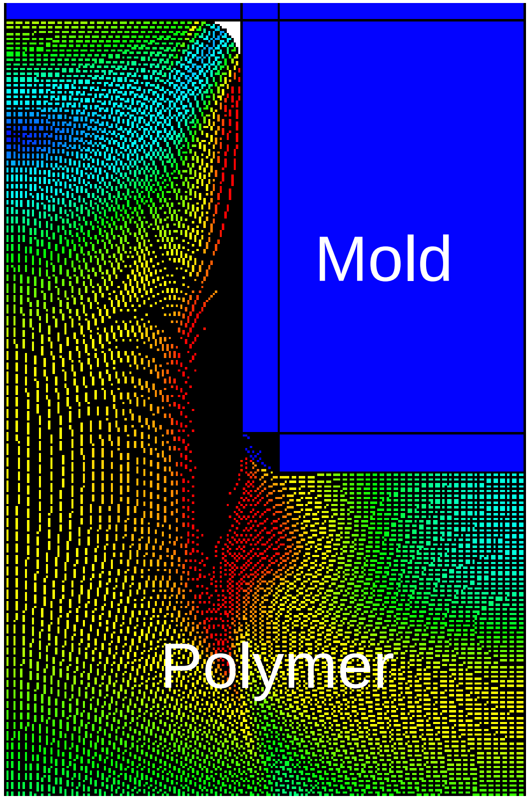
## ***Motivation***

We want to analyze **severely large deformation** problems in solids ***accurately and stably!***
(Target: automobile tire, thermal nanoimprint, etc.)



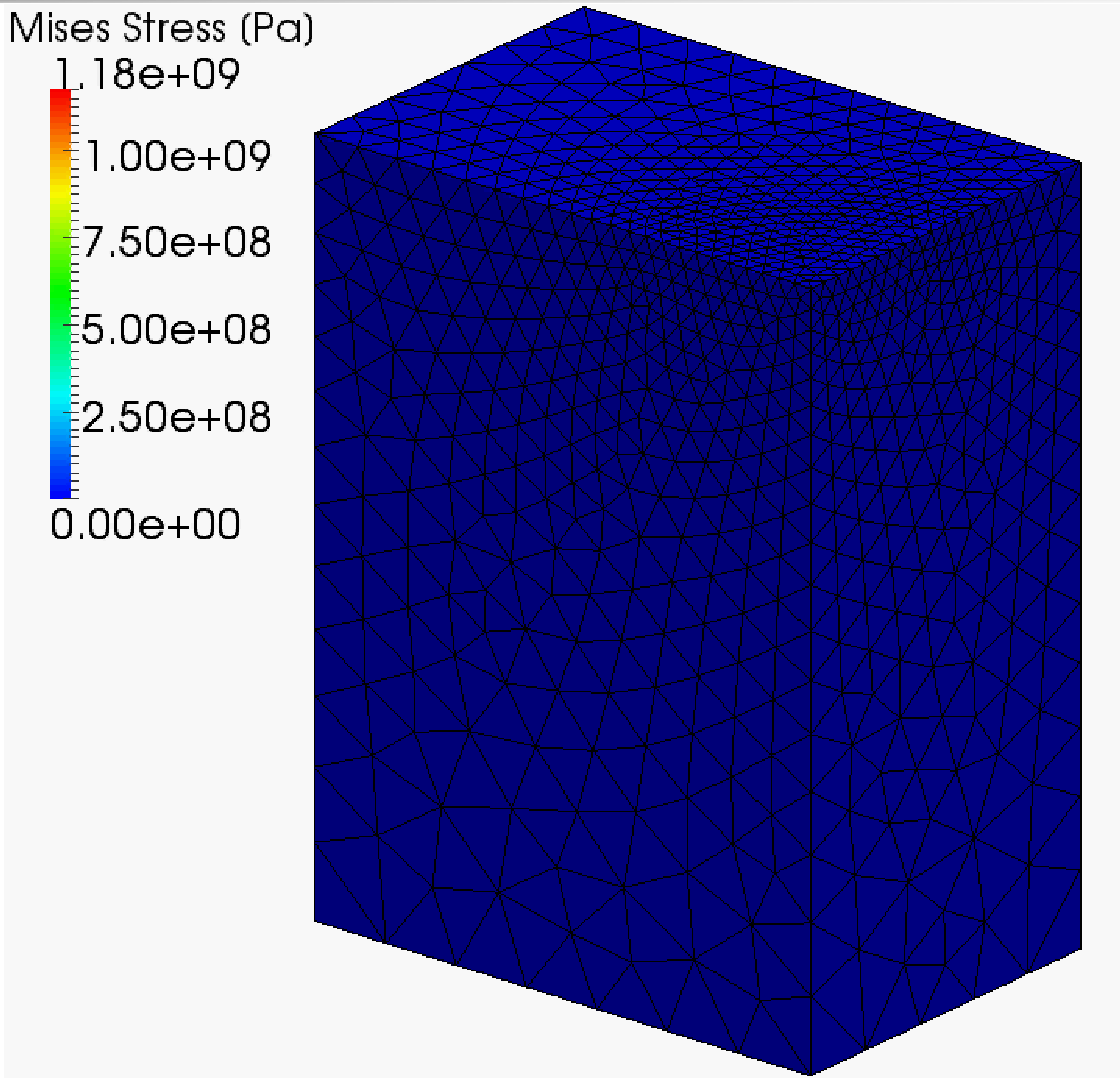

## ***Background***

Finite elements are **distorted** in a short time, thereby resulting in convergence failure.




Mesh rezoning method (*h*-adaptive mesh-to-mesh solution mapping) is indispensable.

# Our First Result in Advance



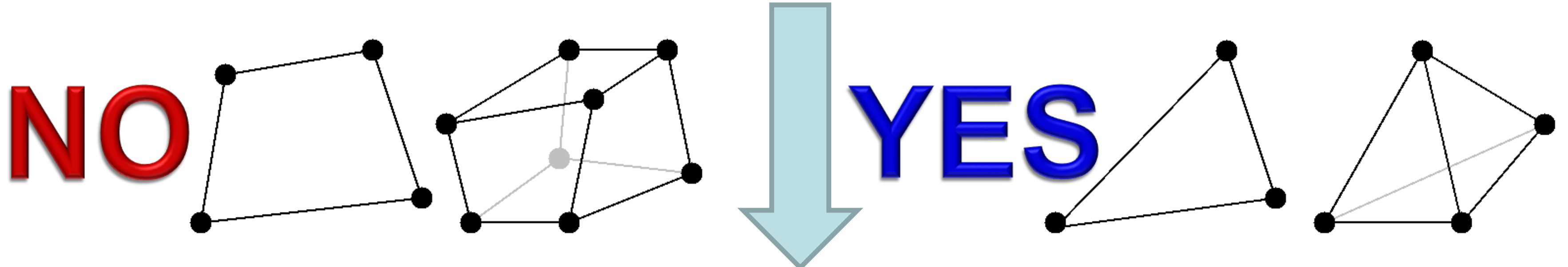

What we want to do:

- Static
- Implicit
- Large deformation
- Mesh rezoning

# Issues

***The biggest issue***
***in large deformation mesh rezoning***

It is impossible to remesh arbitrary deformed 2D or 3D domains with quadrilateral or hexahedral elements.




We have to use triangular or tetrahedral elements...

However, the *standard* (constant strain) triangular or tetrahedral elements induce shear and volumetric locking easily, which leads to inaccurate results.

# Conventional Methods

- Higher order elements:
  - ✗ Not volumetric locking free; Not effective in large deformation due to intermediate nodes.
- EAS elements:
  - ✗ Unstable.
- B-bar, F-bar and selective integration elements:
  - ✗ Not applicable to triangular/tetrahedral mesh.
- F-bar patch elements:
  - ✗ Difficult to construct good patches
- u/p hybrid (mixed) elements ← prime candidate
  - ✗ No sufficient formulation for triangular/tetrahedral mesh is presented so far. (There are almost acceptable hybrid elements such as C3D4H or C3D10H of ABAQUS.)
- Selective smoothed finite elements:
  - ? Unknown potential. Performance evaluation is necessary.

# Objective

Performance evaluation
of the selective S-FEMs
for large deformation problems

***Table of Body Contents***

- Part 1: Brief Introduction to the Formulation of the Selective S-FEMs with Dev/Hyd Split
- Part 2: Results of Performance Evaluation
- Part 3: Demonstration with Mesh Rezoning
- Summary

Part 1:

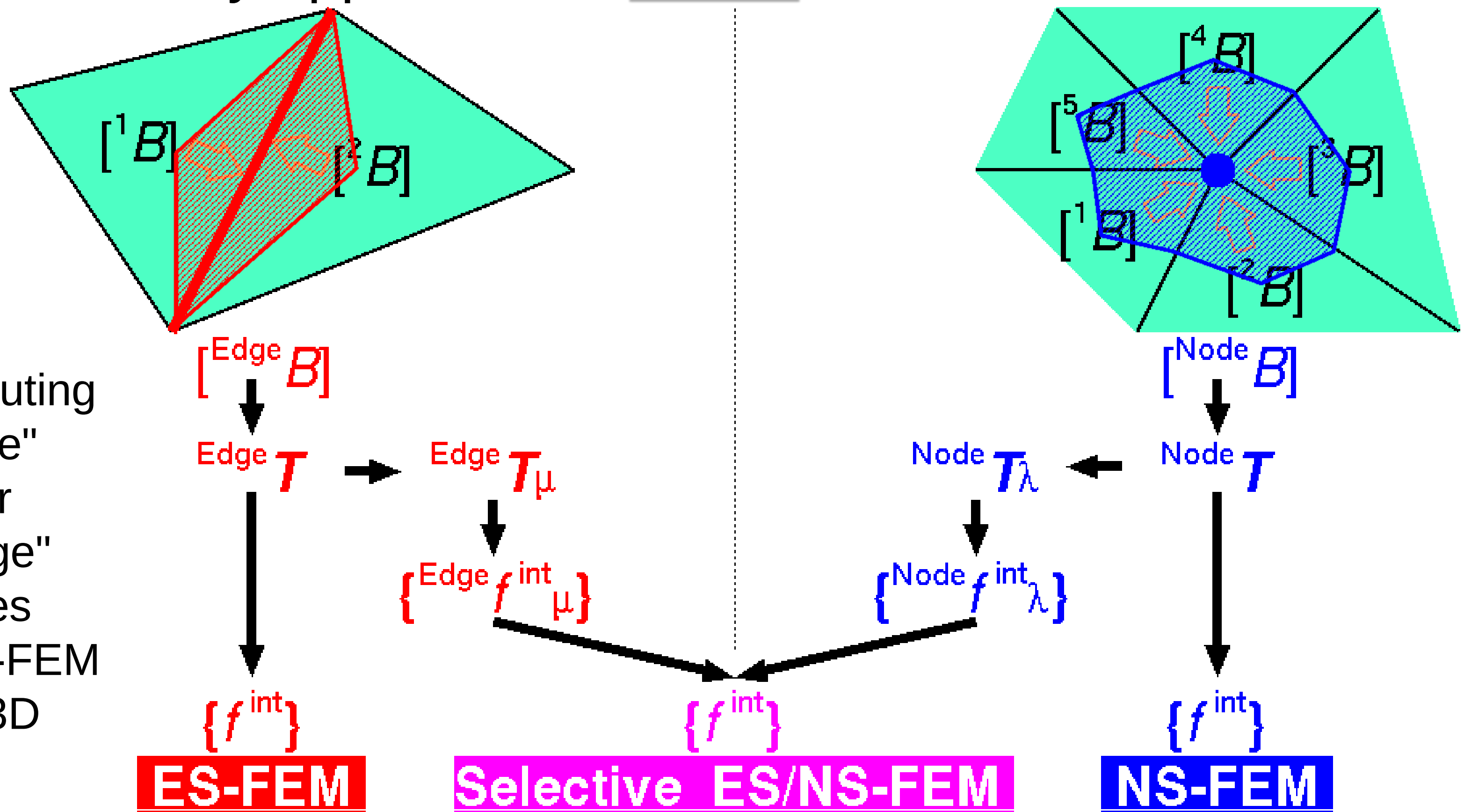
Brief Introduction to the Formulation of the Selective S-FEMs with Dev/Hyd Split

# Review of Selective ES/NS-FEM

- Separate stress into "$\mu$ part" and "$\lambda$ part", where $\mu$ and $\lambda$ are the Lame's parameters.
- ***F*, *T*** etc and {$f^{\text{ int}}$} are calculated on both smoothed domains.

Only applicable to elastic constitutive models.

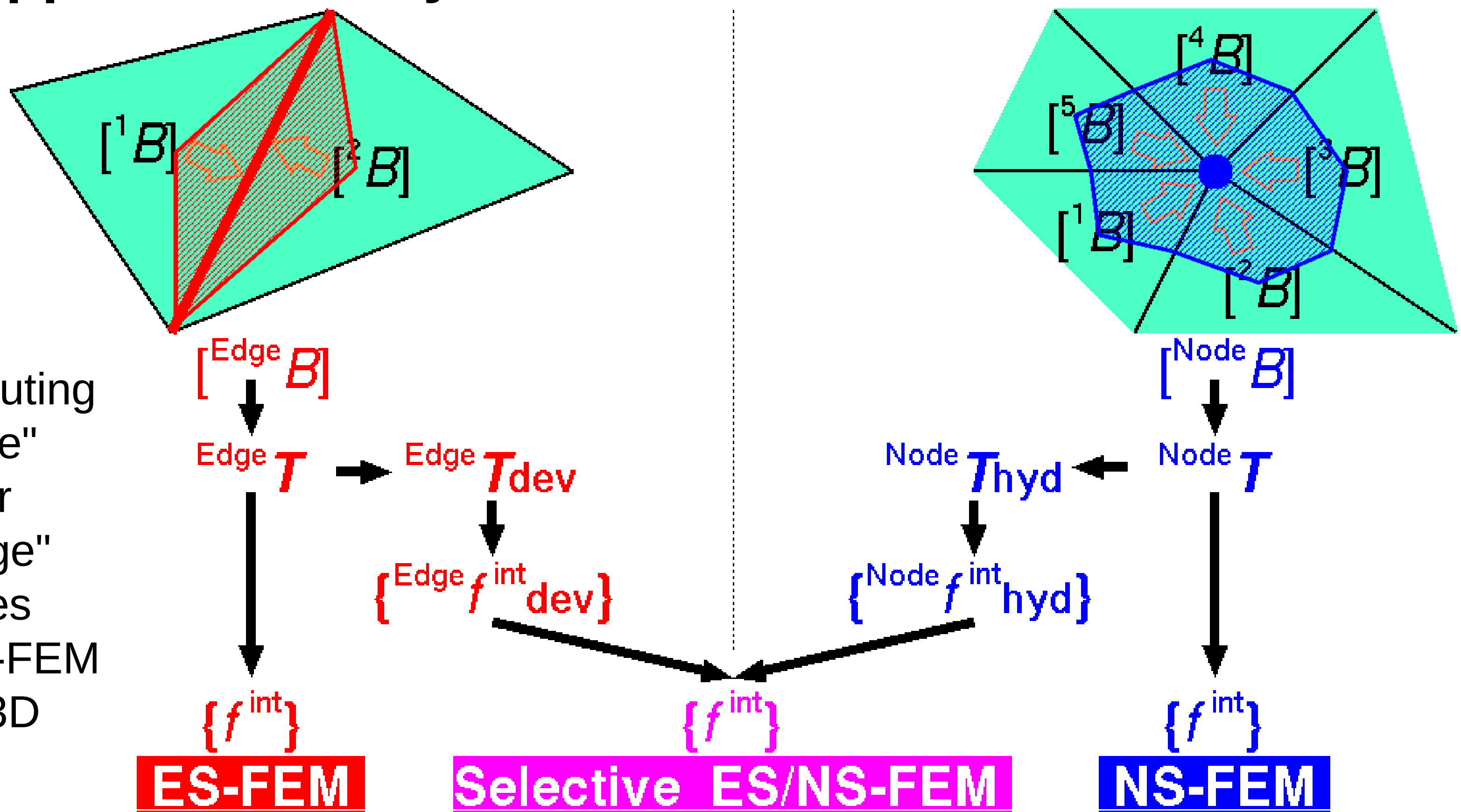
Substituting "face" for "edge" gives FS/NS-FEM for 3D



# Modified Selective ES/NS-FEM

- Separate stress into "deviatoric part" and "hydrostatic part" instead of "$\mu$ part" and "$\lambda$ part".
- ***F***, ***T*** etc and $\{f^{\mathrm{int}}\}$ are calculated on both smoothed domains.

**Applicable to any kind of material constitutive models.**




Substituting "face" for "edge" gives FS/NS-FEM for 3D

# 3 Types of Selective S-FEMs

| Method | Deviatoric Part | Hydrostatic Part |
|---|---|---|
| 2D<br>ES/NS-FEM-T3 | ES-FEM | NS-FEM |
| 3D<br>**ES**/NS-FEM-T4 | **ES**-FEM | NS-FEM |
| 3D<br>**FS**/NS-FEM-T4 | **FS**-FEM | NS-FEM |

<u>No increase in DOF!!</u>

Displacement vector $\{u\}$ is only the unknown.

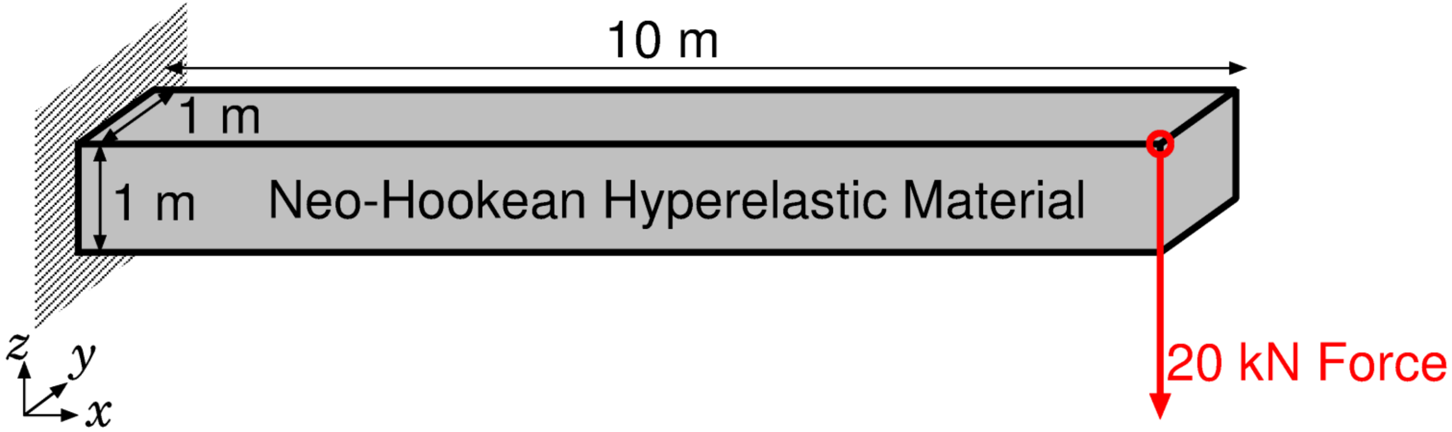
Part 2:

Results of Performance Evaluation

focused on the analysis
for hyperelastic materials
without mesh rezoning

# Verification ~ Bending of Cantilever ~

## Outline (Bending of Cantilever)




- Neo-Hookean hyperelastic material,

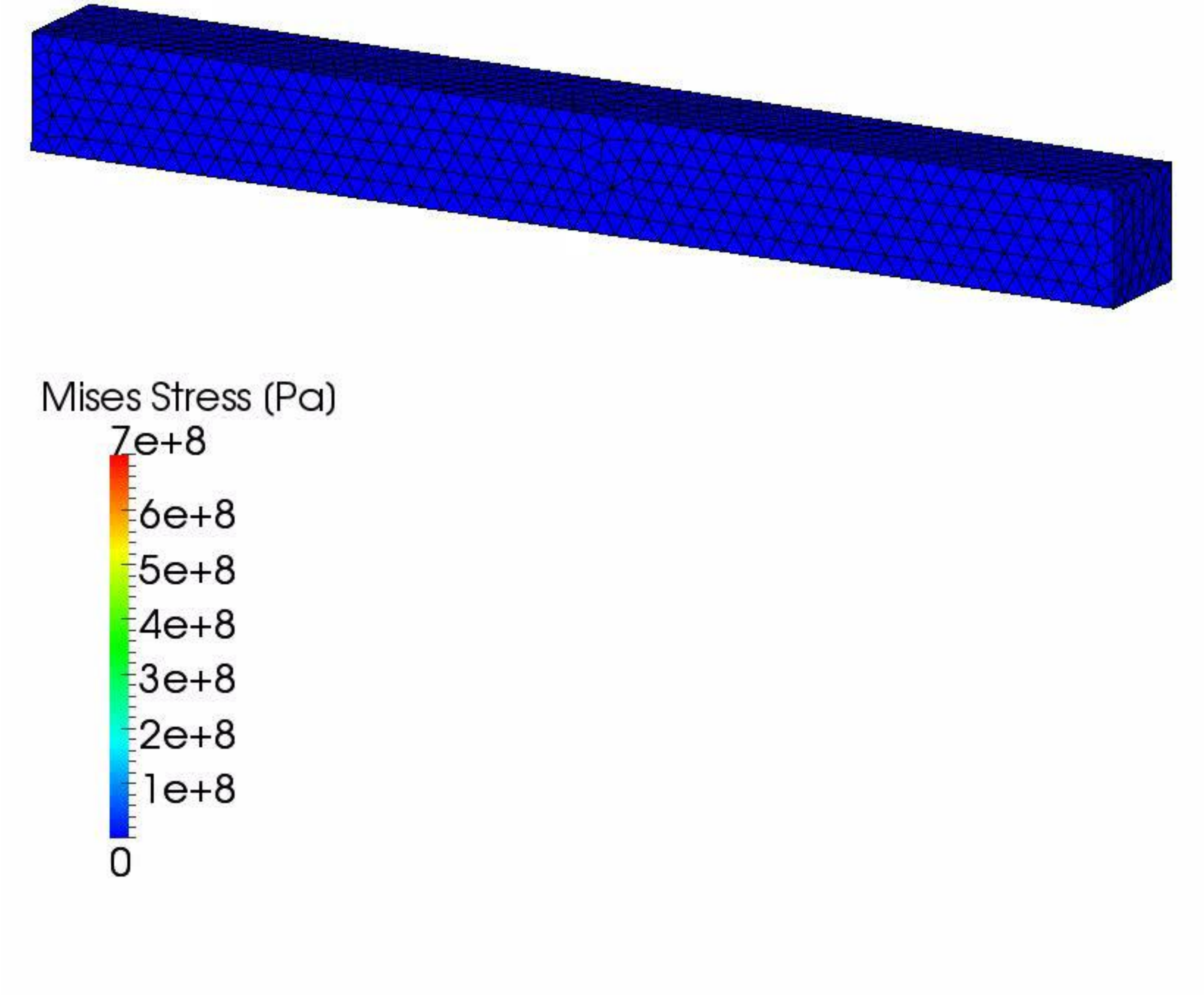
$$[T] = 2C_{10}\frac{\mathrm{Dev}(\bar{B})}{J} + \frac{2}{D_1}(J-1)[I]$$

with a constant $C_{10}$(=1 GPa) and various $D_1$s.

- Compared to ABAQUS with various elements.

# Verification ~ Bending of Cantilever ~

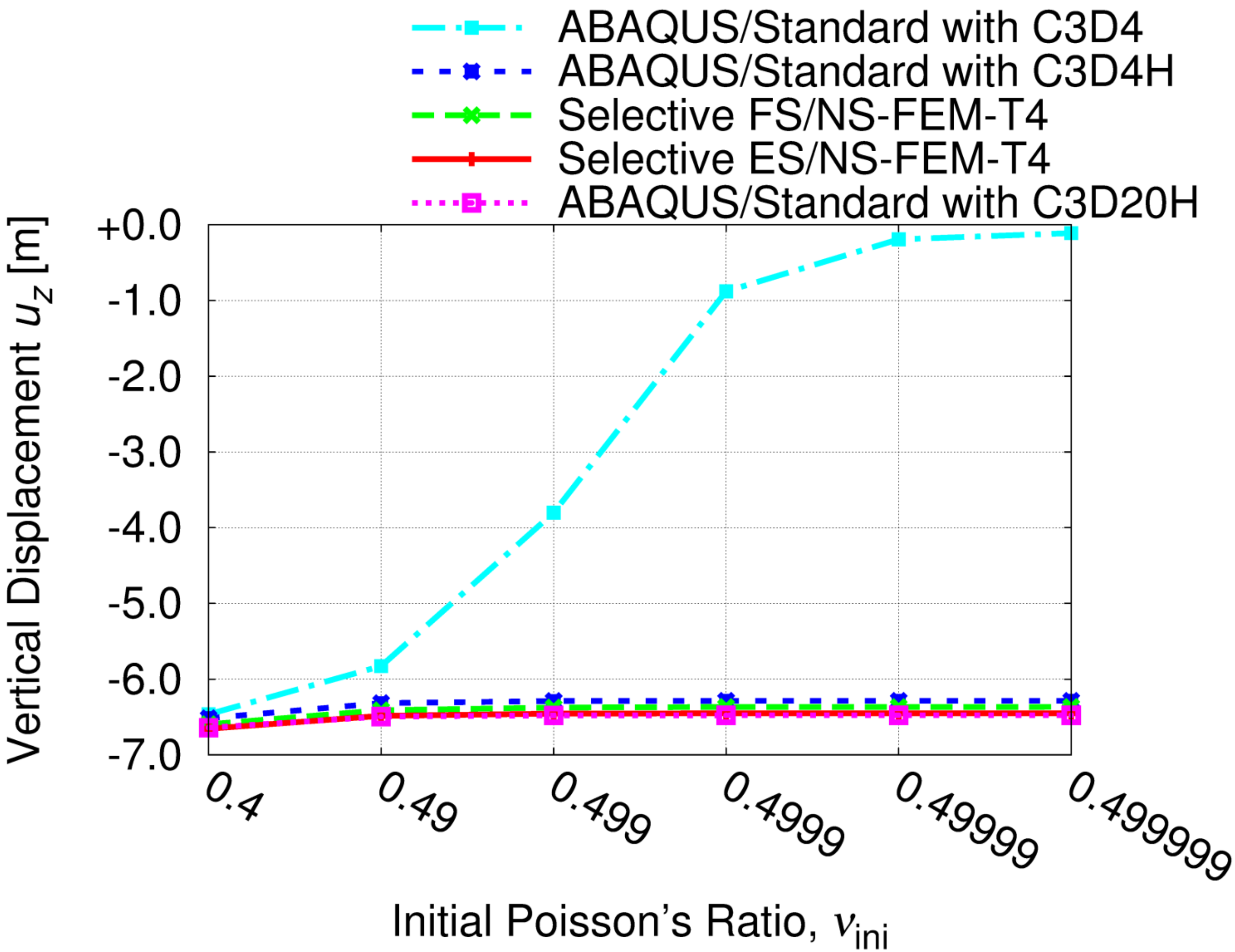
**Results with $D_1 = 2 \times 10^{-15}$ [Pa$^{-1}$] ($\nu_{\mathrm{ini}}$=0.499999)**




The amount of vertical deflection is about 6.5 m.

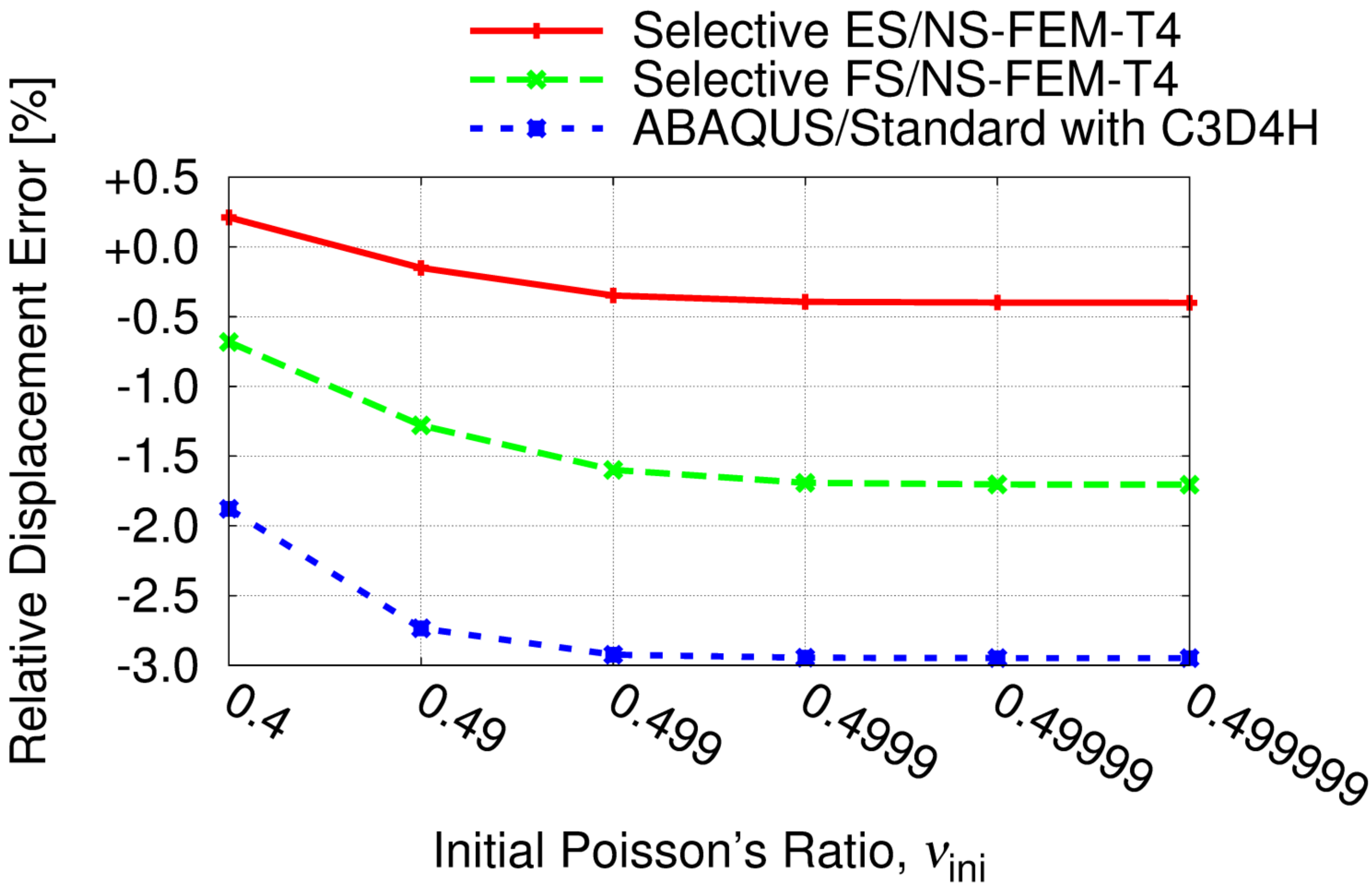
# Verification ~ Bending of Cantilever ~

## ***Comparison of Deflection Displacements***



# Verification ~ Bending of Cantilever ~

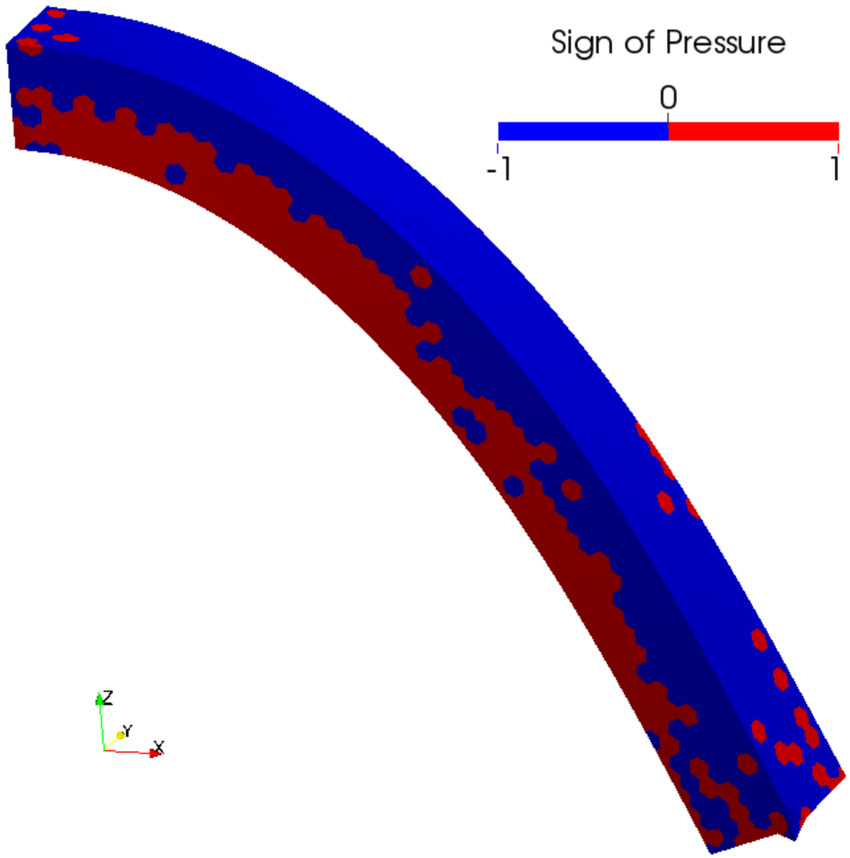
## ***Comparison to 2nd-order Hybrid Hex Element (C3D20H)***



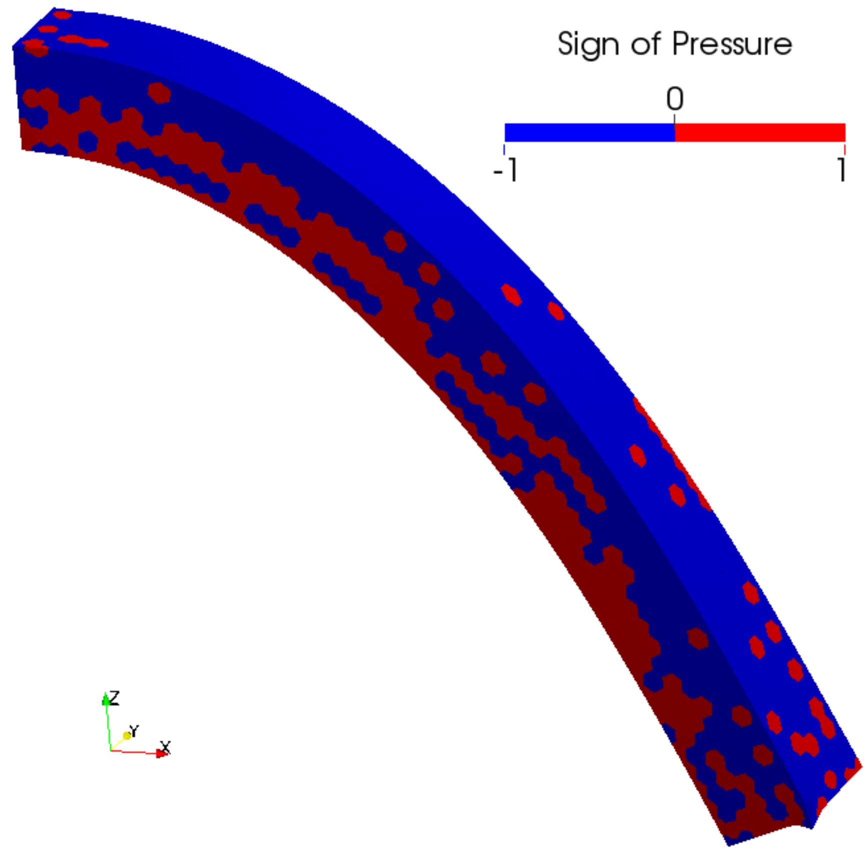

Selective S-FEM is ***locking-free***
in large deflection analysis!!

# Verification ~ Bending of Cantilever ~

## Sign of Pressure at Nodes ($\nu_{ini}$=0.499999)




Selective **ES**/NS-FEM-T4

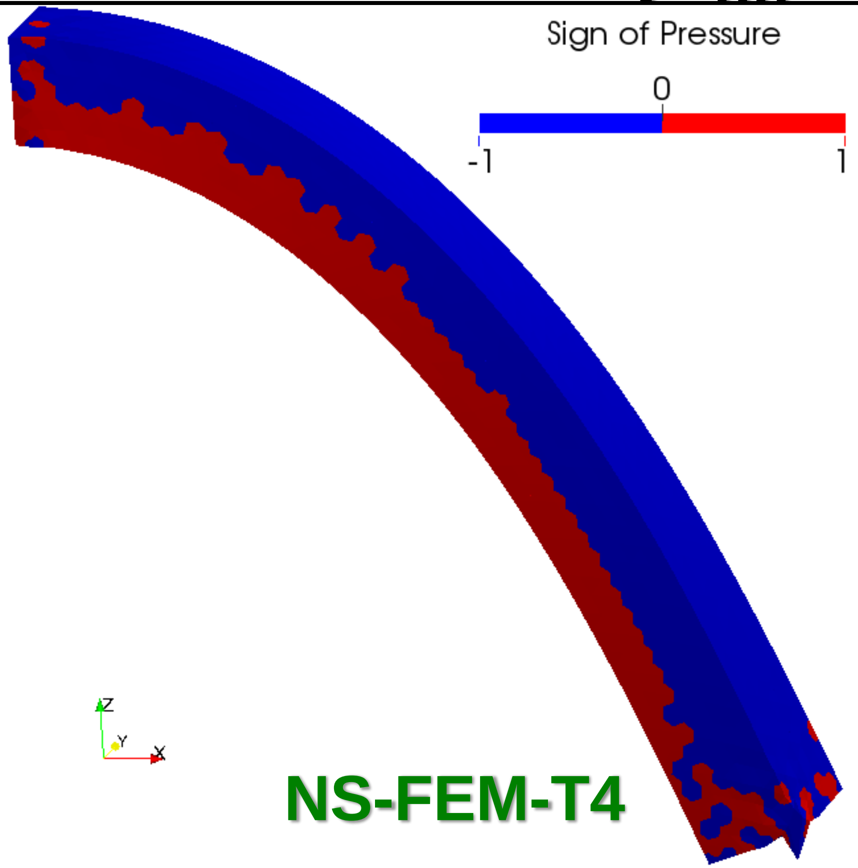



Selective **FS**/NS-FEM-T4

Selective S-FEMs suffer from ***pressure oscillation***…

# Verification ~ Bending of Cantilever ~

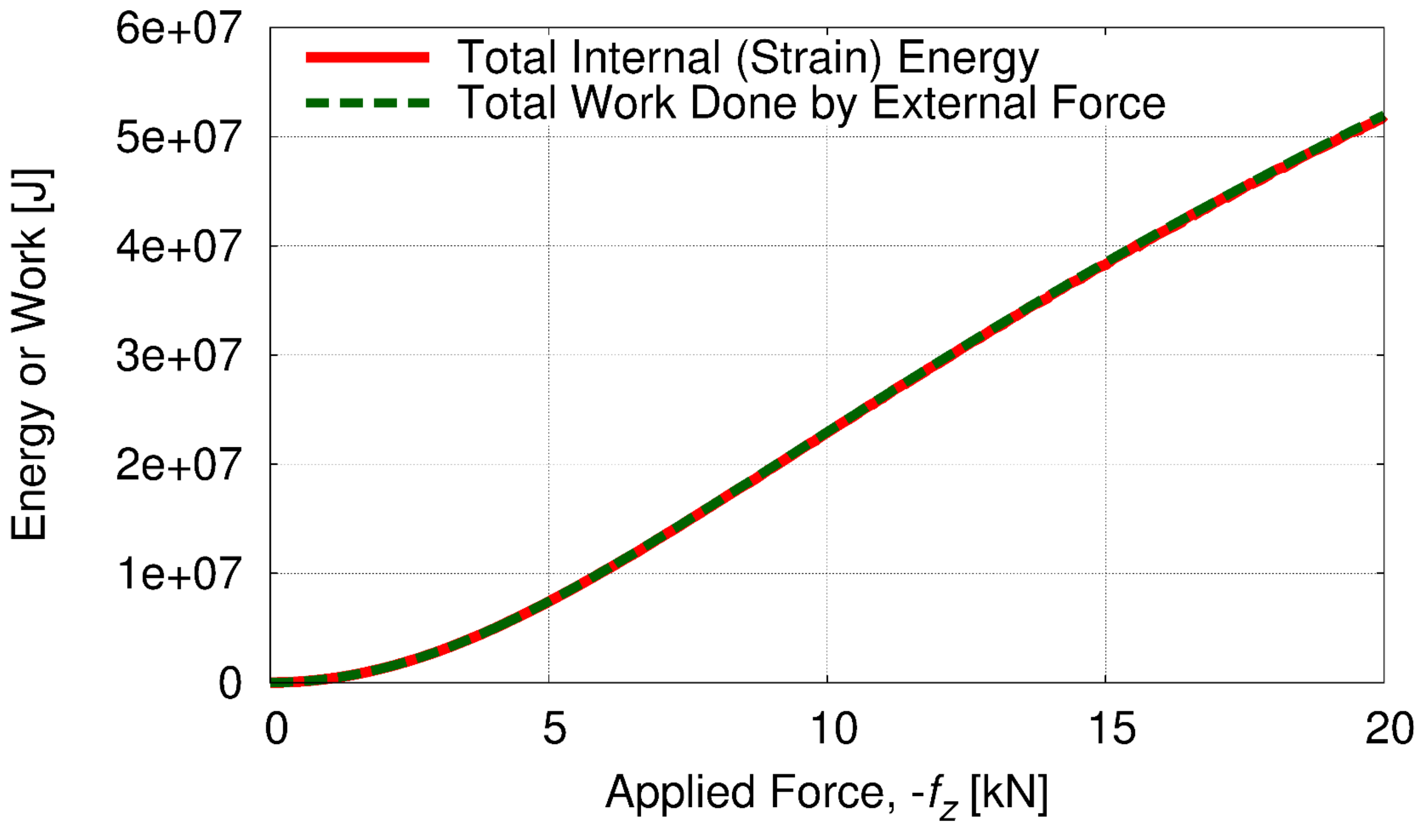
## ***Sign of Pressure at Nodes ($\nu_{ini}$=0.499999)***




NS-FEM already suffers from minor pressure oscillation.
The oscillation is magnified a little in Selective S-FEMs.

# Verification ~ Bending of Cantilever ~

## ***Internal Energy and External Work ($\nu_{\mathrm{ini}}$=0.499999)***

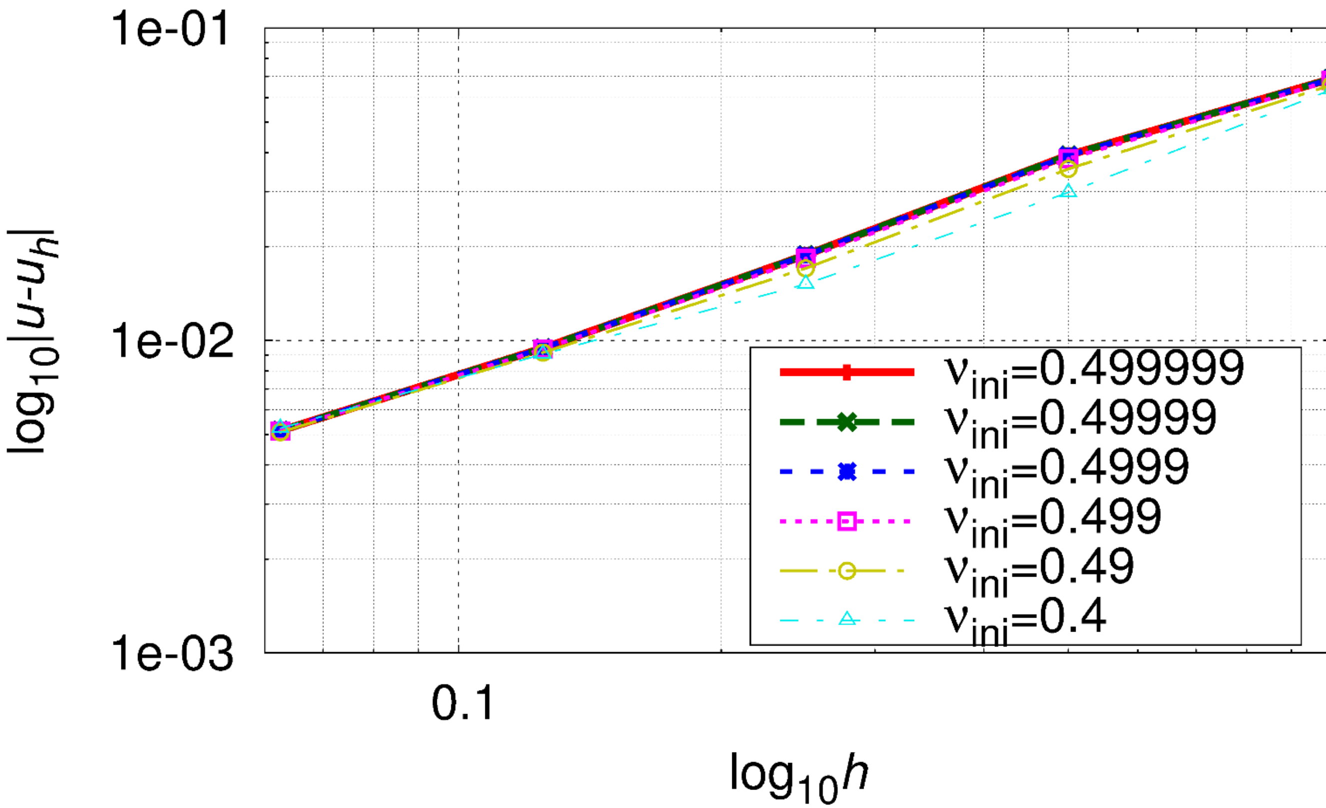



Conservation of energy is exactly satisfied.

⟹ No energy divergence but pure pressure oscillation.

# Verification ~ Bending of Cantilever ~

## ***Mesh Size Convergence Rate (Error of Displacement at the Tip)***

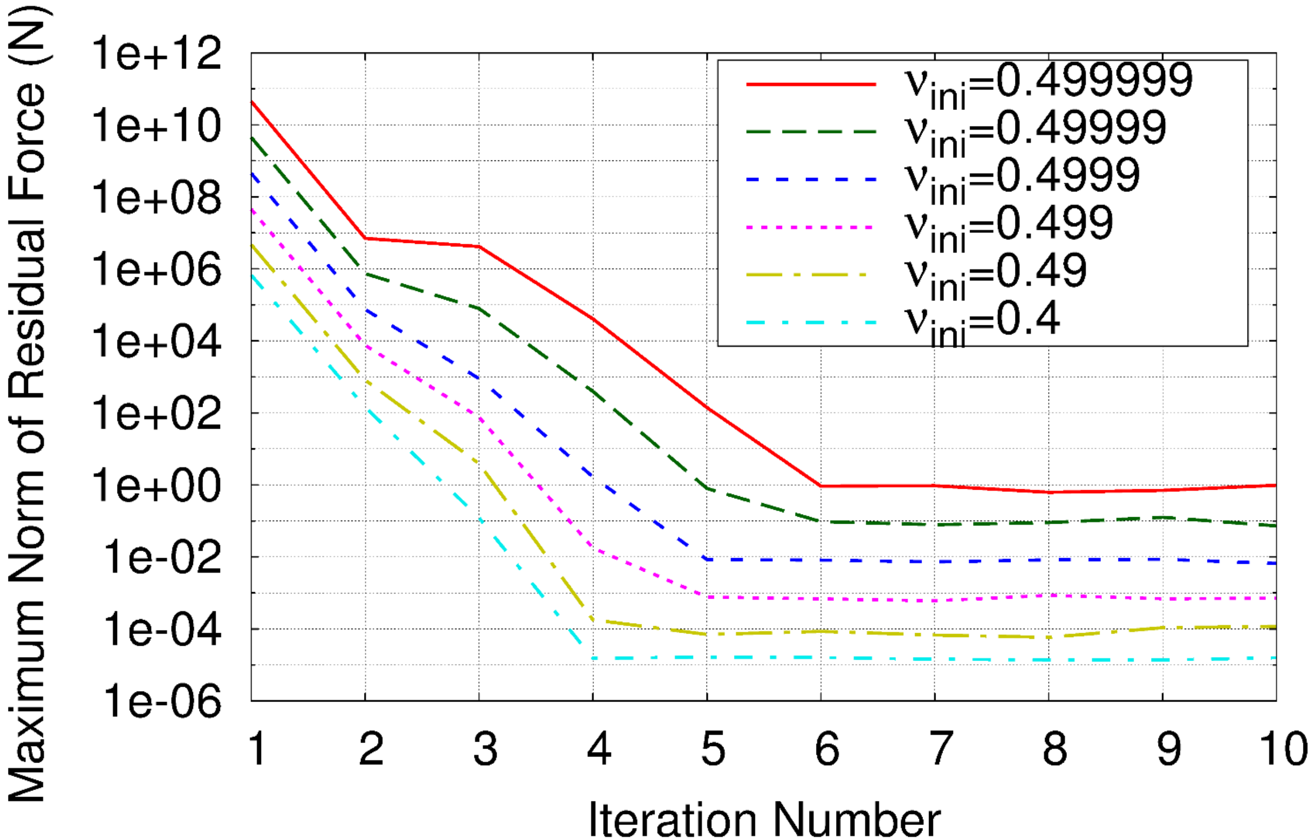



Linear convergence as same as the standard 1st-order FEM.

# Verification ~ Bending of Cantilever ~

## ***Convergence behavior in Newton-Raphson Loop***

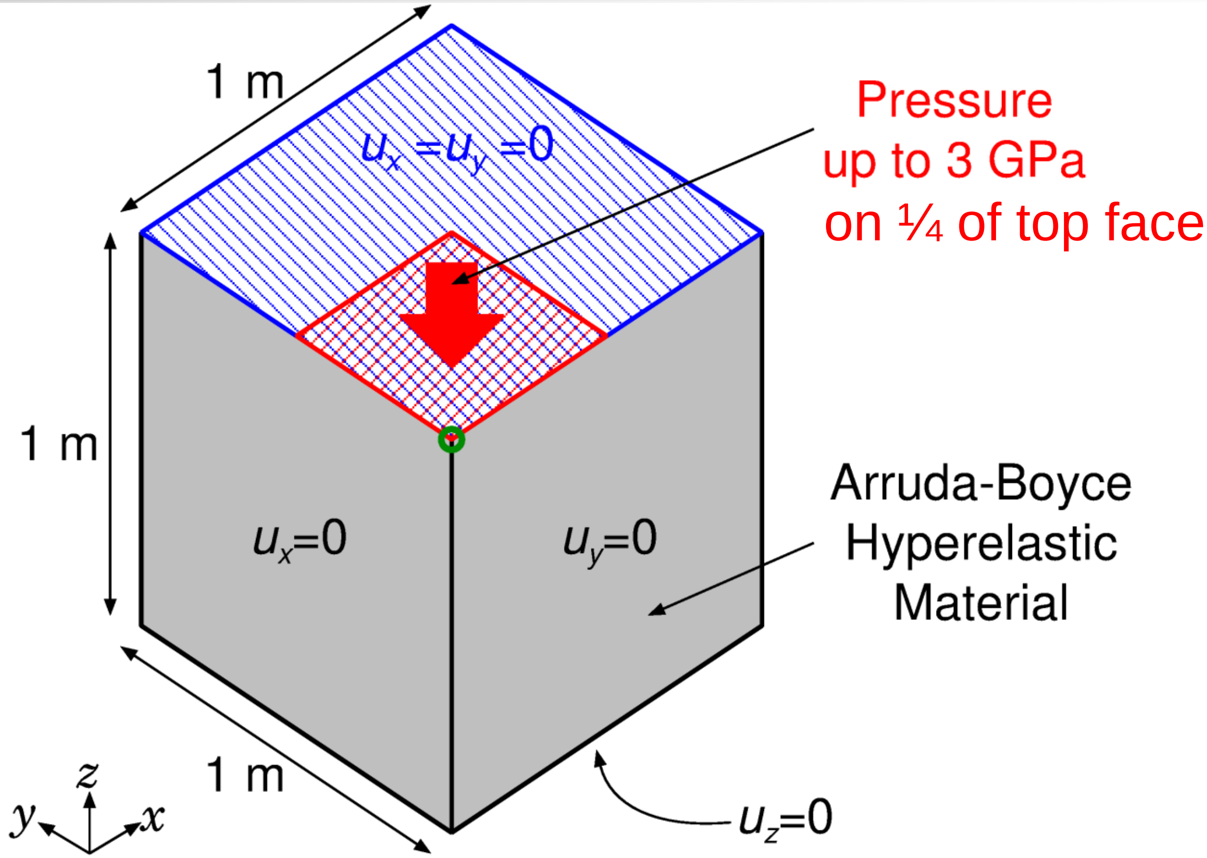
## ***(In the 1st timestep)***




Convergence rate slightly depends on Poisson's ratio.
(Probably due to the loss/cancellation of digits in floating-point calculation.)

# Verification ~Partial Compression of Block ~

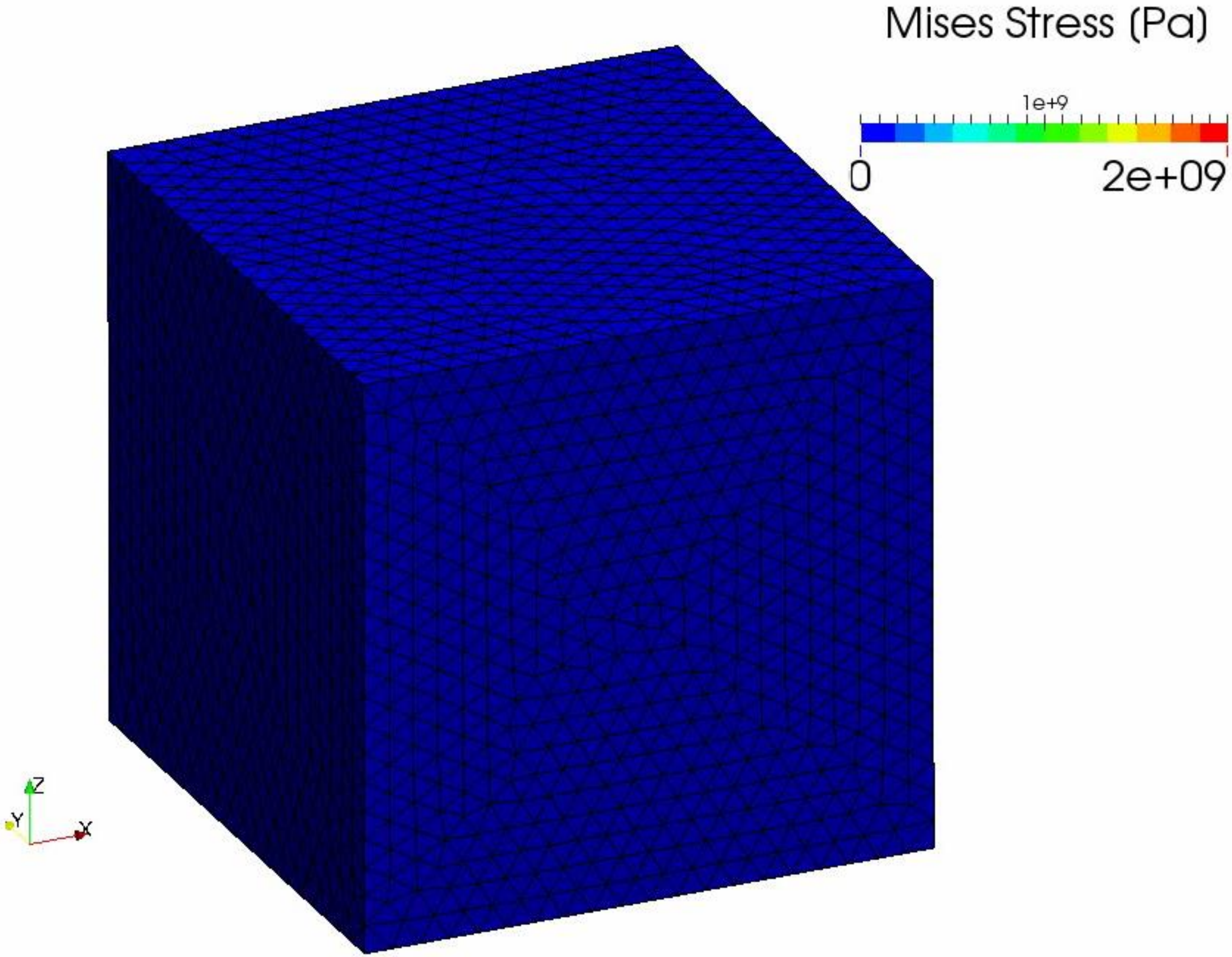
## Outline




- Arruda-Boyce hyperelastic material with $\nu_{\mathrm{ini}} = 0.4999$.
- Compared to ABAQUS with various elements.

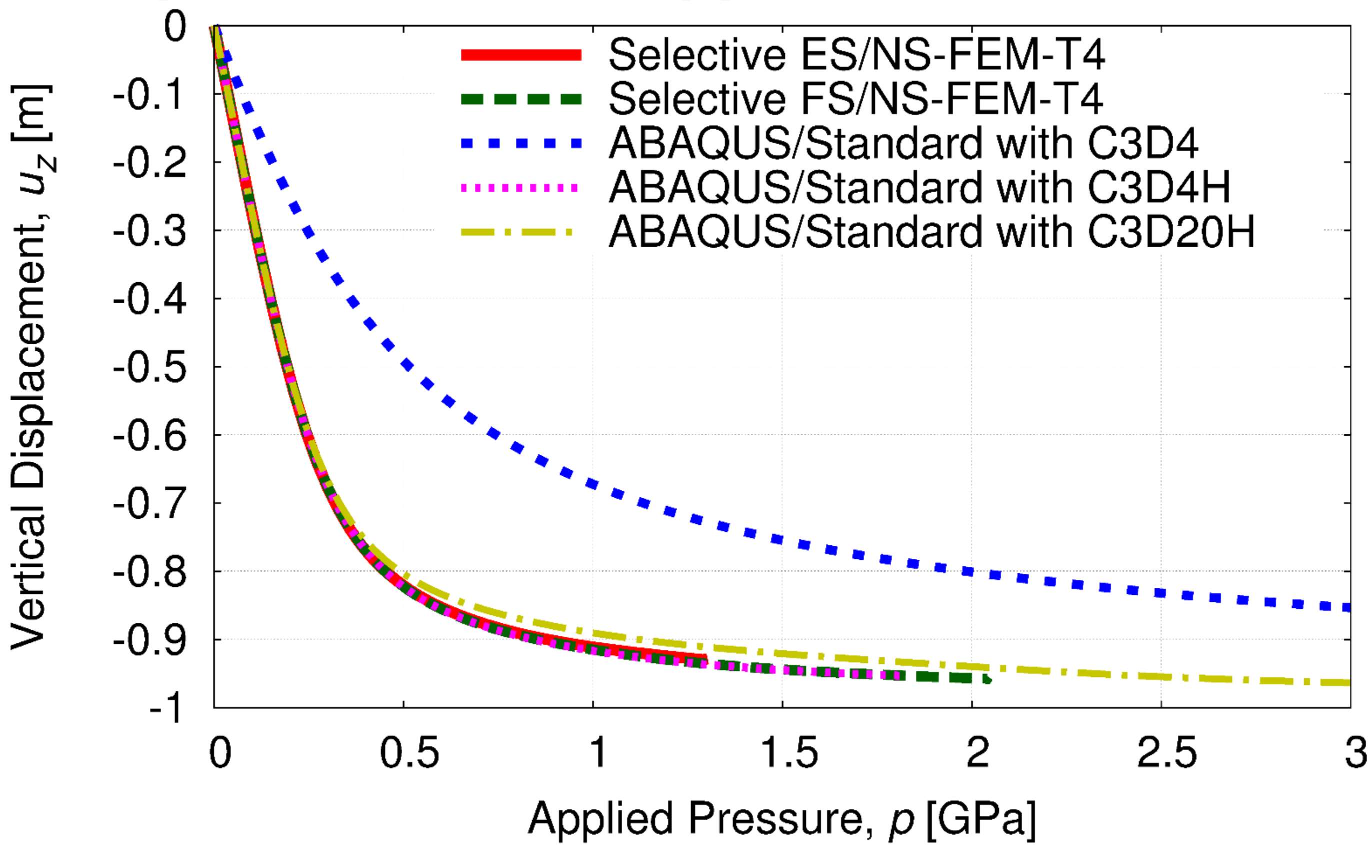
# Verification ~Partial Compression of Block ~

***Result of Selective FS/NS-FEM-T4***



# Verification ~Partial Compression of Block ~

## *Vertical Displacements vs. Applied Pressure*

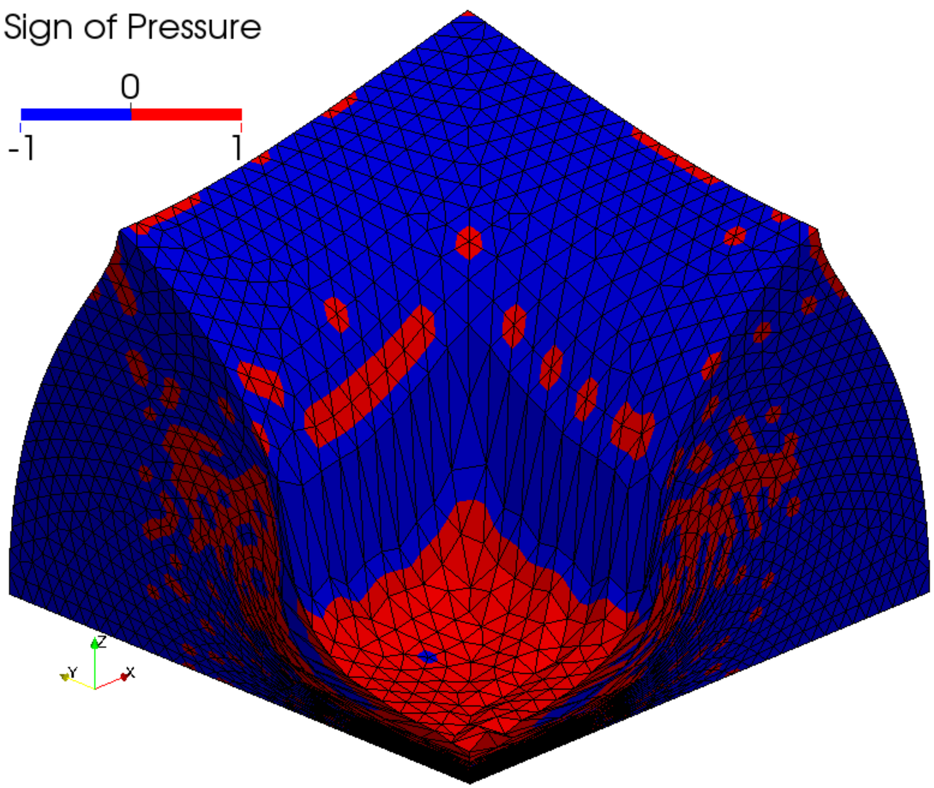


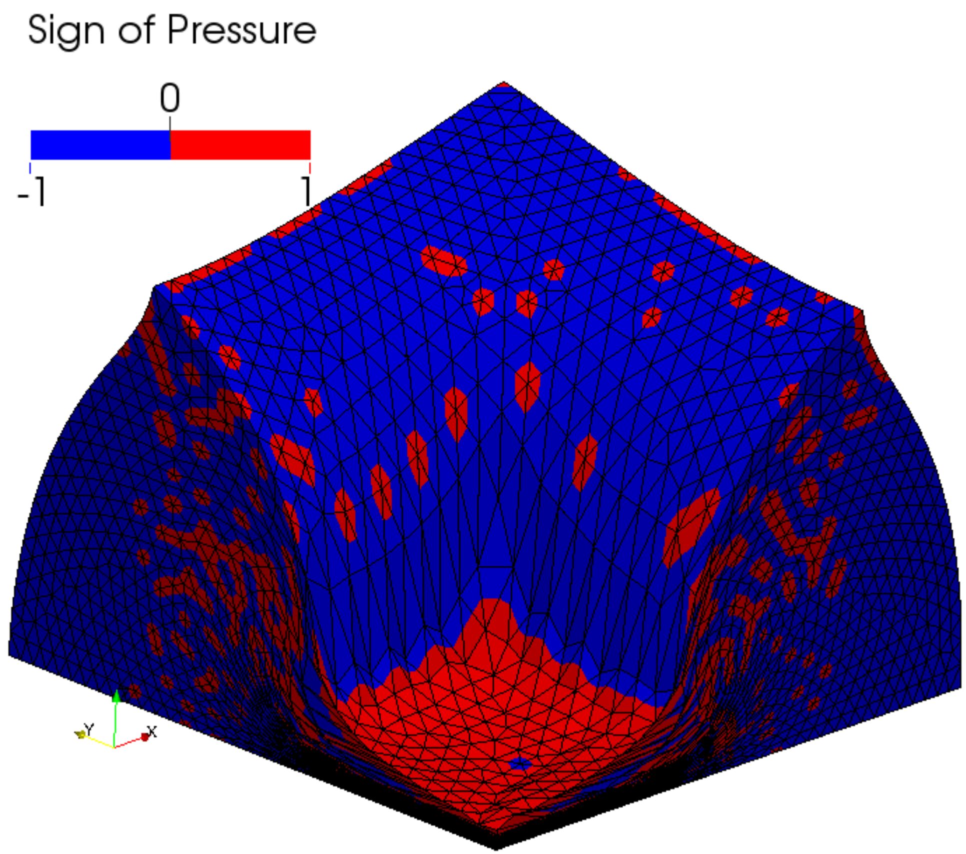

- Constant strain element (C3D4) locks quickly.
- Other elements including selective S-FEMs do not lock.

Selective S-FEMs are **locking-free** in large strain analysis!!

# Verification ~Partial Compression of Block ~

## ***Sign of Pressure***



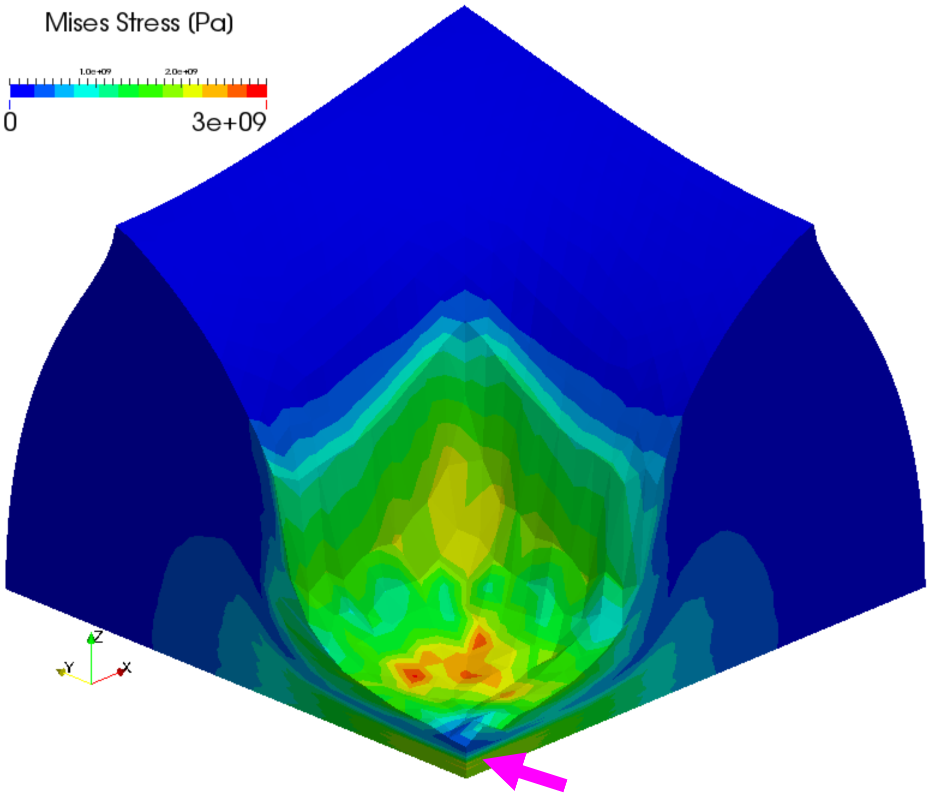




Selective **ES**/NS-FEM-T4 Selective **FS**/NS-FEM-T4

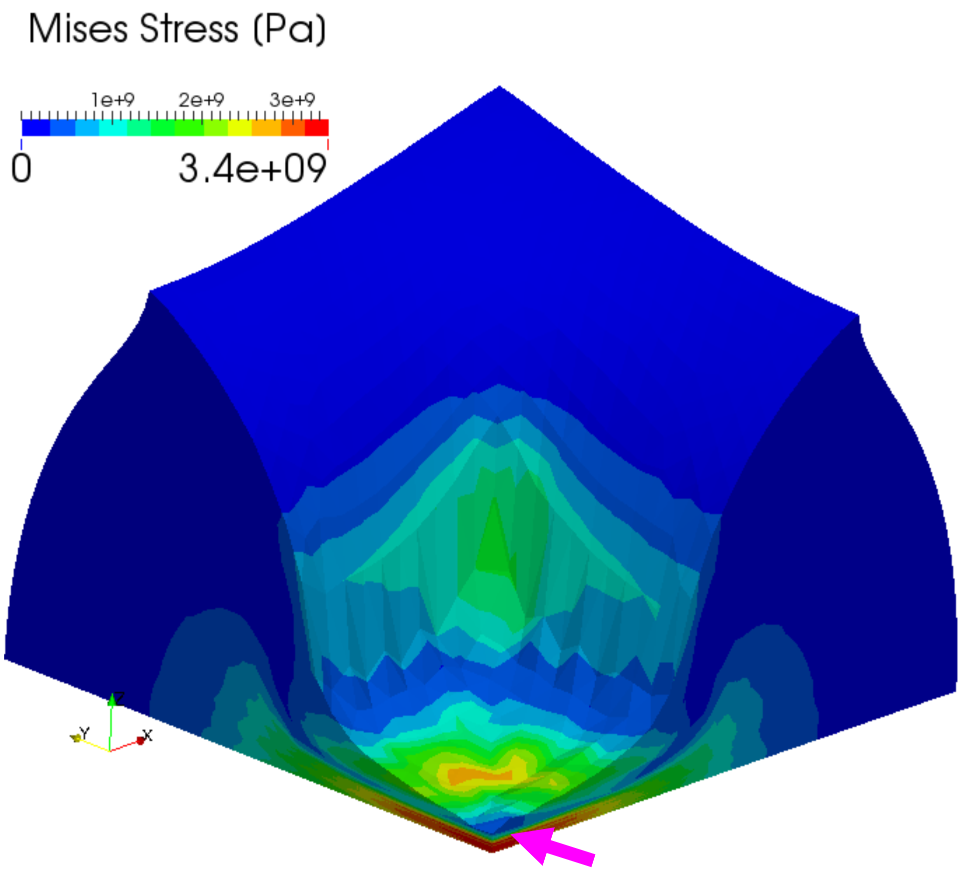
Pressure oscillation is present as well.

# Verification ~Partial Compression of Block ~

## ***Mises Stress***




Selective **ES**/NS-FEM-T4

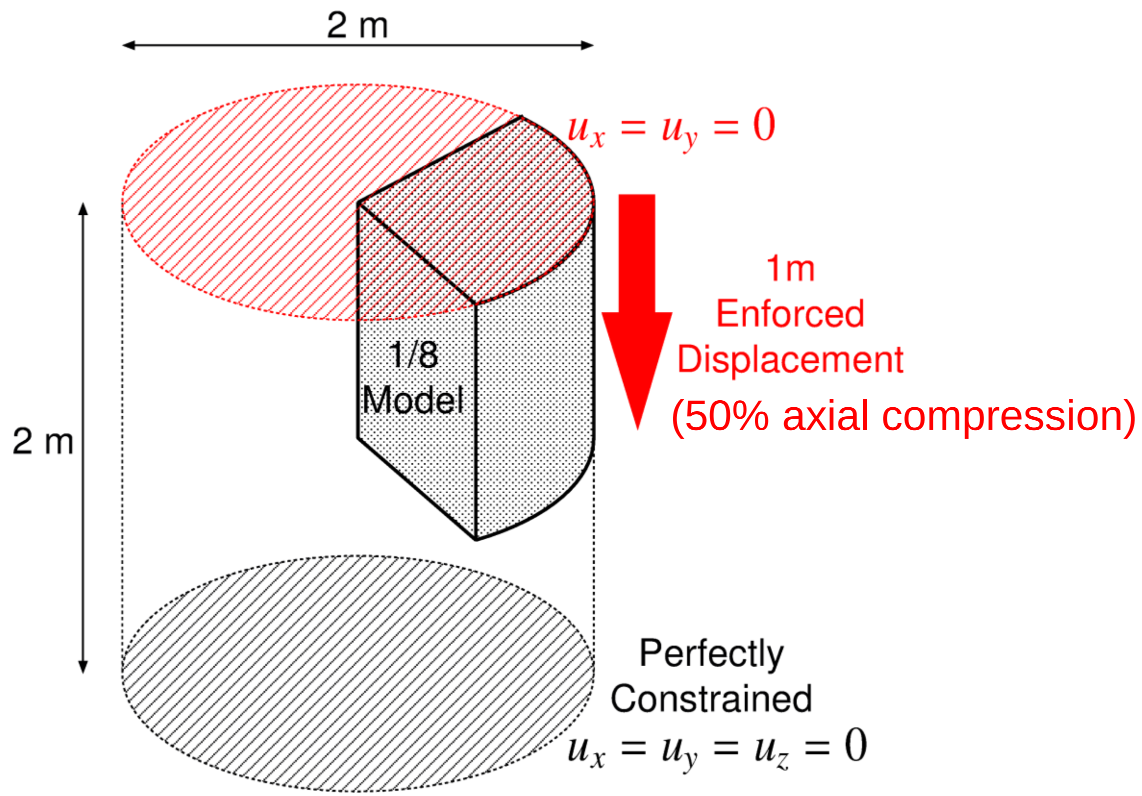



Selective **FS**/NS-FEM-T4

**Selective S-FEMs cannot prevent locking of corner elements.**

# Verification ~ Compression of 1/8 Cylinder ~

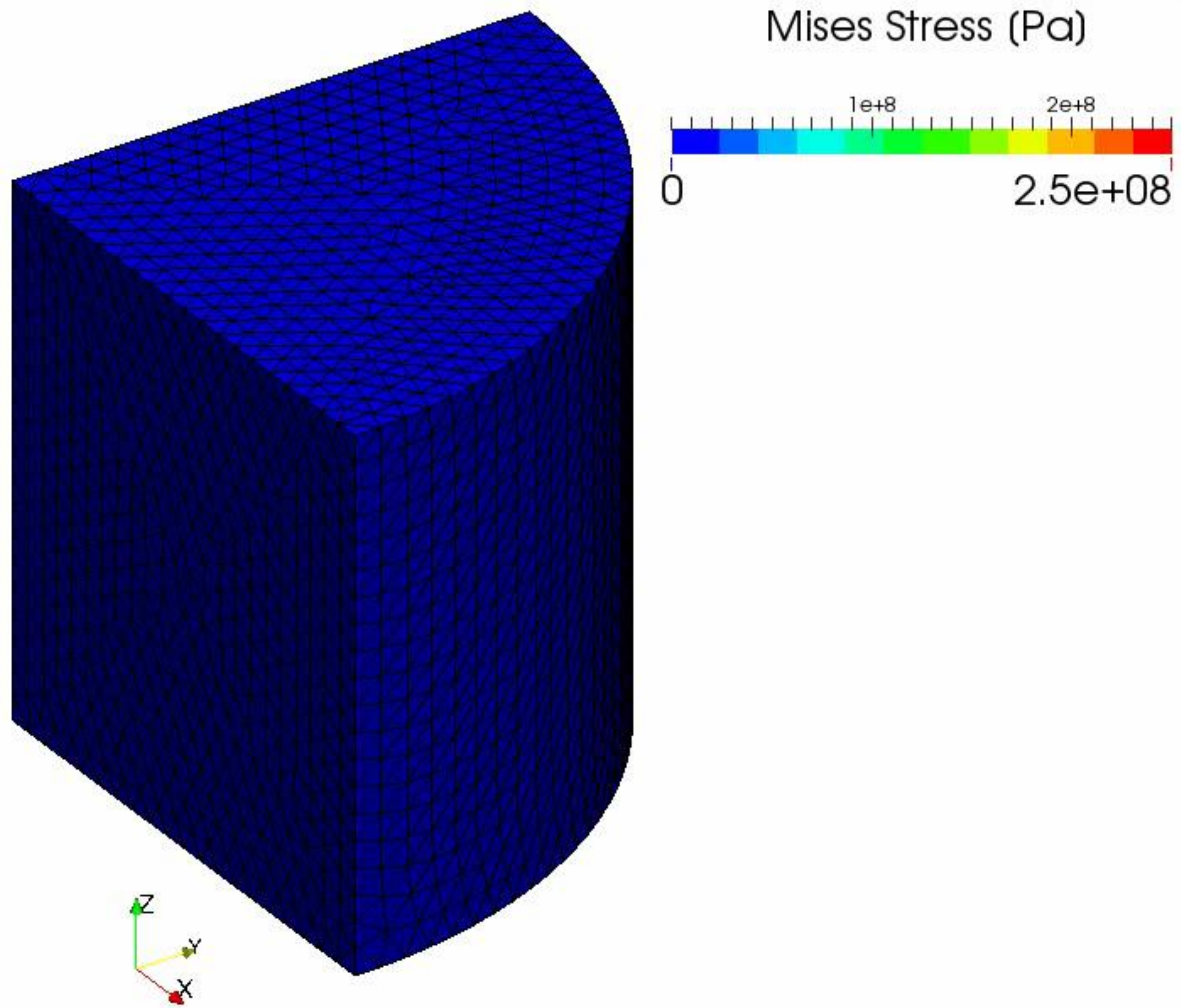
## Outline




- Neo-Hookean hyper elastic material with $\nu_{\mathrm{ini}} = 0.4999$.
- Compared to C3D4H of ABAQUS with exactly the same mesh.

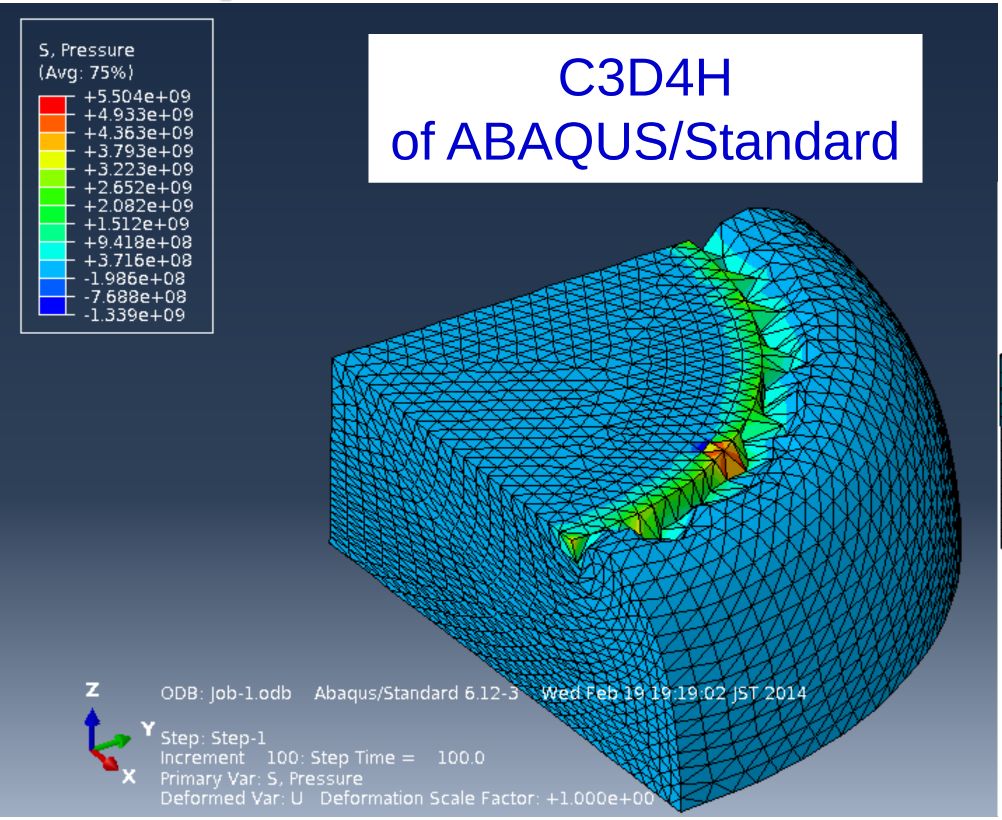
# Verification ~ Compression of 1/8 Cylinder ~

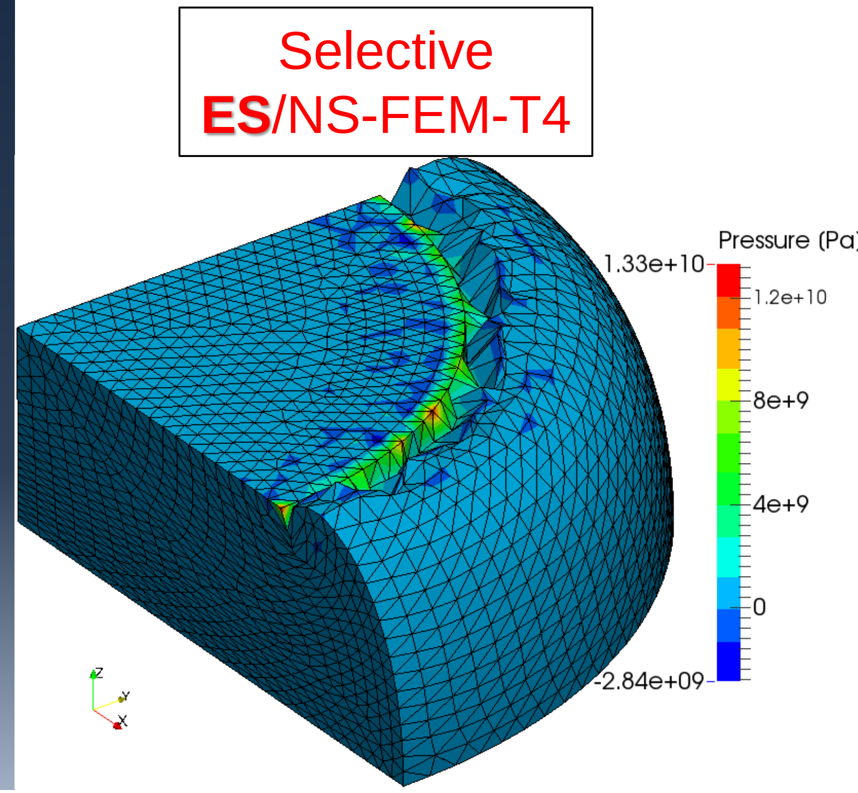
***Result of Selective FS/NS-FEM-T4***



# Verification ~ Compression of 1/8 Cylinder ~

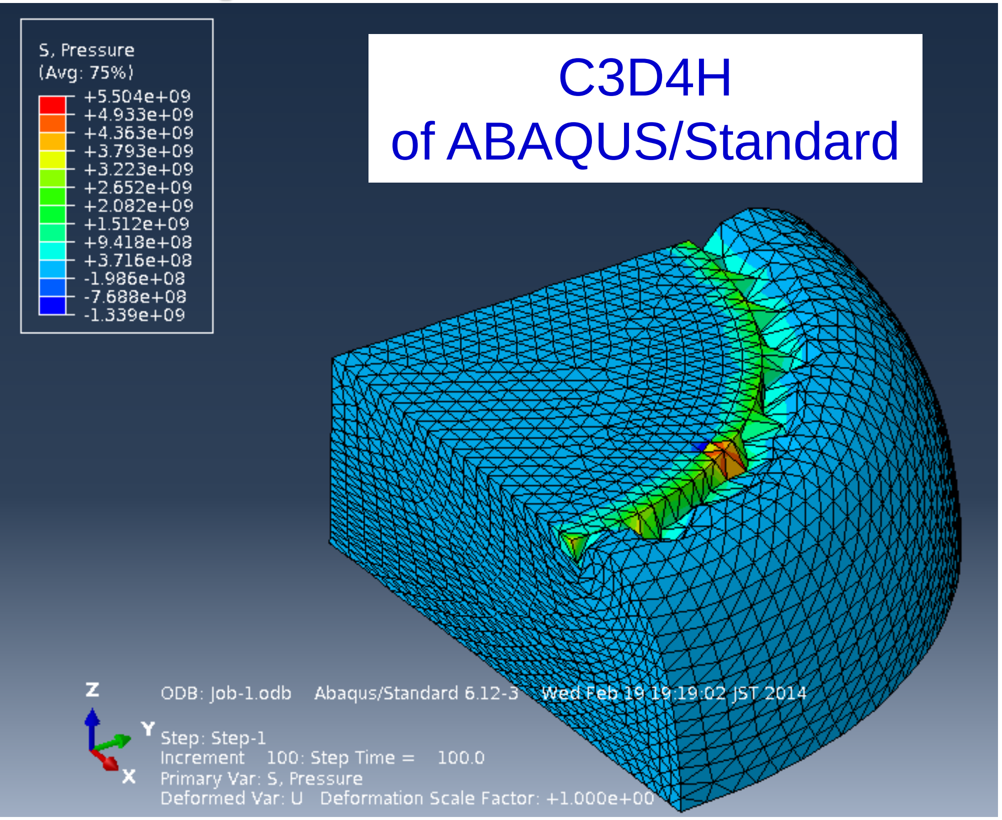
## ***Comparison to ABAQUS***



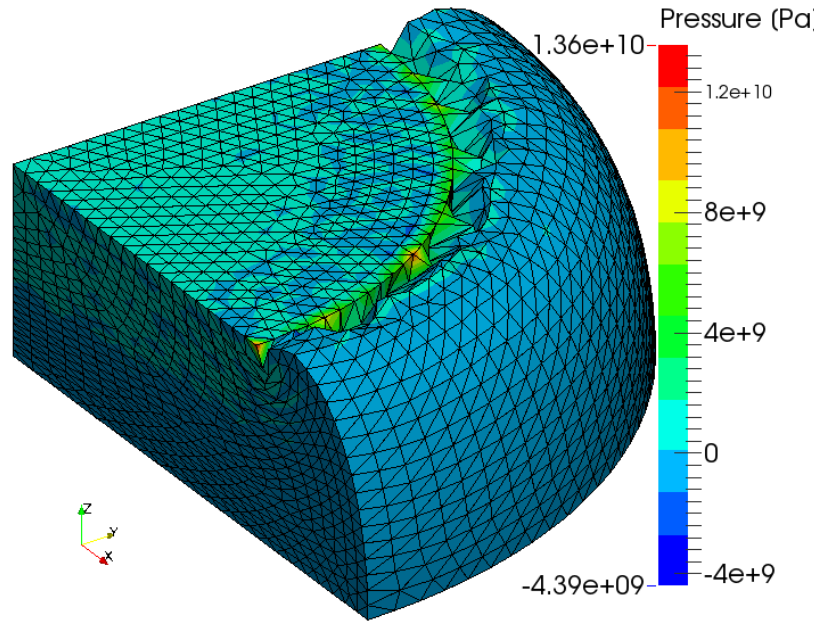




- Deformation is almost the same each other.
- Pressure oscillation is about <u>double</u> in our result.
- Locking of corner/edge elements is observed.

## Comparison to ABAQUS




Selective
**FS**/NS-FEM-T4




- Deformation is almost the same each other.
- Pressure oscillation is about <u>double</u> in our result.
- Locking of corner/edge elements is observed.

# 3 Issues in Selective S-FEMs

1. Selective S-FEMs with dev/hyd split cannot handle material constitutive models with dev/vol coupling terms.

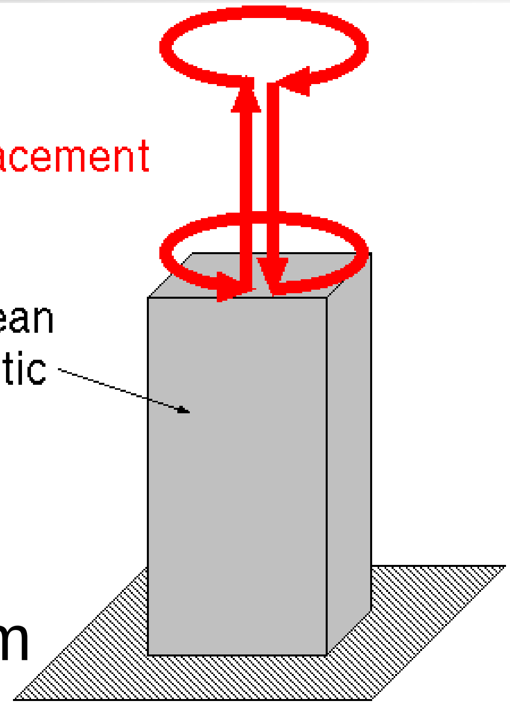
2. Selective S-FEMs cannot prevent pressure oscillation in the analysis with almost incompressible materials.

3. Selective S-FEMs cannot provide sufficient smoothing effect to elements at corners.

Part 3:

<u>Demonstration</u> with Mesh Rezoning

# Demo ~ Twist & Stretch of Rubber Cuboid ~

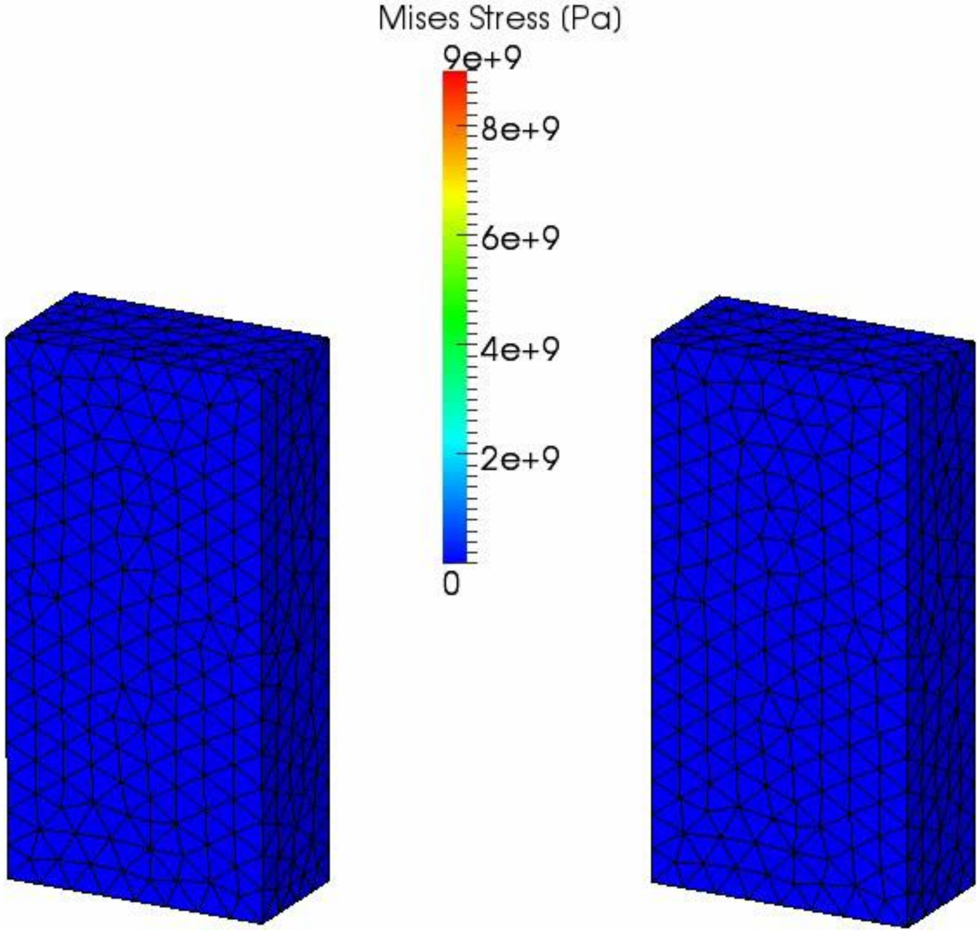



- Static, 1 m x 2 m x 4 m
- Neo-Hookean hyperelastic body of $C_{10} = 1\ \mathrm{GPa}$ and $D_1 = 400\ \mathrm{GPa}^{-1}$ ($\nu_0 = 0.48$)
- Twist up to 360 deg. ⟹ Stretch up to 100% nominal strain ⟹ Twist back ⟹ Shrink back
- Our selective FS/NS-FEM with tetrahedral elements
- Global mesh rezoning every 90 deg. and 50% stretch/shrink

# Demo ~ Twist & Stretch of Rubber Cuboid ~

Our selective FS/NS-FEM with mesh rezoning

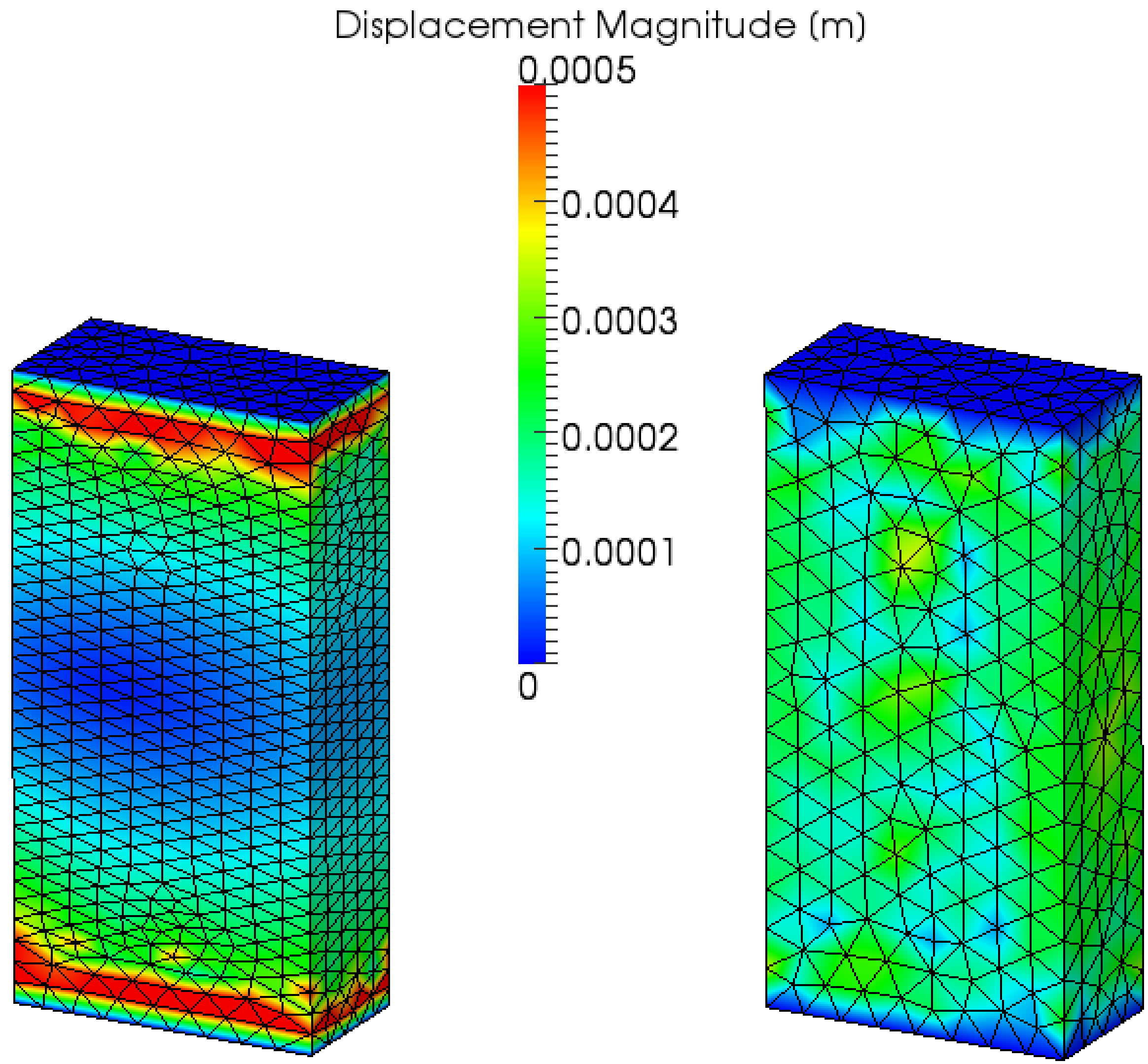
Our selective FS/NS-FEM without mesh rezoning



# Demo ~ Twist & Stretch of Rubber Cuboid ~

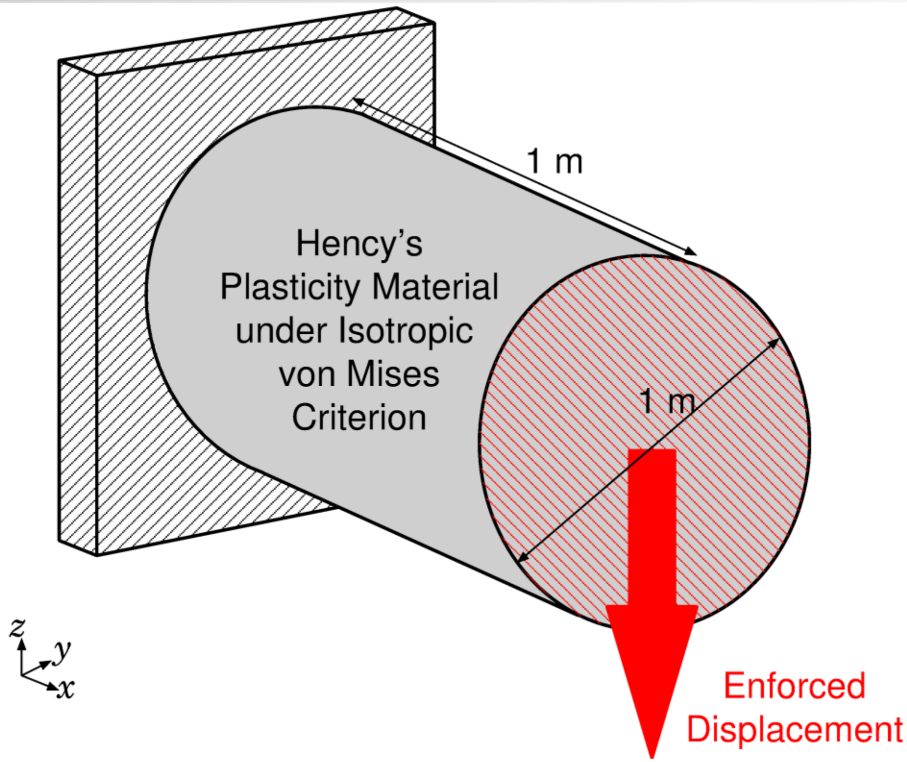
## ***Residual Displacement***

It spring backed almost perfectly.



# Demo ~ Shearing & Necking of Plastic Rod ~

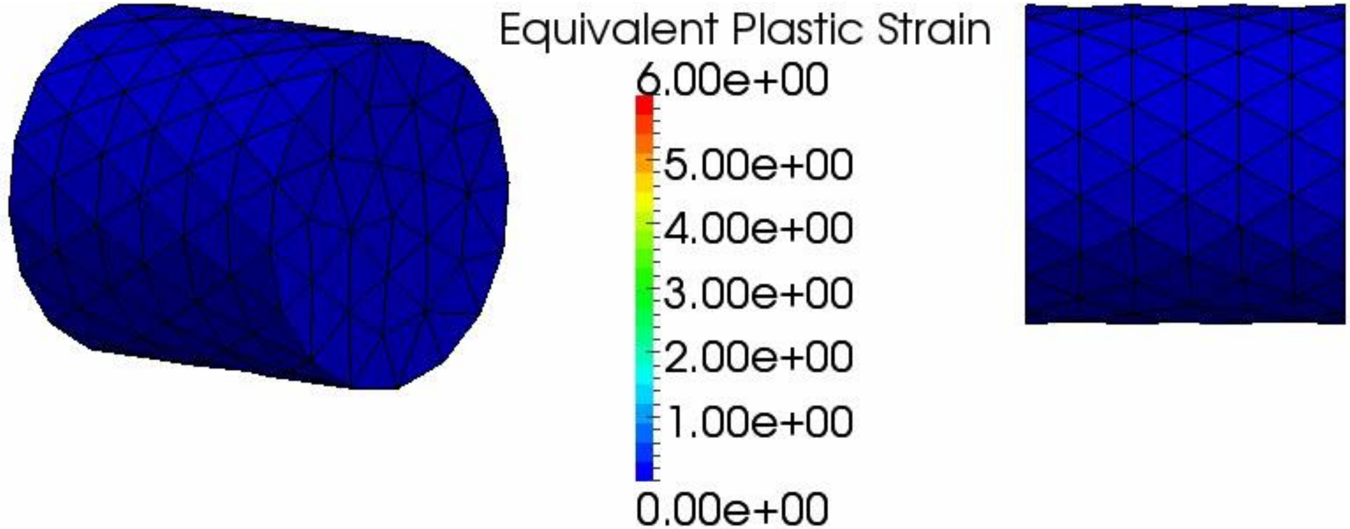
## Outline




- Static, 3D
- Hencky's elasto-plastic material, $\boldsymbol{T} = \boldsymbol{C} : \boldsymbol{h}_{\mathrm{el}}/J$,
  with von Mises yield criterion and isotropic hardening.
  Young's Modulus: 1 GPa, Poisson's Ratio: 0.3,
  Yield Stress: 1 MPa, Hardening Coeff.: 0.5 MPa.

# Demo ~ Shearing & Necking of Plastic Rod ~

***Result with Selective FS/NS-FEM-T4***

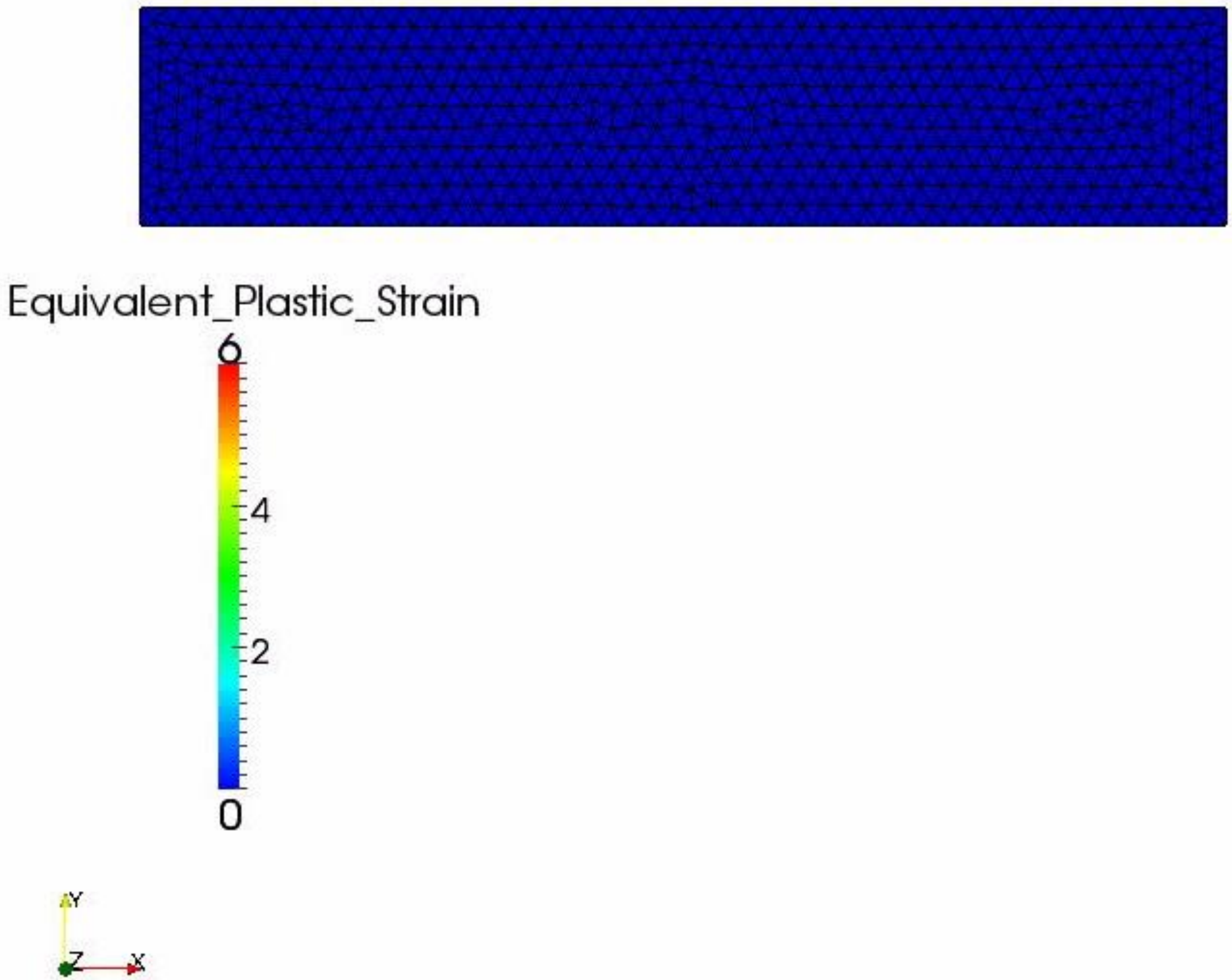



After 2.8 m disp., mesh rezoning error occurred.

The final nominal stretch is >7000% at the neck.

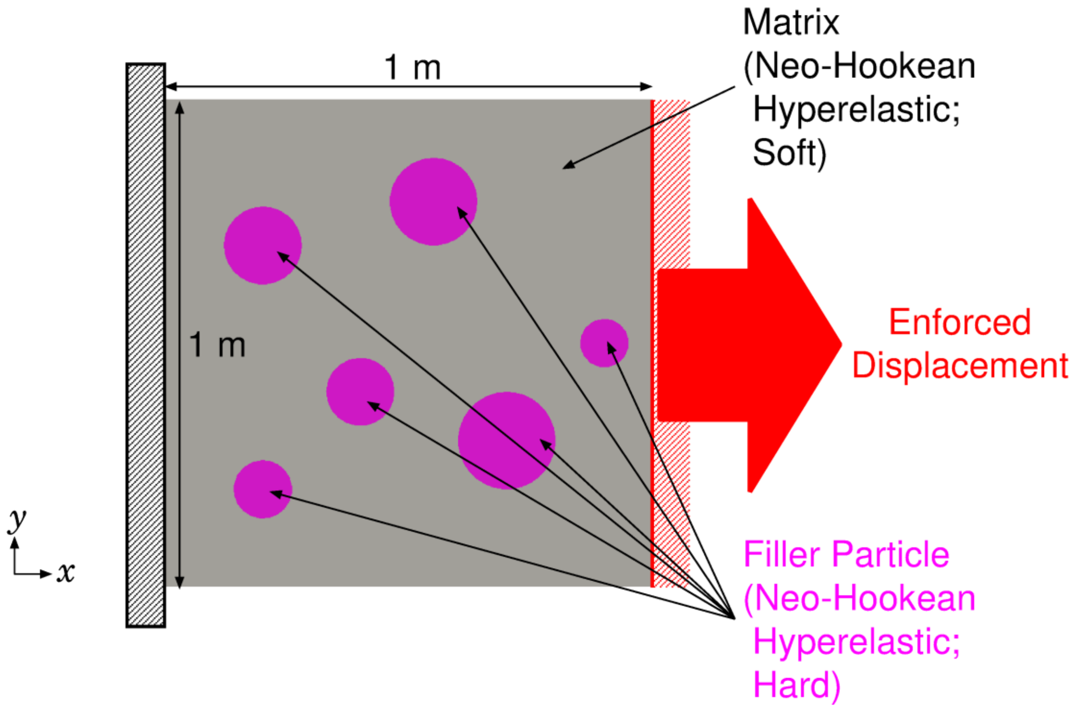
# Demo ~ Shearing & Necking of Plastic Rod ~

***Result of Similar Analysis in 2D with Selective ES/NS-FEM-T3***



# Demo ~Tension of Filler Particle Composite~

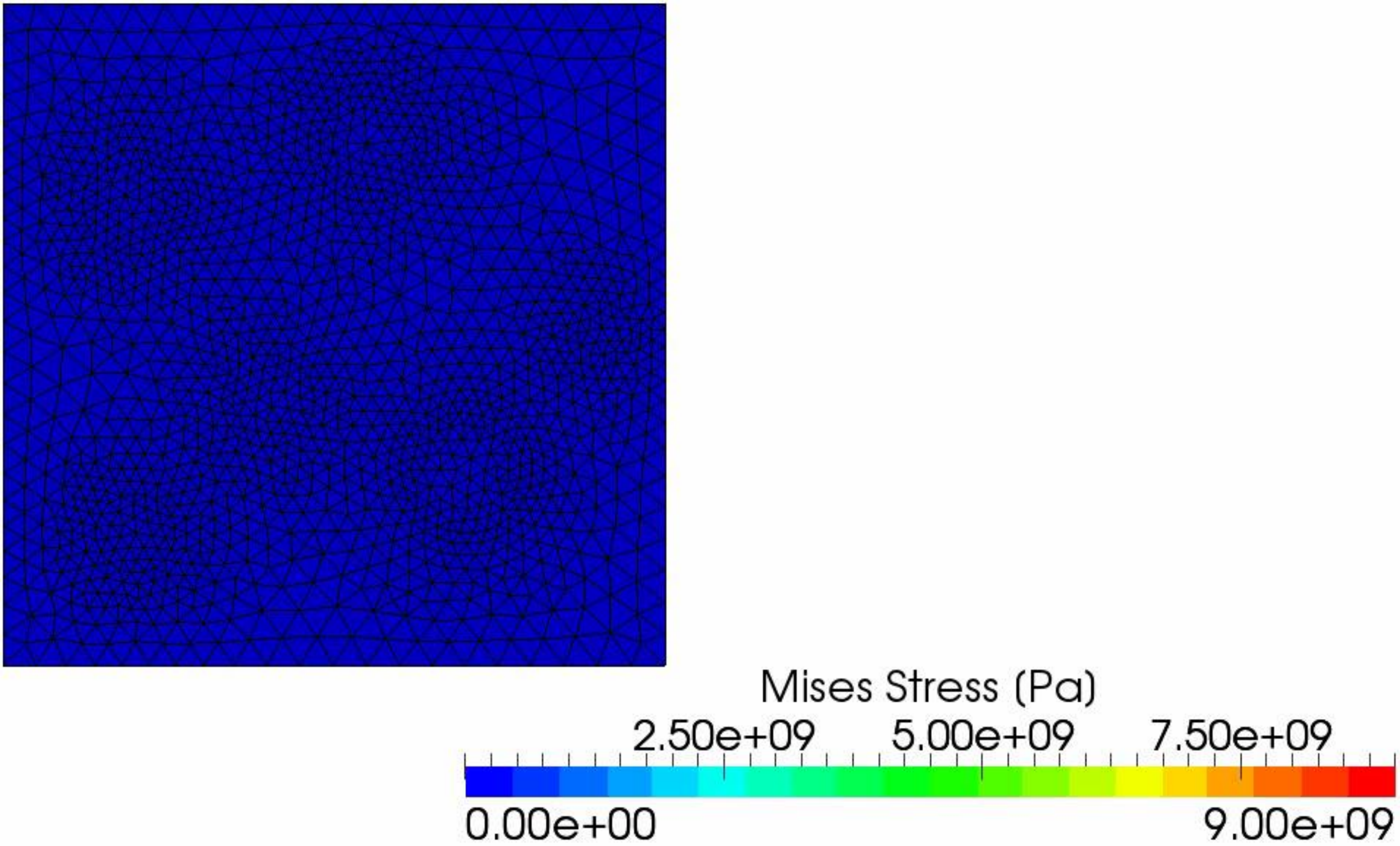
## ***Outline***




- 2D, plane-strain, static
- Neo-Hookean Hyperelastic
  - Filler: hard rubber ($E^{\mathrm{initial}} = 100$ GPa, $\nu^{\mathrm{initial}} = 0.49$)
  - Matrix: soft rubber ($E^{\mathrm{initial}} = 1$ GPa, $\nu^{\mathrm{initial}} = 0.49$)

# Demo ~Tension of Filler Particle Composite~

## ***Result of Selective ES/NS-FEM-T4***




- The deformation seems to be valid.
- After 180% stretch, analysis stopped due to mesh rezoning error.

# Summary

# Characteristics of S-FEMs & C3D4H

| | Shear Locking | Volumetric Locking | Zero Energy Mode | No Increase in DOF | Pressure Oscillation & Locking at Corner | Dev/Vol Coupled Material |
|---|---|---|---|---|---|---|
| Standard FEM-T4 | ✗ | ✗ | ✓ | - | ✗ | ✓ |
| ES-FEM-T4 & FS-FEM-T4 | ✓ | ✗ | ✓ | ✓ | ✗ | ✗ |
| NS-FEM-T4 | ✓ | ✓ | ✗ | ✓ | ✗ | ✗ |
| Selective FS/NS-FEM-T4 & ES/NS-FEM-T4 | ✓ | ✓ | ✓ | ✓ Benefit | ✗ Issues in Future | ✗ |
| ABAQUS C3D4H | ✓ | ✓ | ✓ | ✗ | ✗ | ✓ |

# Summary and Future Work

## Summary

- A new implicit static mesh rezoning method for severely large deformation analysis was proposed.
- It adopts our modified selective S-FEM, which separates stress into deviatoric part and hydrostatic part.
- Its performance was evaluated and then 3 issues were revealed.

## Future Work

- Resolve the 3 issues
- Apply to contact forming, crack propagation, etc.
- Explicit dynamic formulation
- Local mesh rezoning

Thank you for your kind attention.
I appreciate your question in slow and easy English!!