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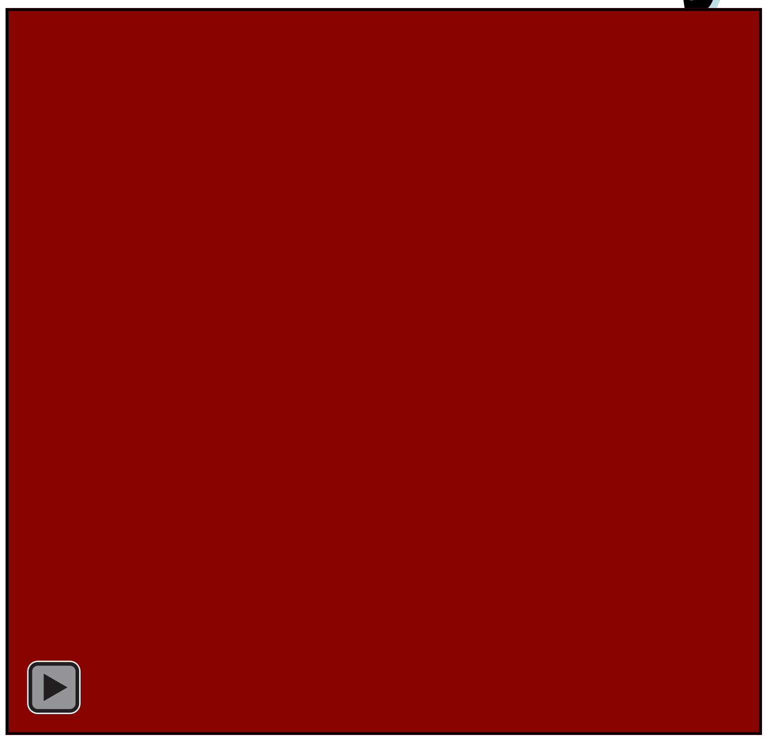
An Accurate and Stable Static-Implicit Finite Element Rezoning Method for Large Deformation Elasto-Plastic Analysis



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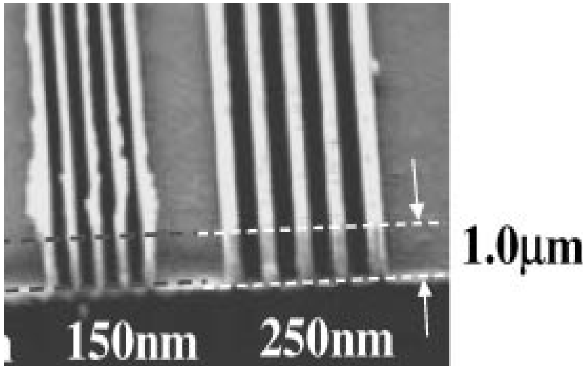
Motivation and Background

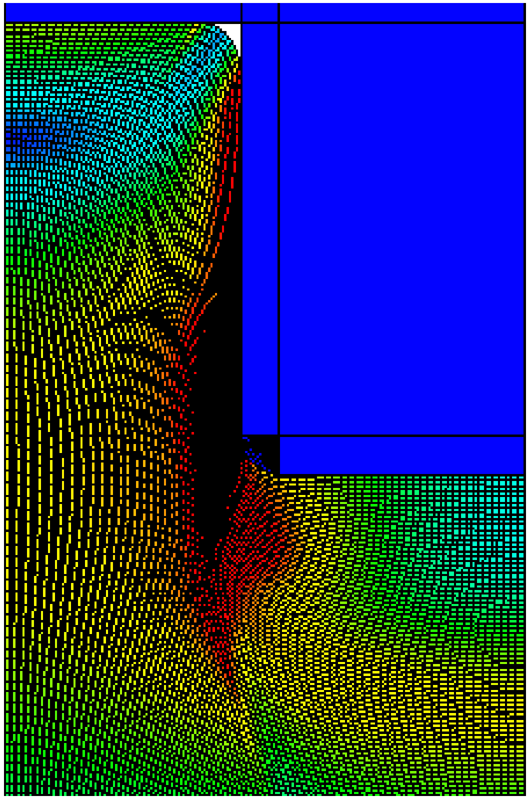
Motivation

We want to solve **severely large deformation** problems **accurately and stably**!
(Final target: thermal nanoimprinting)



Background

Finite elements are **distorted** in a short time, thereby resulting in convergence failure.

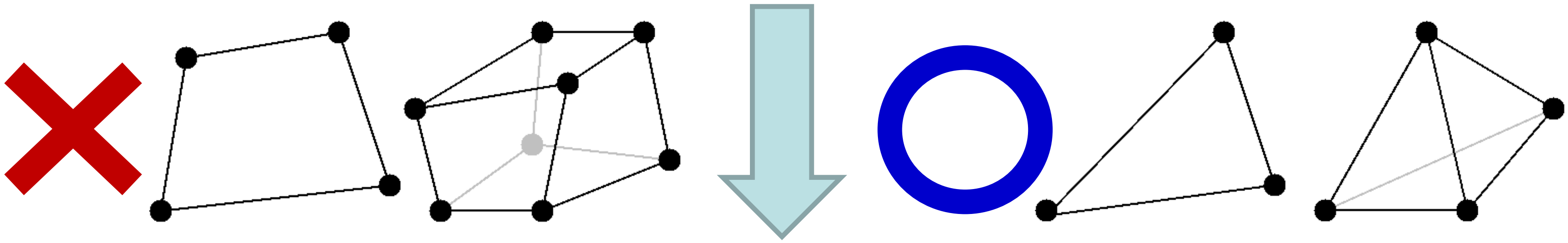


Mesh rezoning method (*h*-adaptive mesh-to-mesh solution mapping) is indispensable.

2 Major Problems in Mesh Rezoning

Problem 1: accuracy

It is impossible to remesh arbitrary deformed 2D or 3D domains with quadrilateral or hexhedral elements.



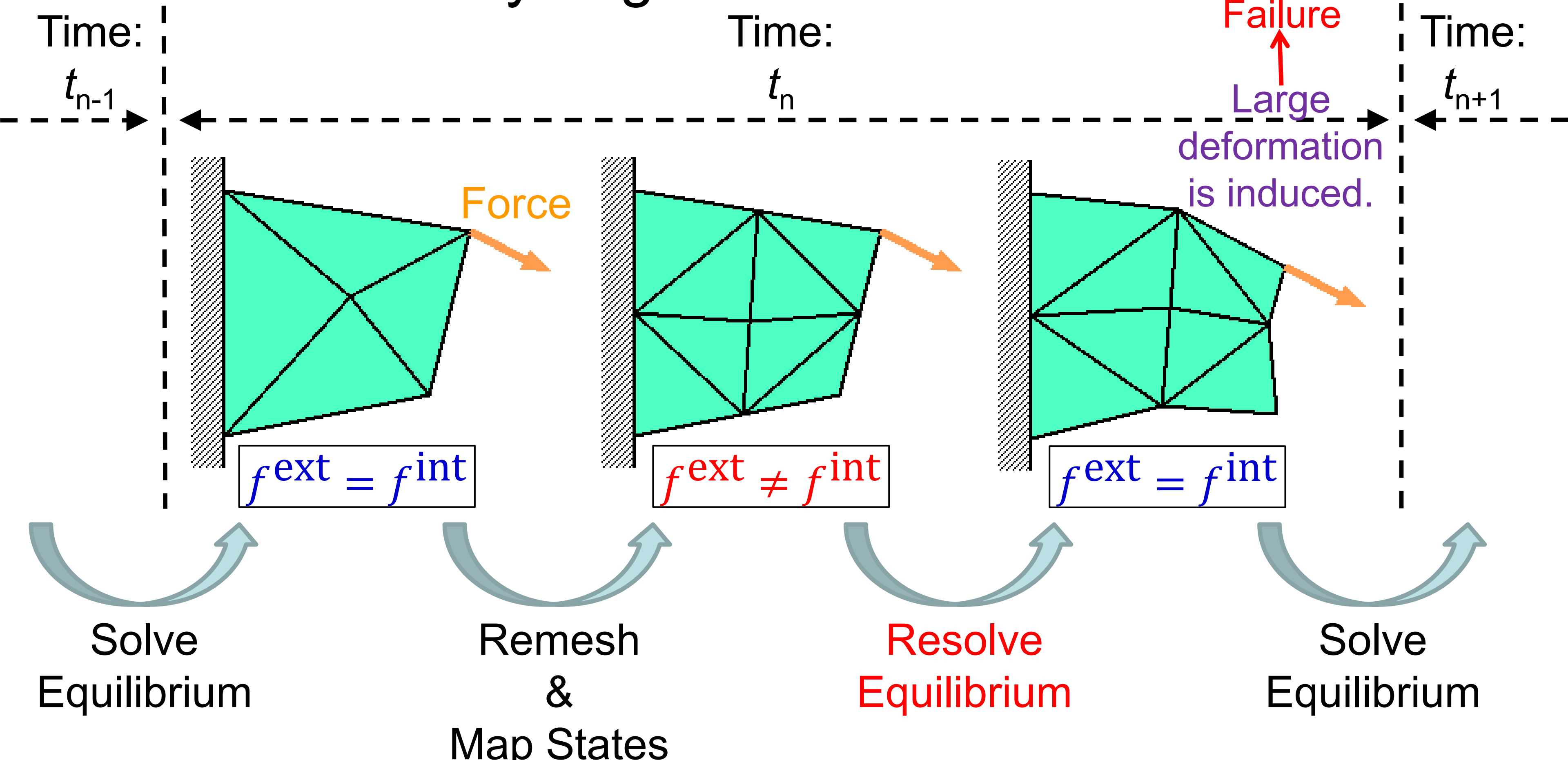
We have to use triangular or tetrahedral elements...

However, the *standard* (constant strain) triangular or tetrahedral elements induce shear and volumetric locking easily, which leads to inaccurate results.

2 Major Problems in Mesh Rezoning

Problem 2: stability

The resolving process in mesh rezoning sometimes induces extremely large deformation.



Our Ideas

Idea for accuracy improvement

We adopt smoothed finite element method (S-FEM) to avoid shear and volumetric locking even with use of triangular or tetrahedral elements.

Idea for stability improvement

We adopt the incremental implicit equilibrium equation (IIEE) as the equation to solve.

In this talk today,
I focus on Problem 1 and the idea of S-FEM.

Objective

Develop an accurate
mesh rezoning method
for large deformation problems
with our modified S-FEM formulation

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- Part 1: Introduction of our modified S-FEM formulation
- Part 2: Procedure of our mesh rezoning method
- Part 3: Examples analysis
- Summary

Part 1:

Introduction of Our Modified S-FEM Formulation

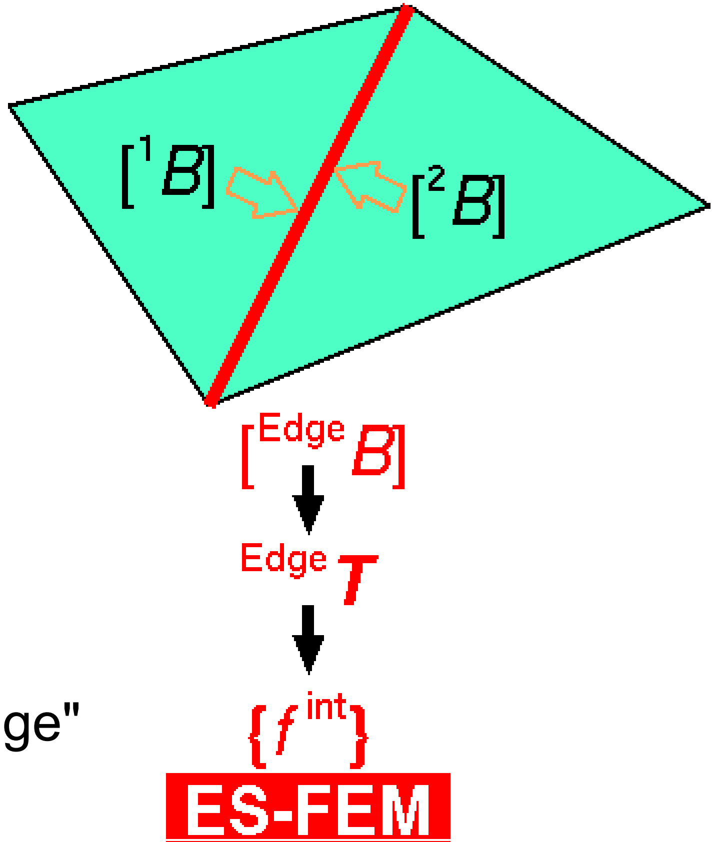
What is S-FEM?

- One of strain smoothing techniques.
- There are several types of S-FEM.
  - Edge-based (ES-FEM) for 2D
  - Face-based (FS-FEM) for 3D
  - Node-based (NS-FEM) for both 2D and 3D
  - Selective edge/node-based (ES/NS-FEM) for 2D
  - Selective face/node-based (FS/NS-FEM) for 3D
- Selective S-FEMs are thought to be the best choice because they can avoid both shear and volumetric locking even with use of triangular or tetrahedral elements.

I will explain ES-FEM, NS-FEM,
and selective ES/NS-FEM one by one.

Edge-based S-FEM (ES-FEM)

- Calculate $[B]$ at element.
- Distribute $[B]$ to the connecting edges and make $[{}^{\text{Edge}}B]$
- ***F***, ***T***, $\{f^{\text{int}}\}$ etc. are calculated at edges
- Generally accurate but induces volumetric locking.

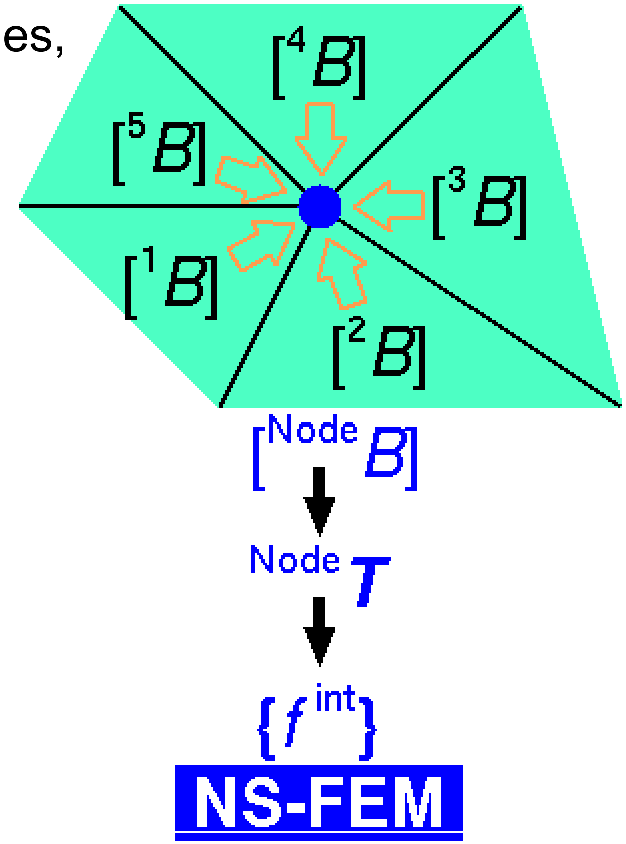


Substituting "face" for "edge" gives FS-FEM for 3D

Node-based S-FEM (NS-FEM)

- Calculate $[B]$ at element.
- Distribute $[B]$ to the connecting nodes and make $[{}^{\mathrm{Node}}B]$
- ***F***, ***T***, $\{f^{\mathrm{int}}\}$ etc. are calculated at nodes
- Generally <u>not accurate</u> but volumetric locking free.

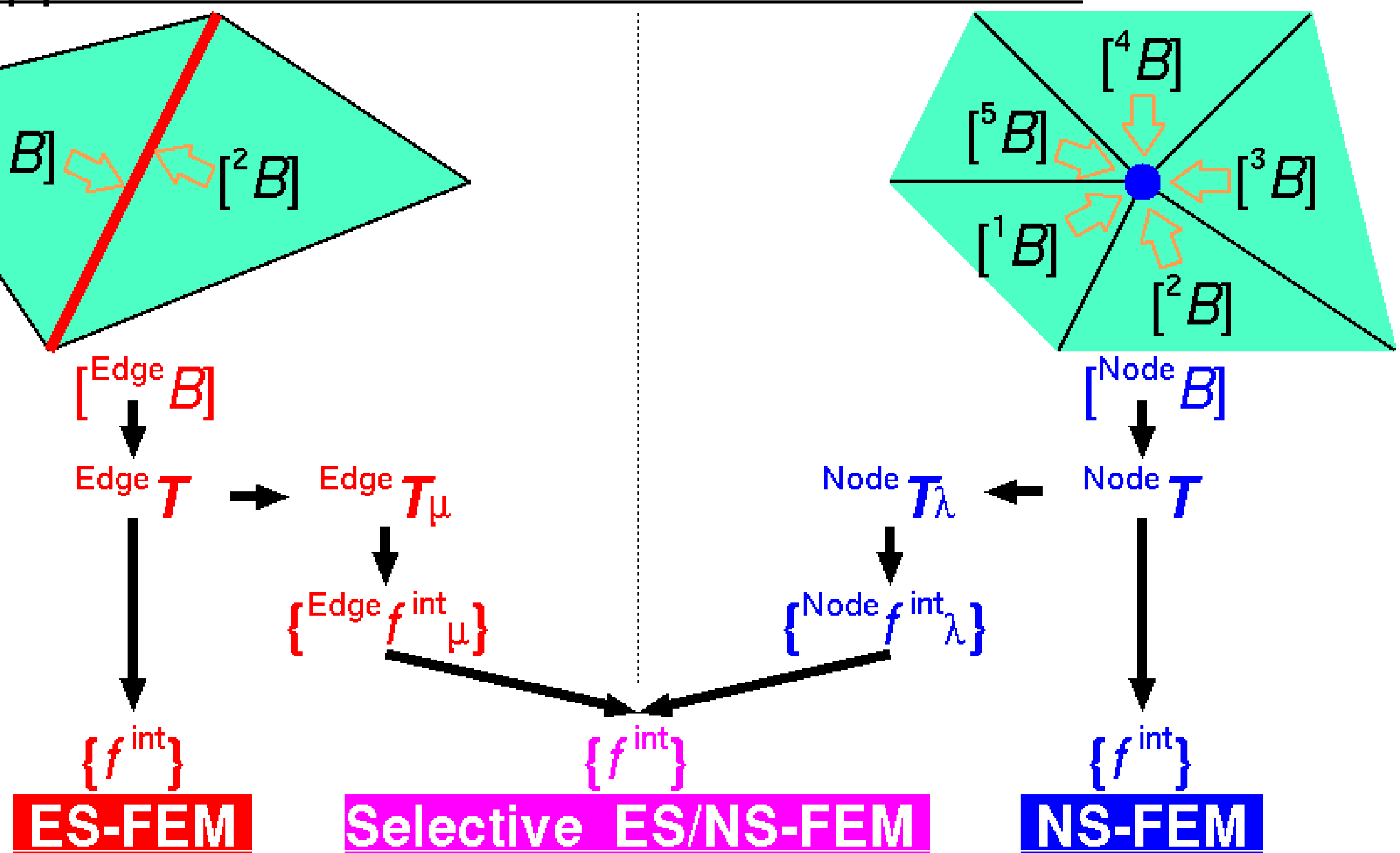
(due to zero-energy modes, which are arisen in reduced integration finite elements as hour-glass modes)



Original Selective ES/NS-FEM

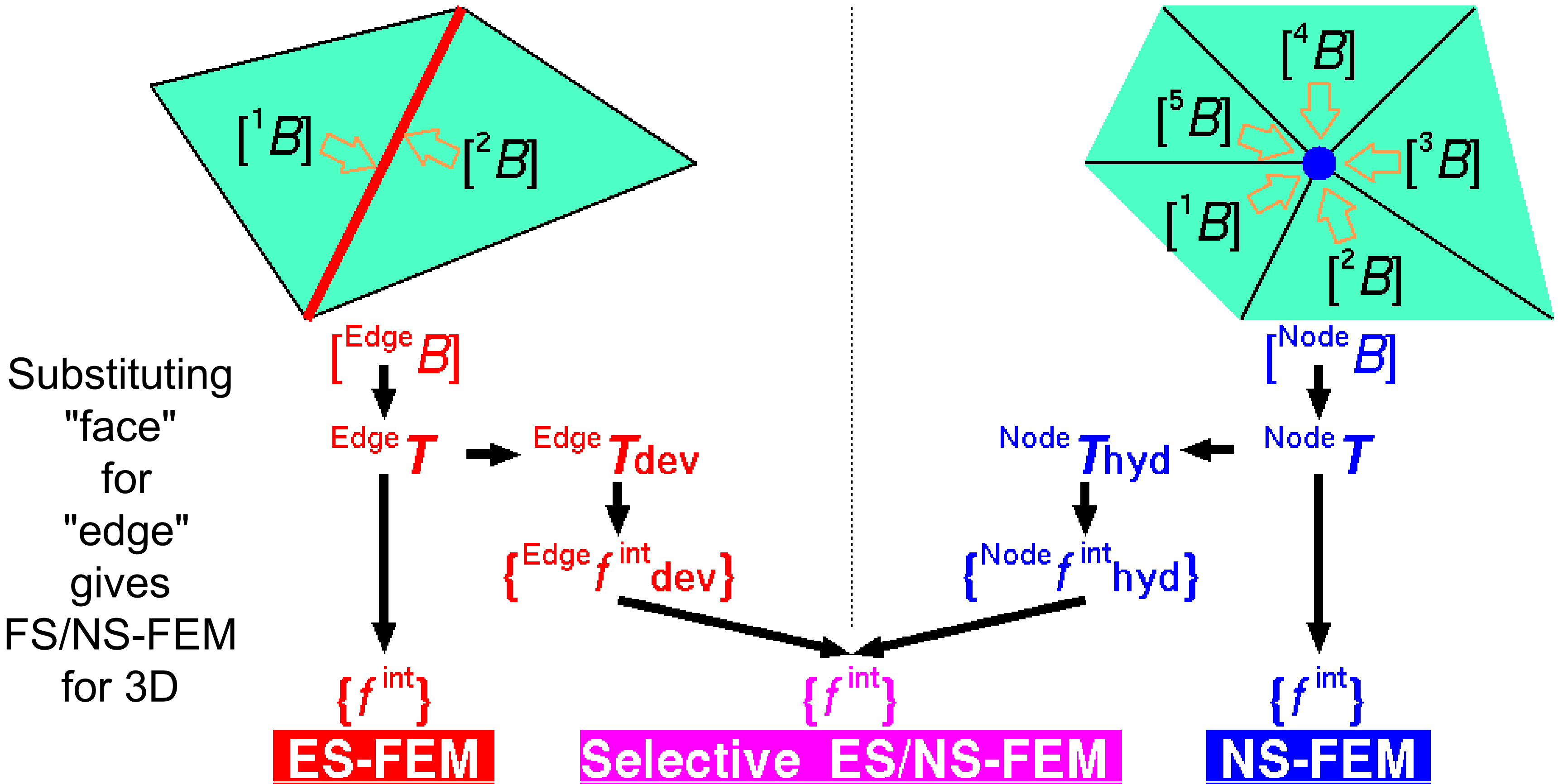
- Separate stress into "μ part" and "λ part", where μ and λ are the Lame's parameters.
- ***F***, ***T***, {*f* int} etc. are calculated at both edges and nodes.
- <u>Only applicable to elastic constitutive models.</u>

Substituting "face" for "edge" gives FS/NS-FEM for 3D



Our Selective ES/NS-FEM

- Separate stress into "deviatoric part" and "hydrostatic part" instead of "μ part" and "λ part".
- **Applicable to any kind of material constitutive models.**



Verification of Our Selective S-FEM

Cantilever Bending Test

- 10m x 1m x 1m cantilever with 20 kN dead load
- Neo-Hookean hyperelastic material

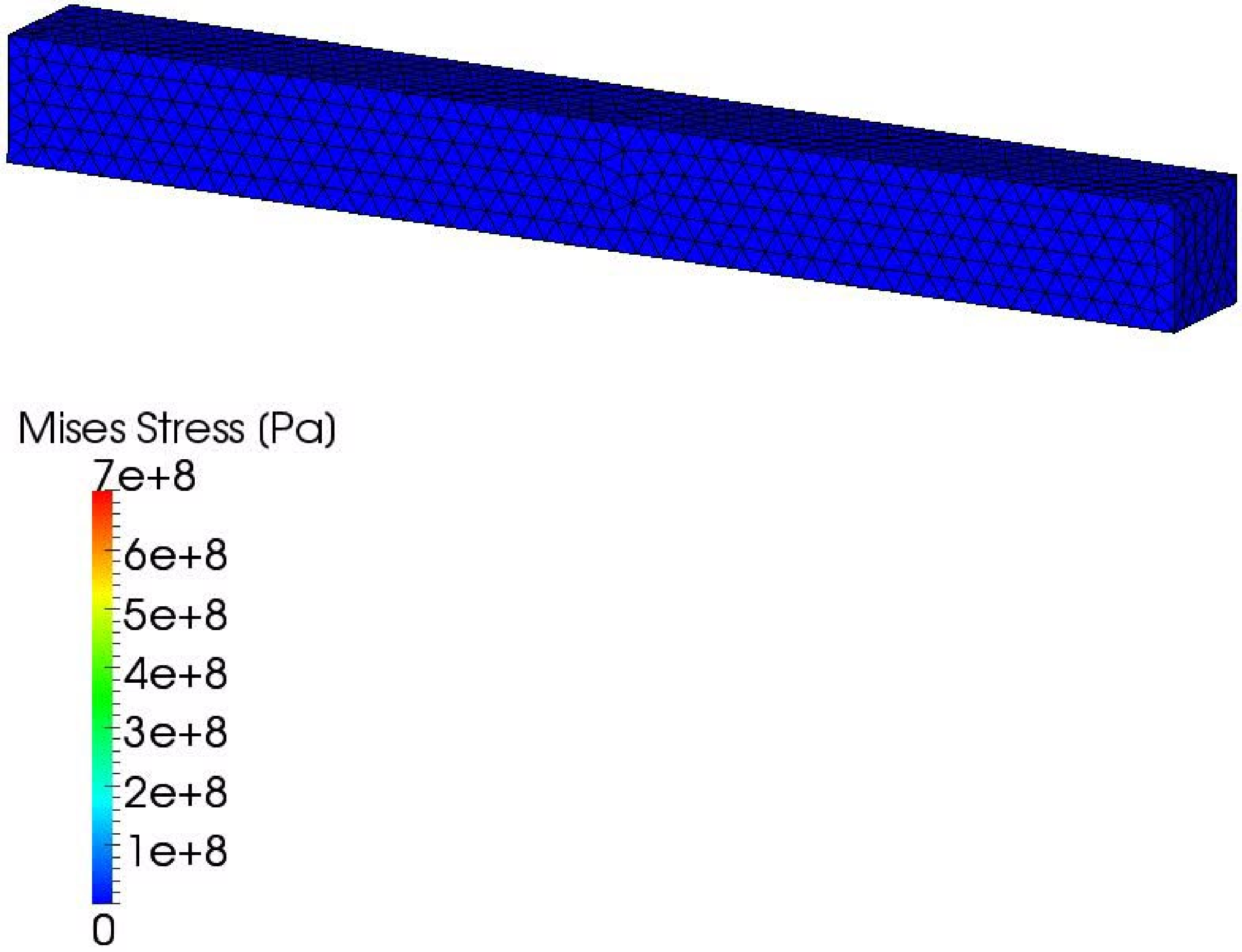
$$[T] = 2C_{10}\frac{\mathrm{Dev}(\bar{B})}{J} + \frac{2}{D_1}(J-1)[I]$$

with a constant C_{10}(=1 GPa) and various D_1s.

- Our selective FS/NS-FEM with 9560 tetrahedral elements (and 2288 nodes) is performed.
- ABAQUS/Standard with 1250 *C3D20H* (2nd-order hybrid hexahedral) elements (and 6696 nodes) is also performed.
- No mesh rezoning is taken place for this test.

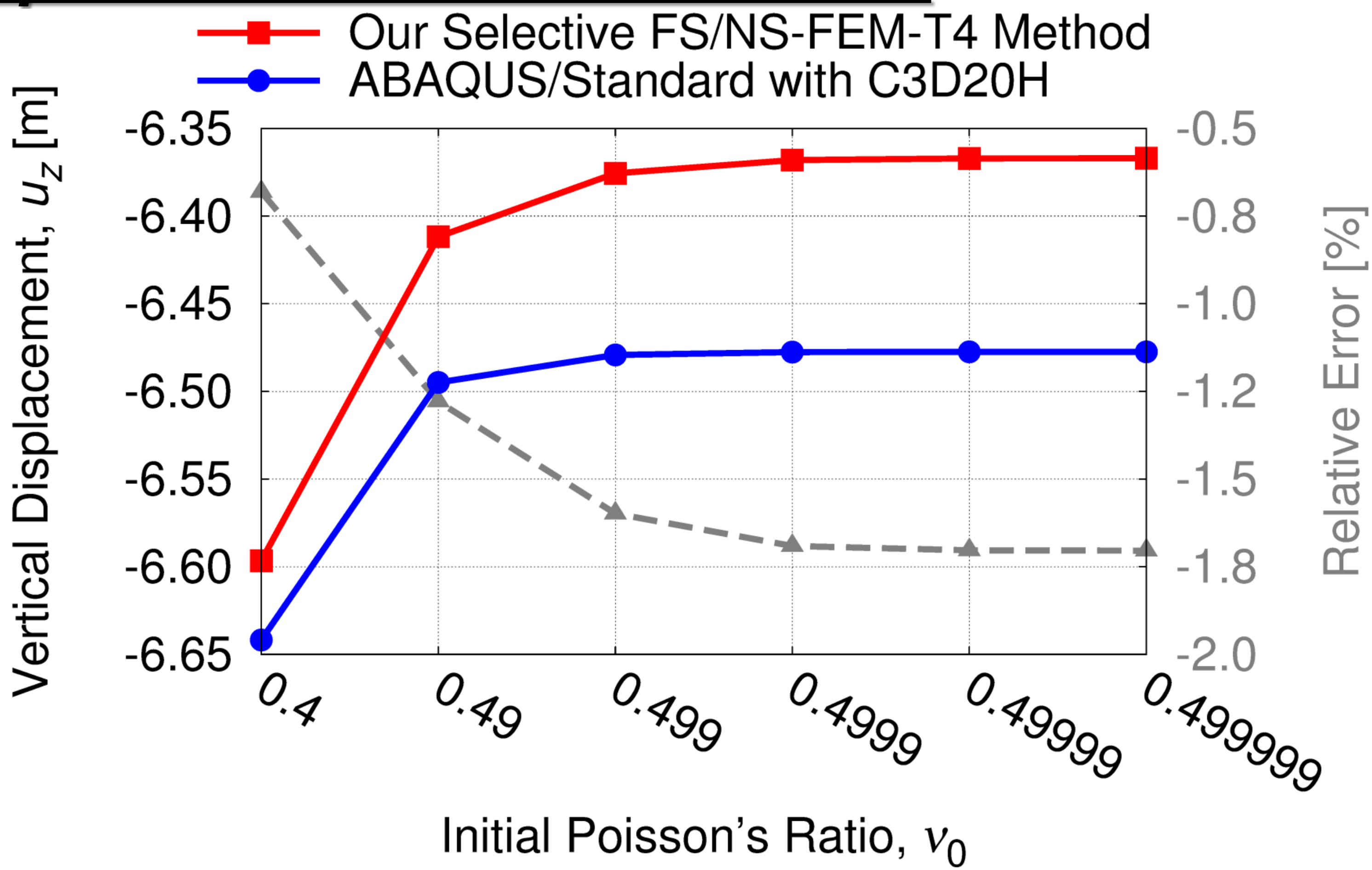
Verification of Our Selective S-FEM

Results with $D_1 = 2\ \mathrm{PPa}^{-1}$ (ν_0=0.499999)



Verification of Our Selective S-FEM

Comparison to Standard Method

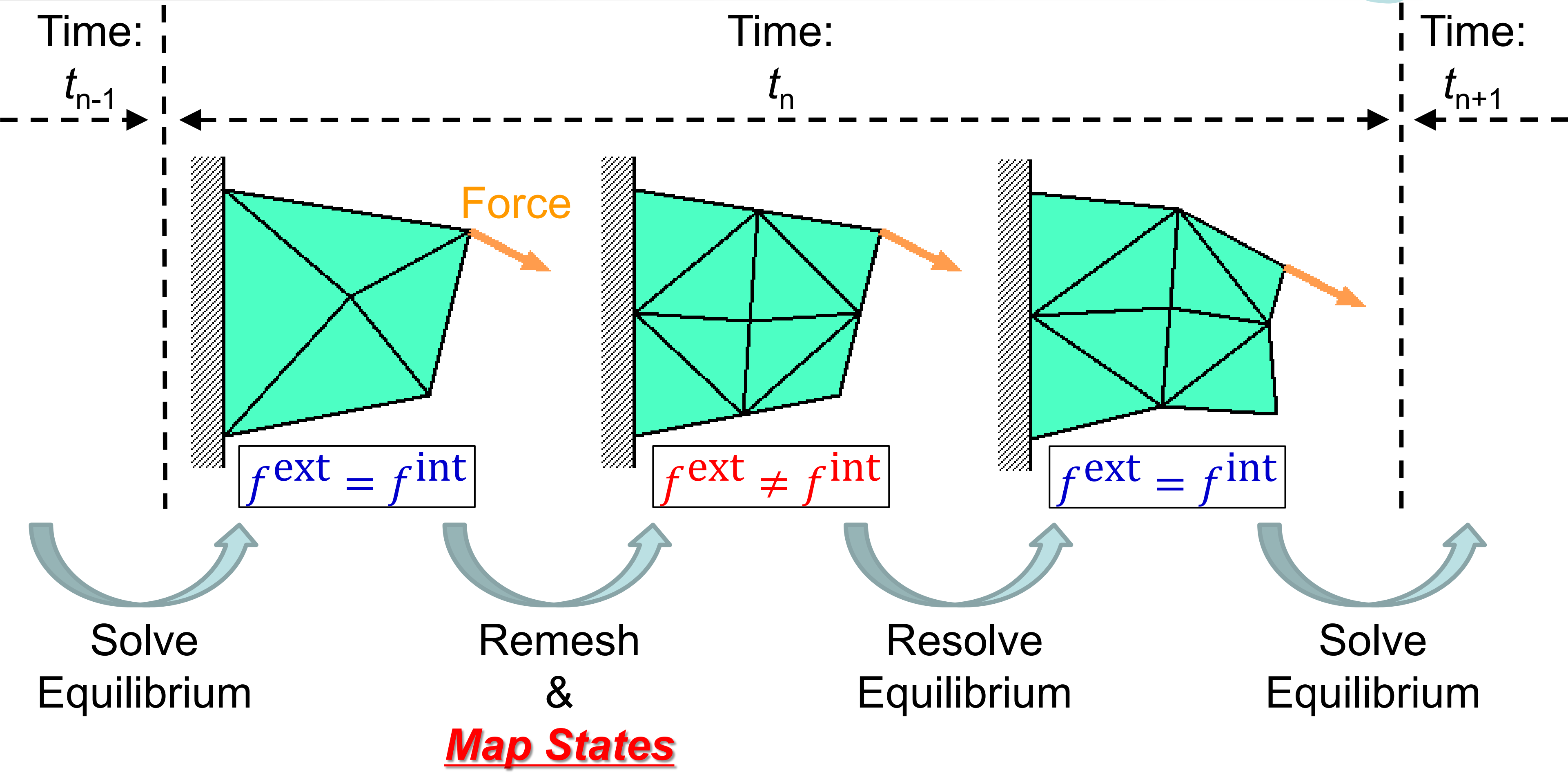


Our selective S-FEM can treat any material model and is free from locking!!

Part 2:

Procedure of our mesh rezoning method

Procedure of Mesh Rezoning



The way of mapping varies with the material constitutive model.
Elasto-plastic models necessitate some kind of correction.

Mapping of Stress/Strain States

For Elastic or Hyperelastic Materials

i.e., $[T] = [T([F])]$

- Map initial position $\{x^{\text{initial}}\}$ at nodes, and then remake deformation gradient $[F]$ at edges & nodes.

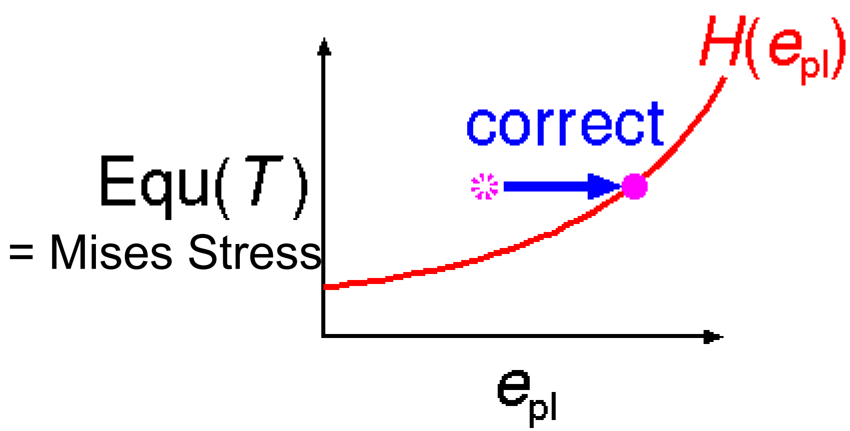
Each node preserve its initial position so that the domain can spring back to the initial shape after unloading.

Mapping of Stress/Strain States

For Elasto-Plastic Material in Total Strain Form

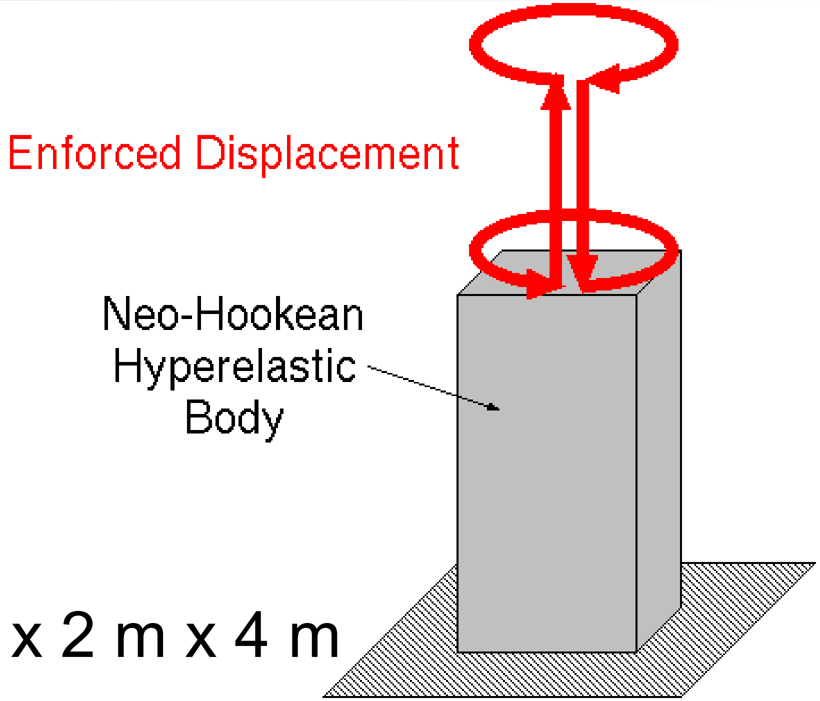
e.g., $[T] = [T([F], [E_{\mathrm{pl}}], e_{\mathrm{pl}}; H(e_{\mathrm{pl}}))]$

- Map initial position $\{x^{\mathrm{initial}}\}$ at nodes, and then remake deformation gradient $[F]$ at edges & nodes.
- Map history dependent variables, plastic strain $[E_{\mathrm{pl}}]$ and equivalent plastic strain e_{pl}
- Correct e_{pl} to satisfy $\mathrm{Equ}([T]) = H(e_{\mathrm{pl}})$



Part 3:
Examples of Analysis

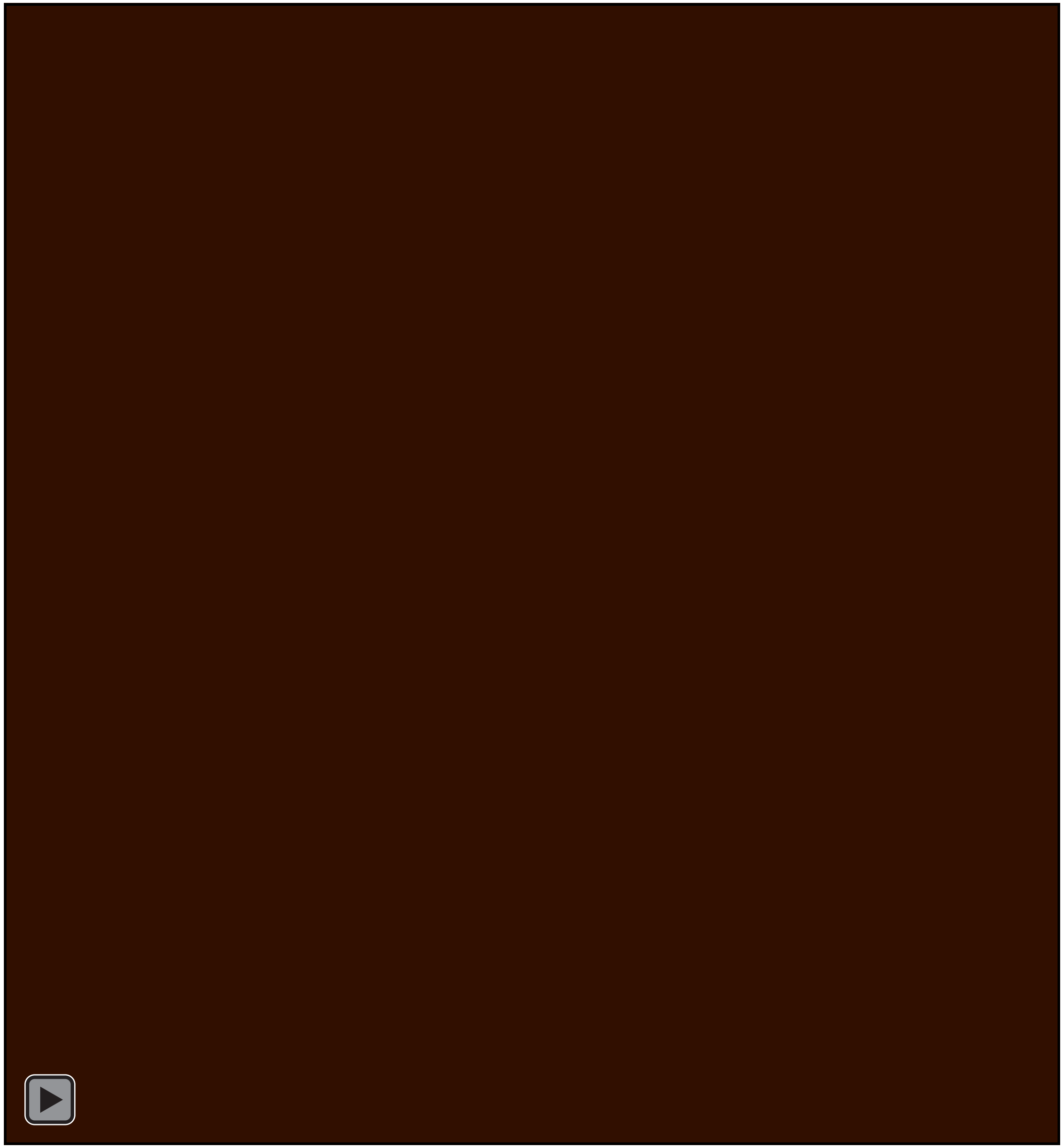
Twist and Stretch of Hyperelastic Body



- Static, 1 m x 2 m x 4 m
- Neo-Hookean hyperelastic body of $C_{10} = 1\ \mathrm{GPa}$ and $D_1 = 400\ \mathrm{GPa}^{-1}$ $(\nu_0 = 0.48)$
- Twist up to 360 deg. ⟹ Stretch up to 100% nominal strain ⟹ Twist back ⟹ Shrink back
- Our selective FS/NS-FEM with tetrahedral elements
- Global mesh rezoning every 90 deg. and 50% stretch/shrink

Twist and Stretch of Hyperelastic Body

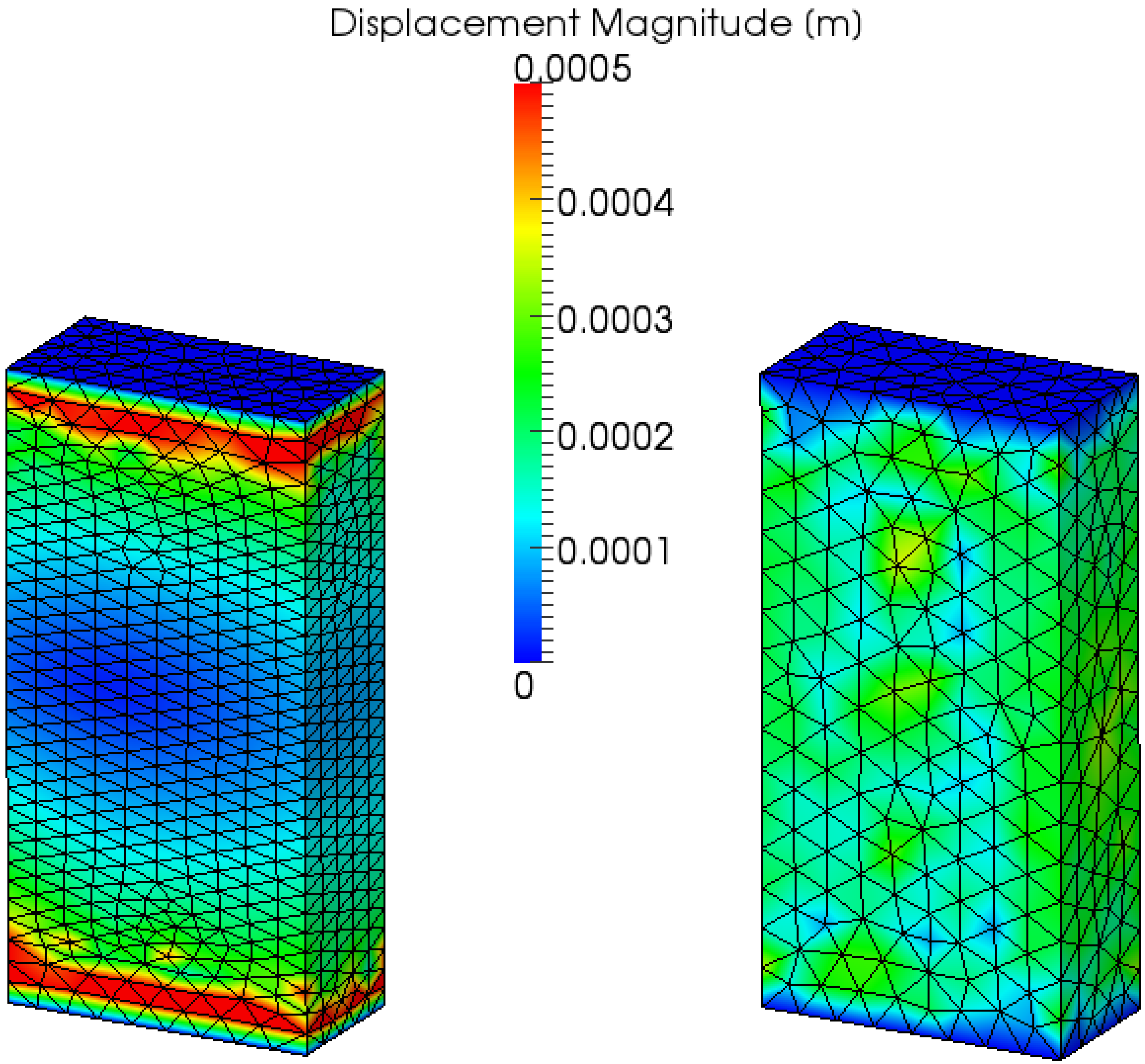
Our selective FS/NS-FEM **with** mesh rezoning



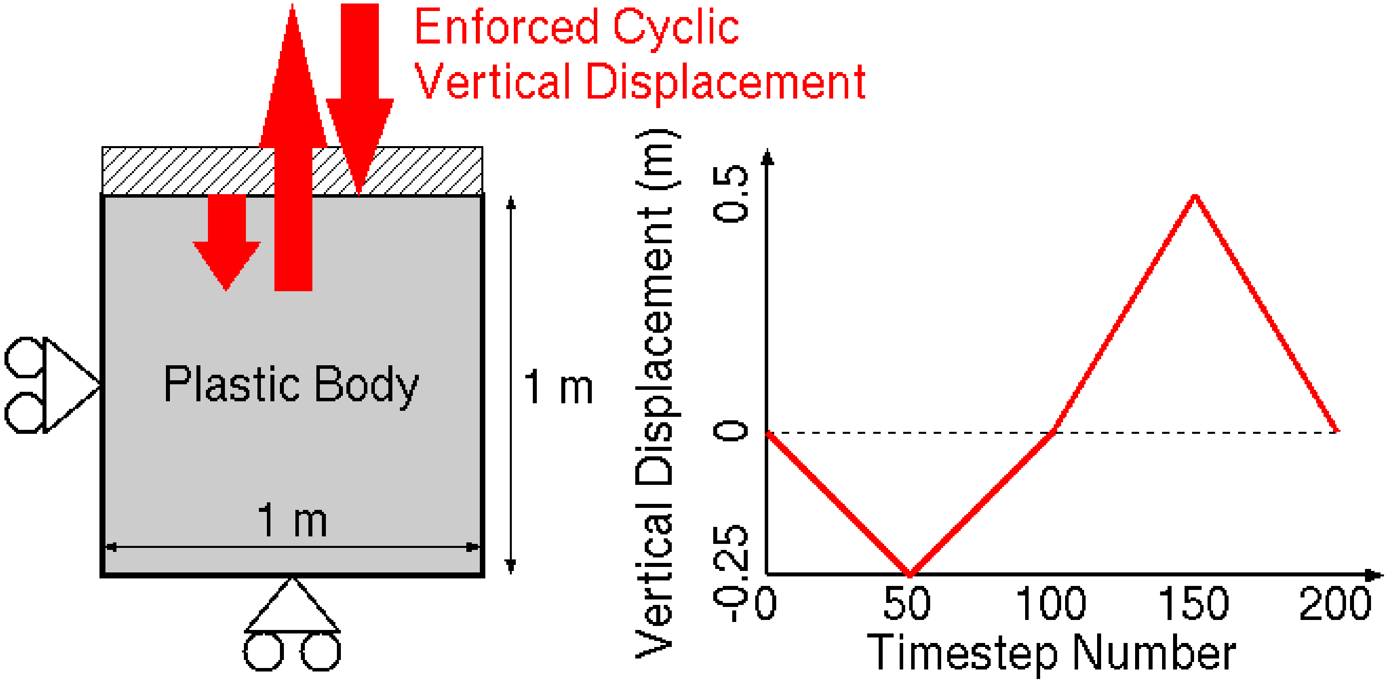
Our selective FS/NS-FEM **without** mesh rezoning

Twist and Stretch of Hyperelastic Body

Residual Displacement



Cyclic Disp. of Elasto-Plastic Body



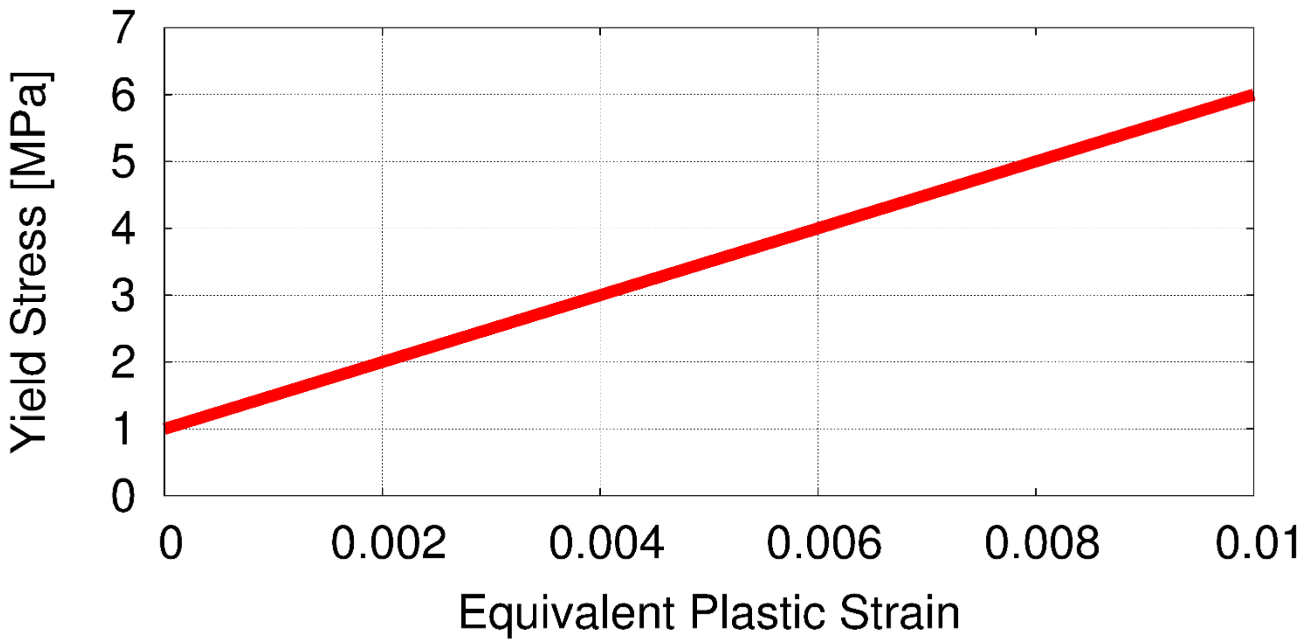
- Static, Plane Strain, 1 m x 1 m
- Hencky's elasto-plastic material
- Horizontal constraint on left edge
- Vertical constraint on bottom edge
- Horizontal constraint and vertical enforced disp. on top edge
- Our selective ES/NS-FEM with triangular elements

Cyclic Disp. of Elasto-Plastic Body

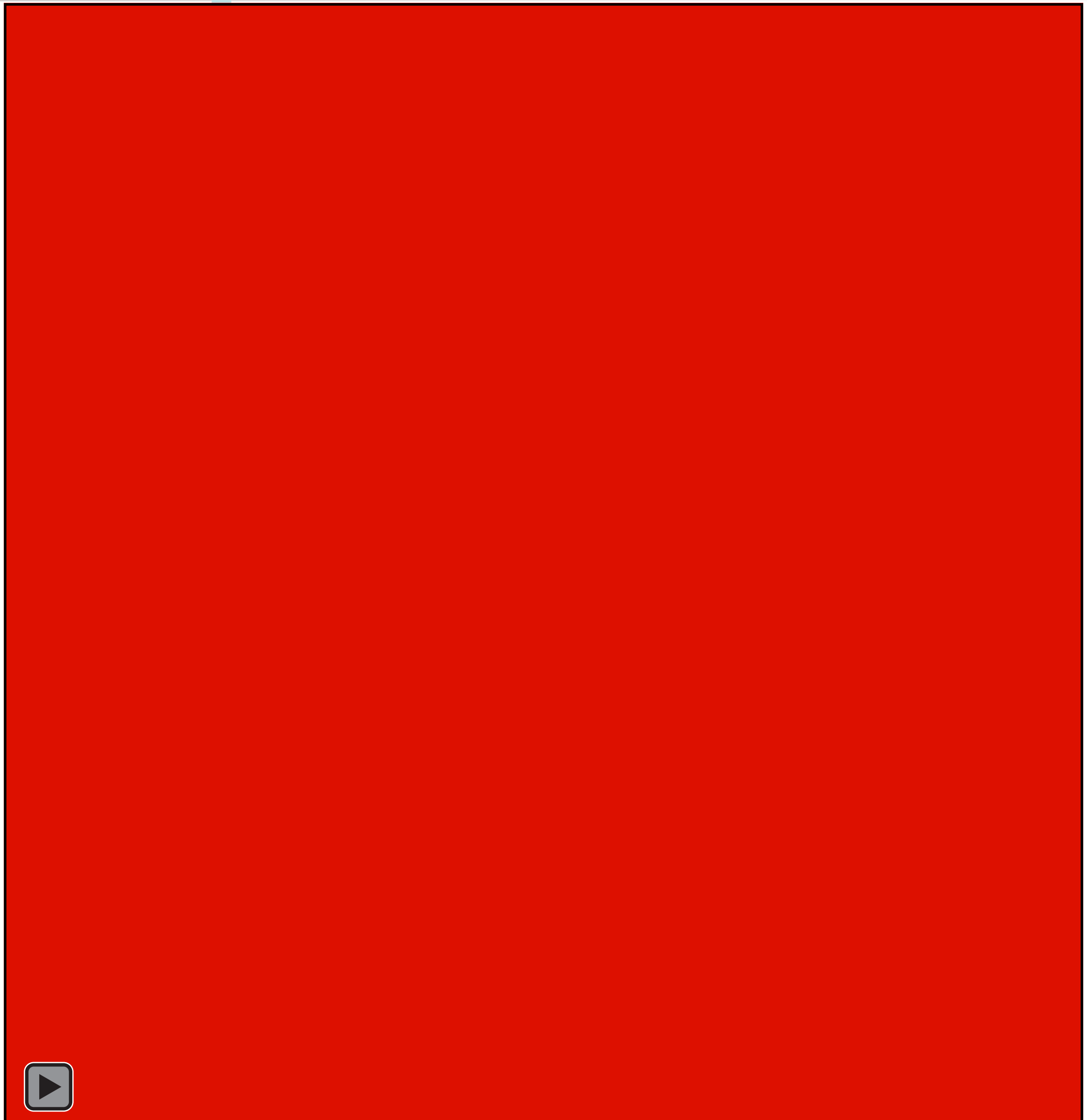
Material Constitutive Model

Hencky's elasto-plastic material, $\boldsymbol{T} = \boldsymbol{C} : \boldsymbol{h}_{\mathrm{el}}/J$,
with von Mises yield criterion and isotropic hardening

- Young's Modulus: 1 GPa, Poisson's Ratio: 0.3
- Yield Stress: 1 MPa, Hardening Coeff.: 0.5 GPa

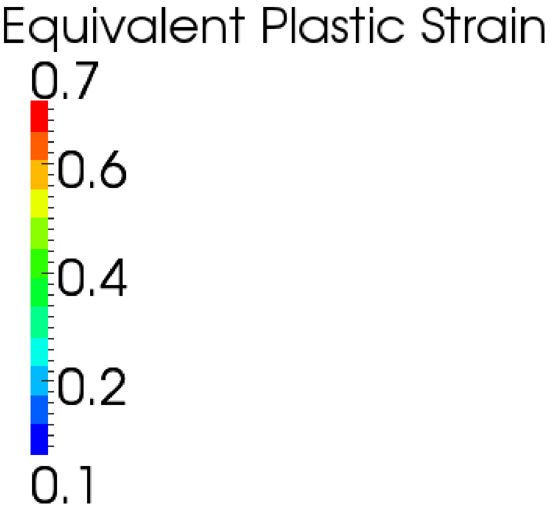


Cyclic Disp. of Elasto-Plastic Body



Deformation is always smooth.

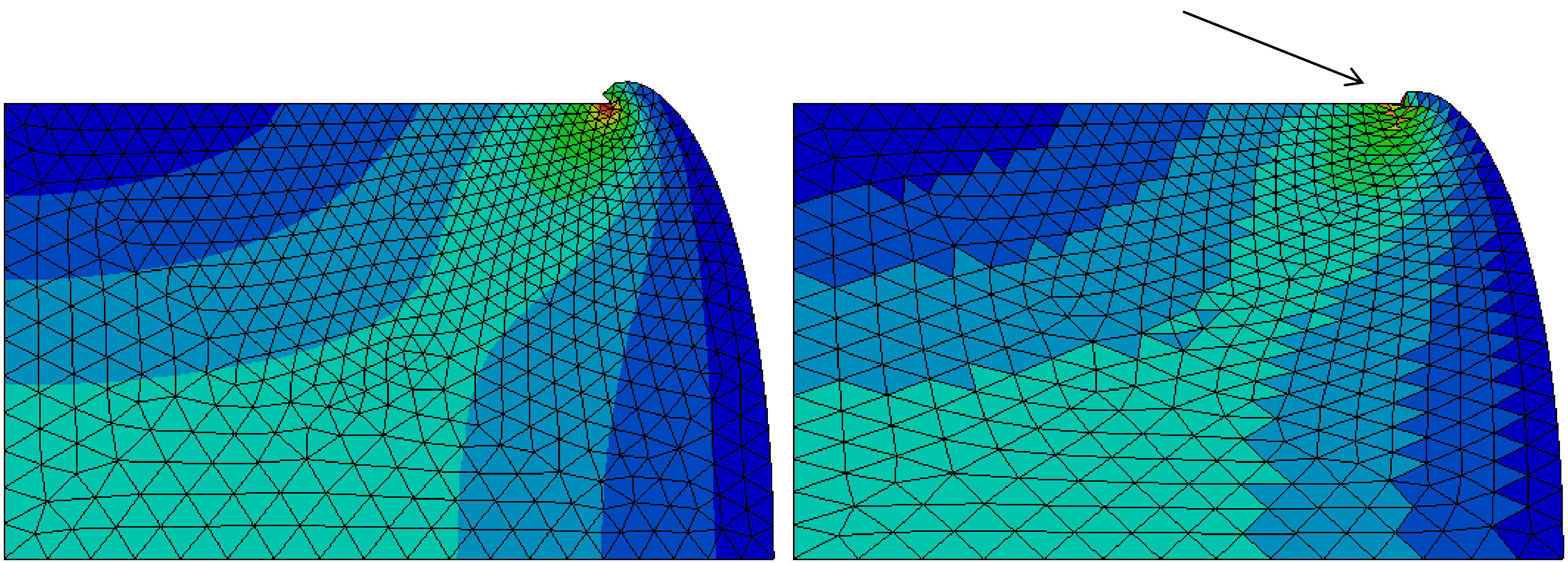
Cyclic Disp. of Elasto-Plastic Body



At the concave corner:

- checkerboard pattern is observed.
- stiffer than S-FEM.

⟹Locking is induced.

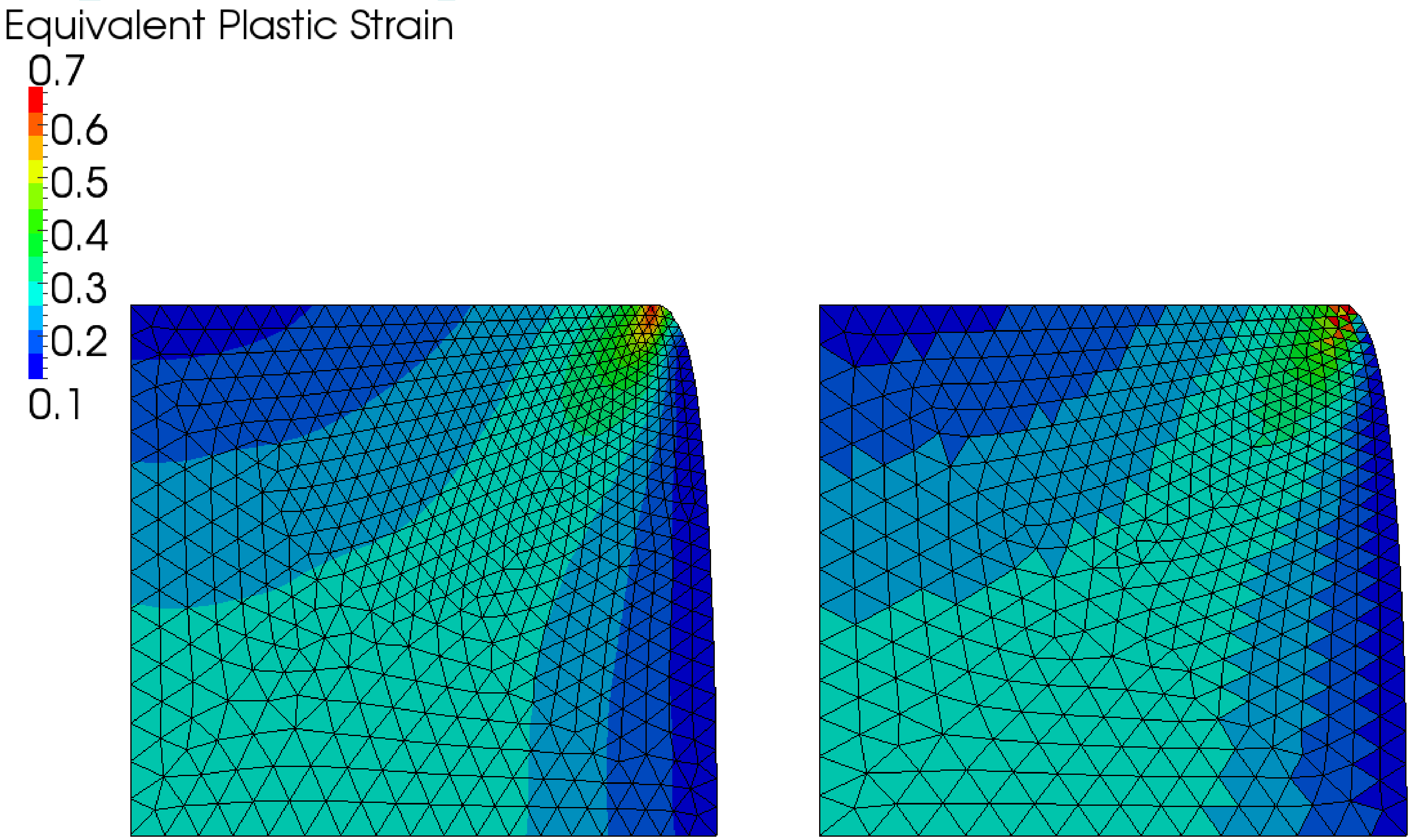


Our Selective ES/NS-FEM
with Mesh Rezoning

Timestep: 50

Standard FEM
without Mesh Rezoning

Cyclic Disp. of Elasto-Plastic Body

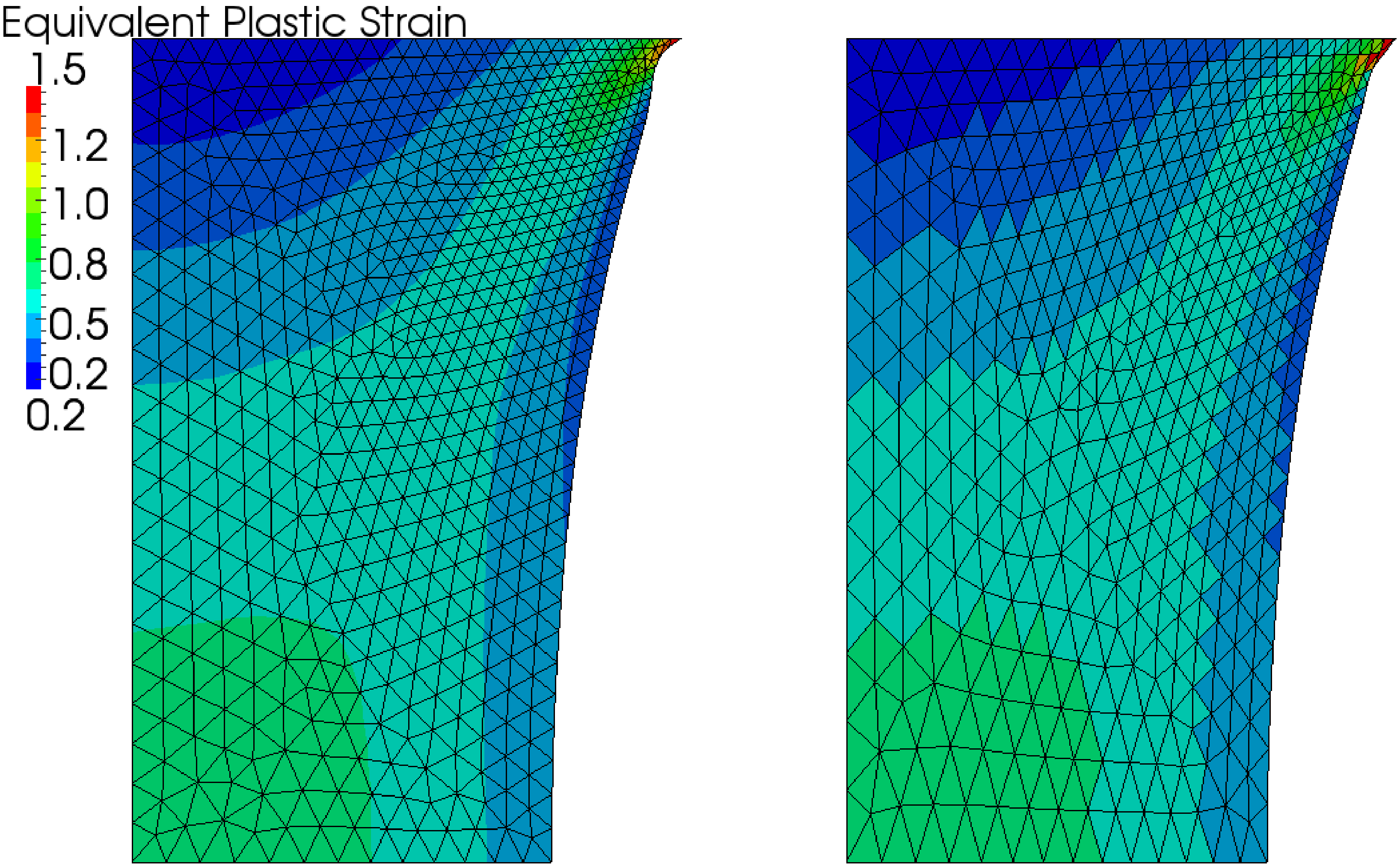


Our Selective ES/NS-FEM with Mesh Rezoning

Standard FEM without Mesh Rezoning

Timestep: 100

Cyclic Disp. of Elasto-Plastic Body

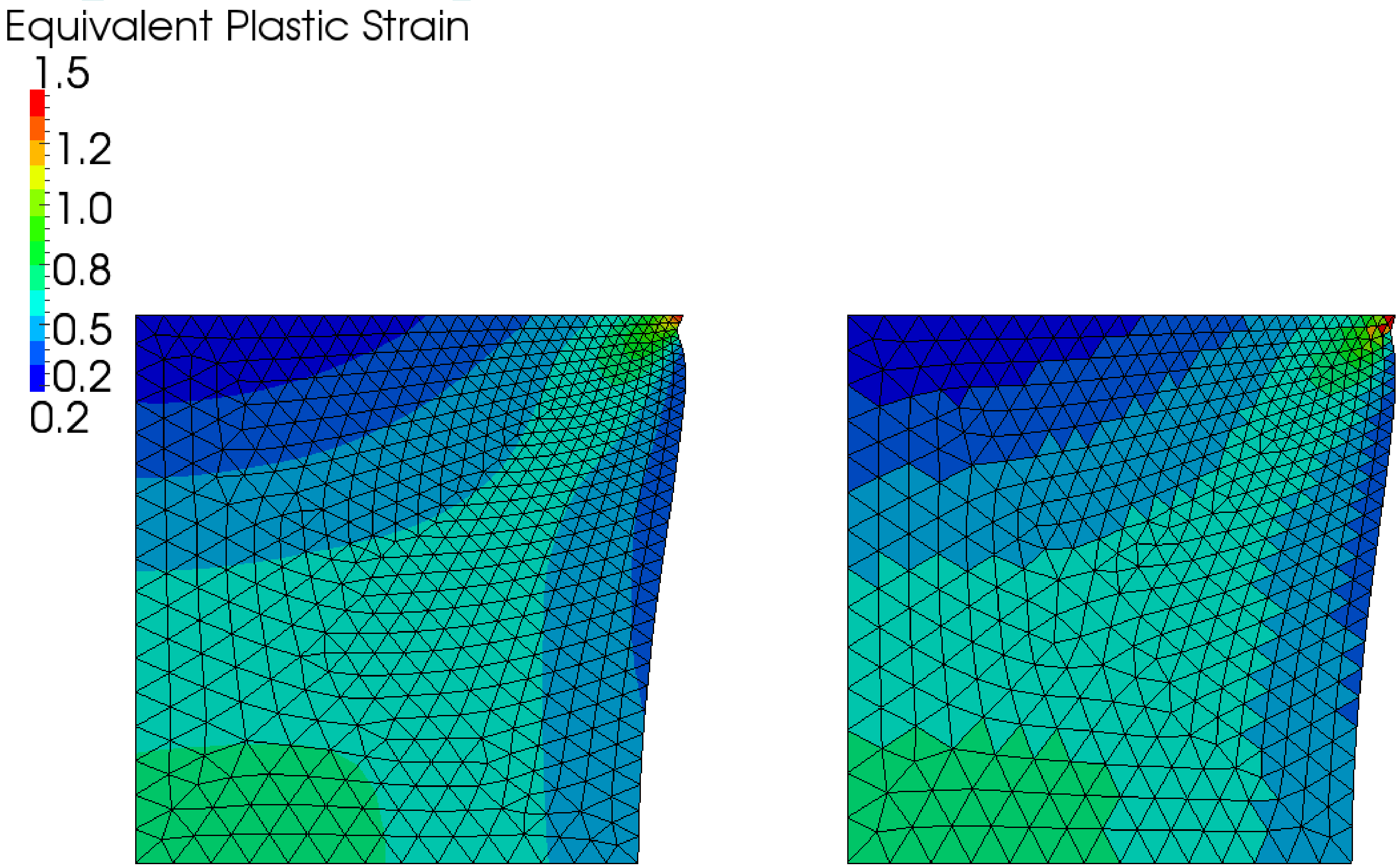


Our Selective ES/NS-FEM
with Mesh Rezoning

Standard FEM
without Mesh Rezoning

Timestep: 150

Cyclic Disp. of Elasto-Plastic Body



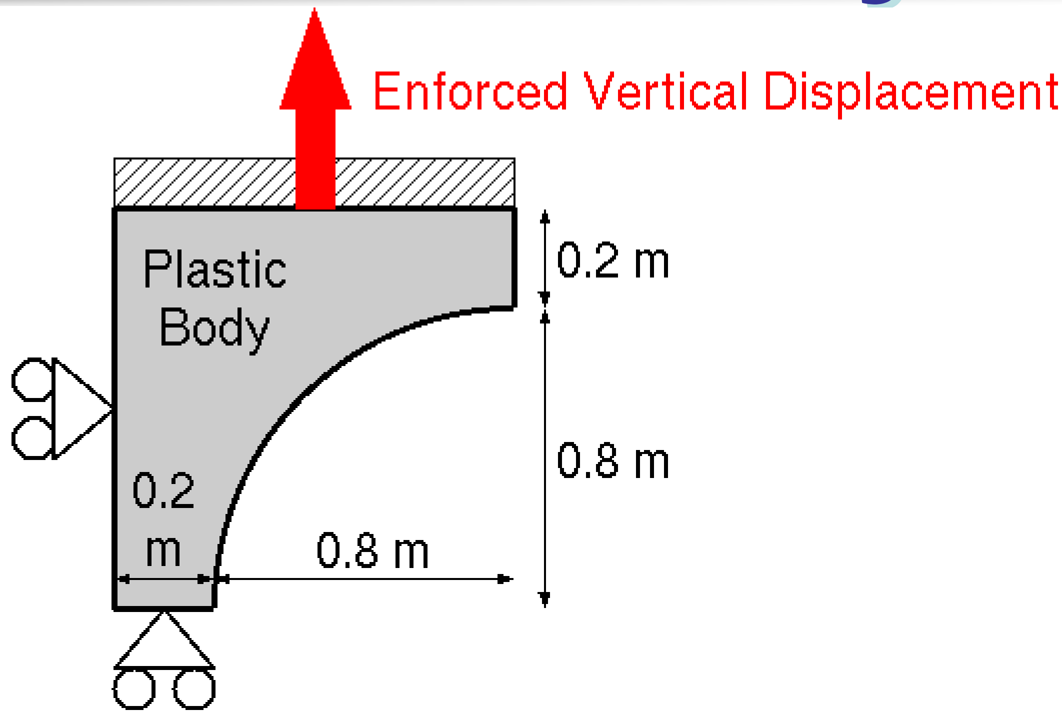
Our Selective ES/NS-FEM
with Mesh Rezoning

Standard FEM
without Mesh Rezoning

Timestep: 200

Necking of Elasto-Plastic Body

Outline



- Static, Plane-strain
- 1/4 of test piece
- horizontal constraint on left edge
- vertical constraint on bottom edge
- horizontal constraint and enforced displacement on top edge
- Mesh rezoning every 0.05 m displacement
- Our selective ES/NS-FEM with triangular elements

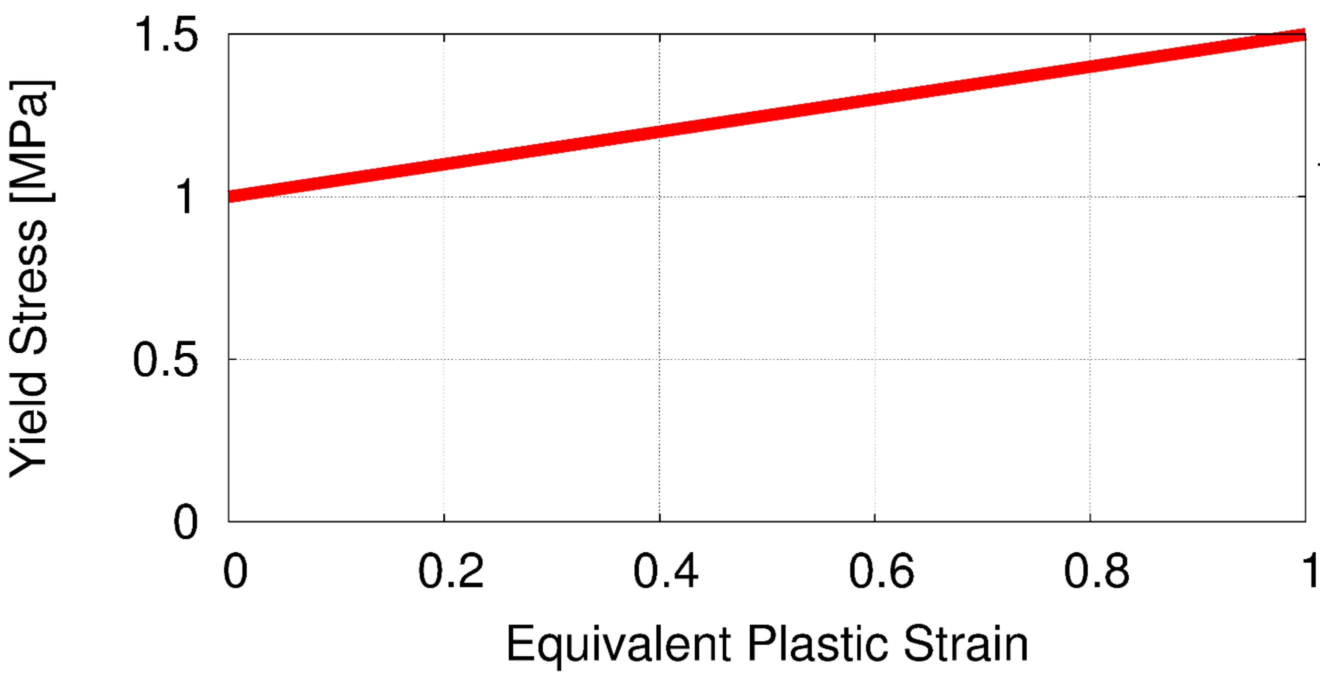
Necking of Elasto-Plastic Body

Material Constitutive Model

Hencky's elasto-plastic material, $\boldsymbol{T} = \boldsymbol{C} : \boldsymbol{h}_{\mathrm{el}}/J$,
with von Mises yield criterion and isotropic hardening

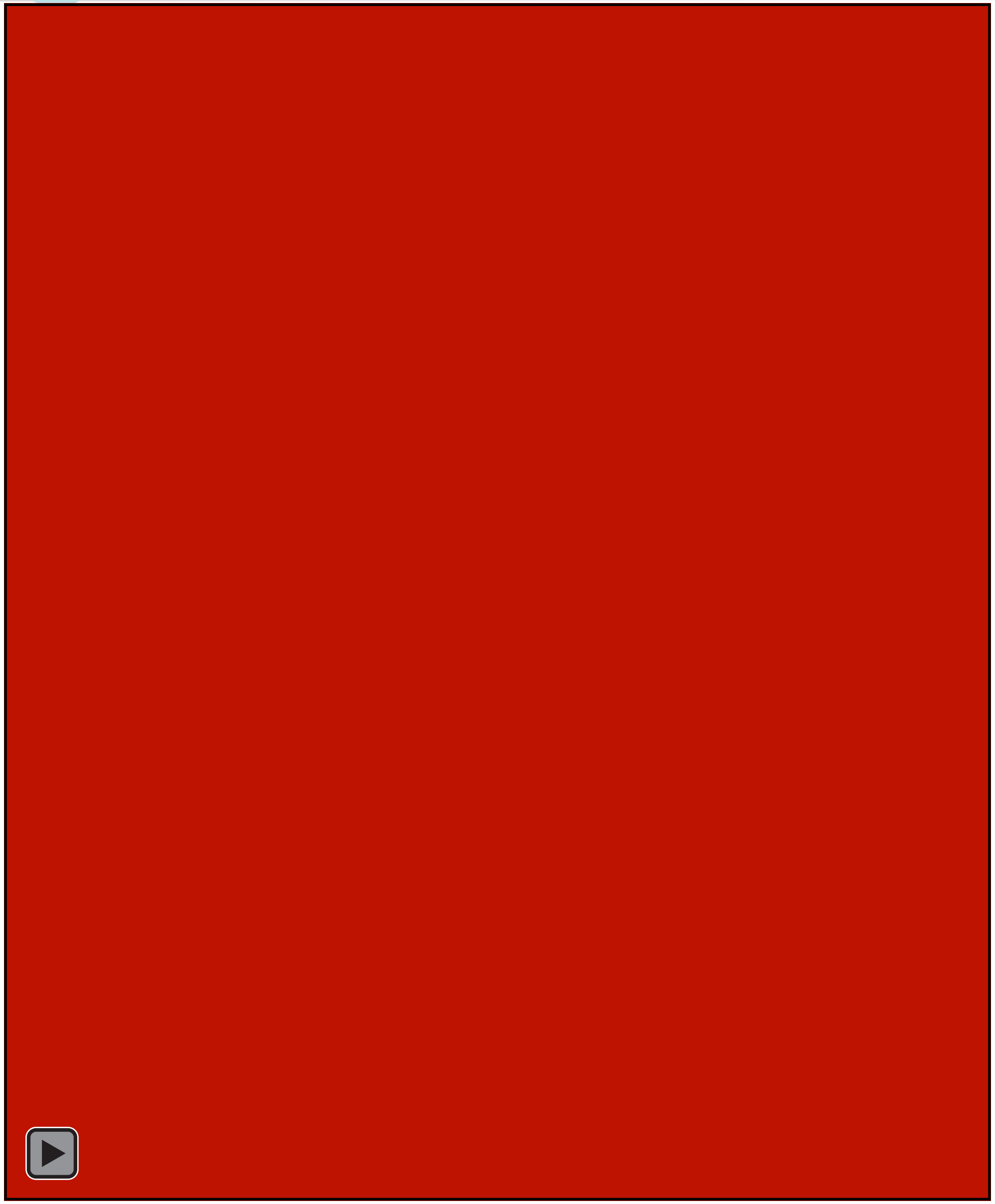
Young's Modulus: 1 GPa, Poisson's Ratio: 0.3

Yield Stress: 1 MPa, Hardening Coeff.: 0.5 MPa



very close to **elastic-perfect plastic** model

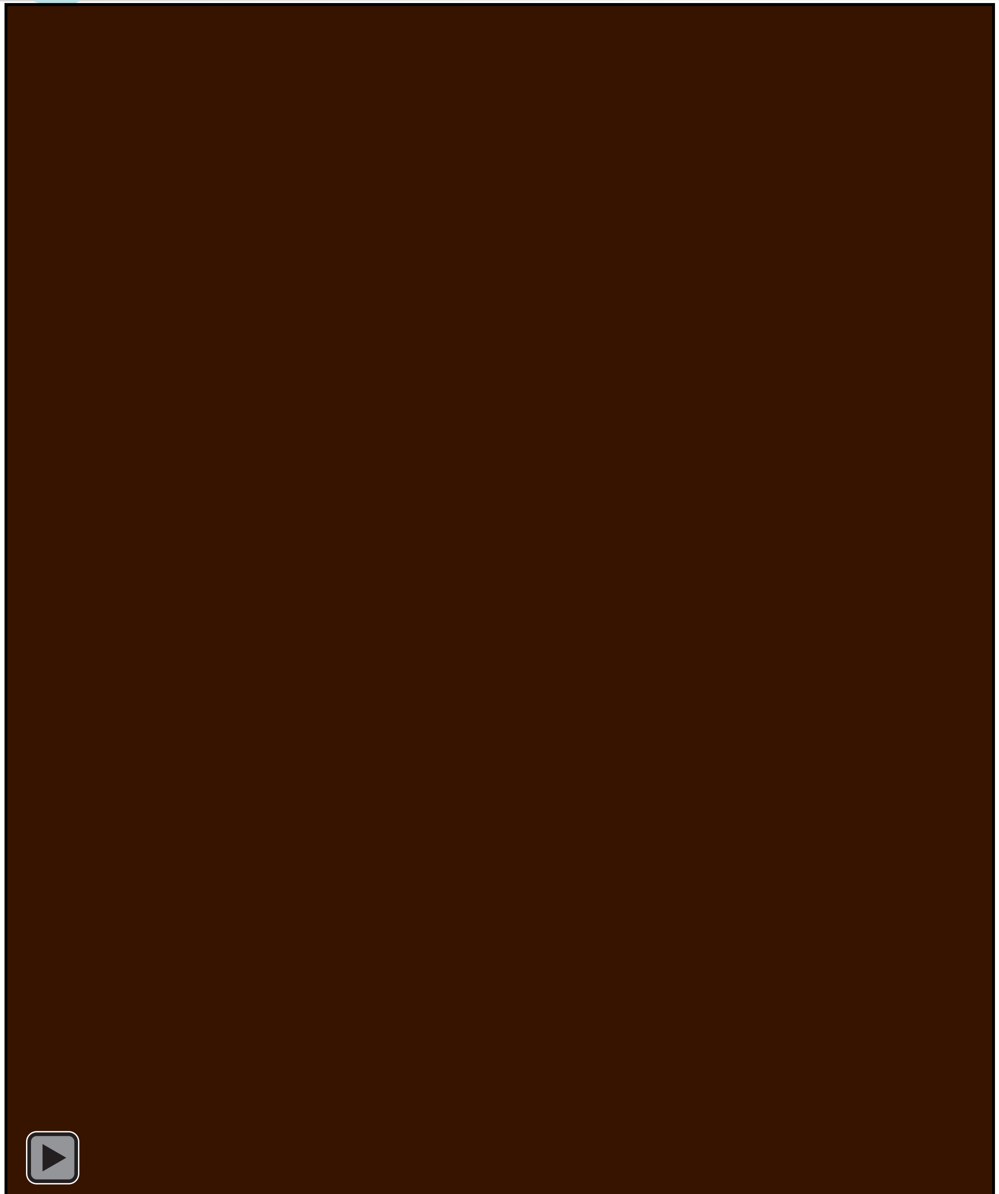
Necking of Elasto-Plastic Body



Typical deformation of necking test is obtained.

Necking of Elasto-Plastic Body

Zoom-in view around the center



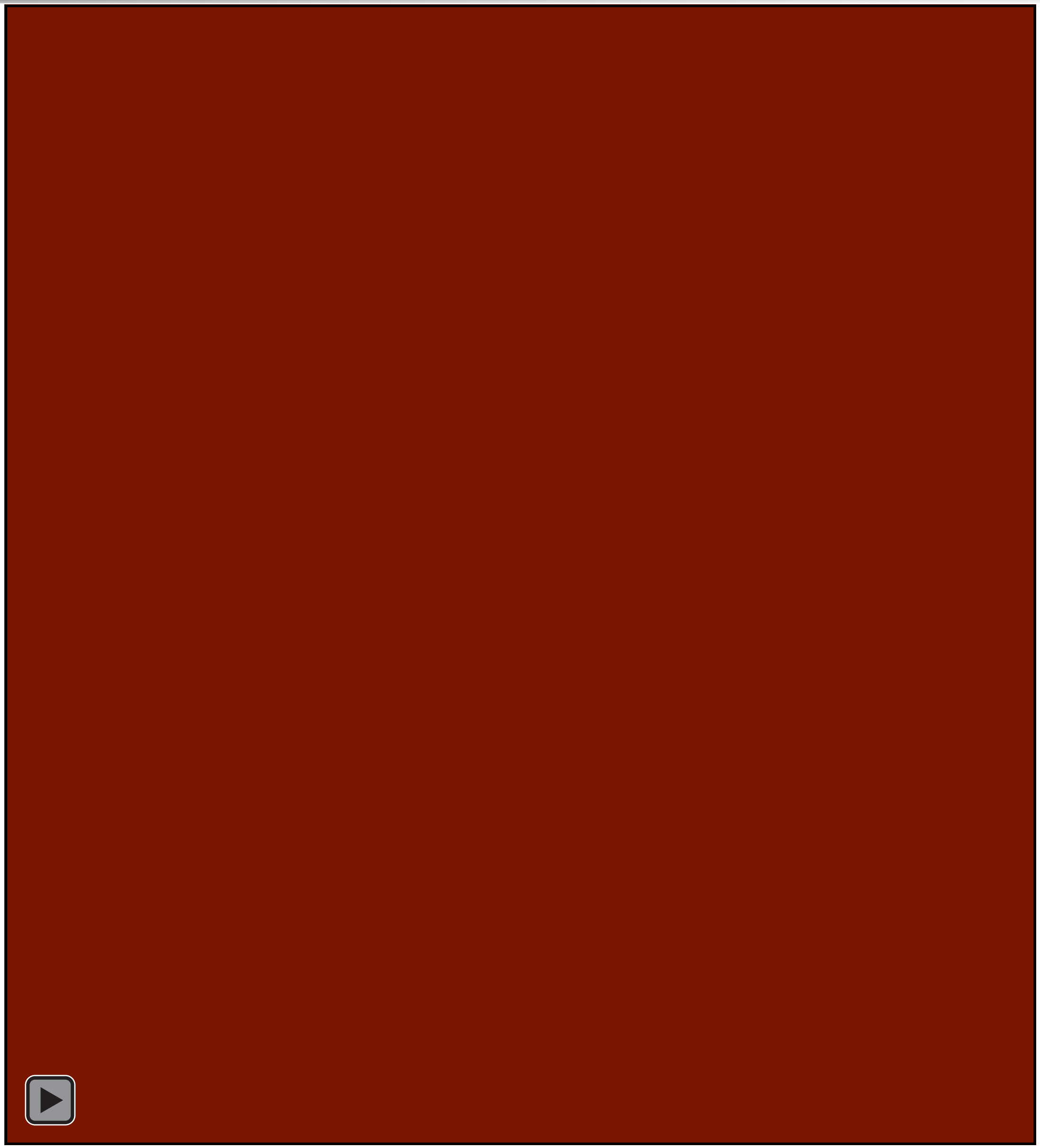
Necking of Elasto-Plastic Body

ABAQUS
/Standard

Strange deformation is obtained due to:

- locking of triangular elements
- absence of mesh rezoning

Twist of Elasto-Plastic Body



- Static, 3D
- 1 m x 2 m x 4 m
- Elasto-plastic body with 0.5 GPa
- perfect constraint on bottom face
- Horizontally twist top face up to 360 deg. and twist back
- mesh rezoning every 90 deg. twist
- Our selective FS/NS-FEM with tetrahedral elements

Summary

Take-Home Messages

1. Accurate (implicit) and stable mesh rezoning is *almost* a reality!!
2. Our modified selective S-FEM are worth using even without mesh rezoning!!

Summary and Future Work

Summary

- A new static-implicit mesh rezoning method for severely large deformation analysis is proposed.
- It adopts our modified selective S-FEM, which separates stress into deviatoric part and hydrostatic part.
- Its accuracy are verified with hyperelastic material and elasto-plastic material.

Future Work

- More V&V
- Local mesh rezoning
- Apply to contact forming, crack propagation, etc.

Thank you for your kind attention!!