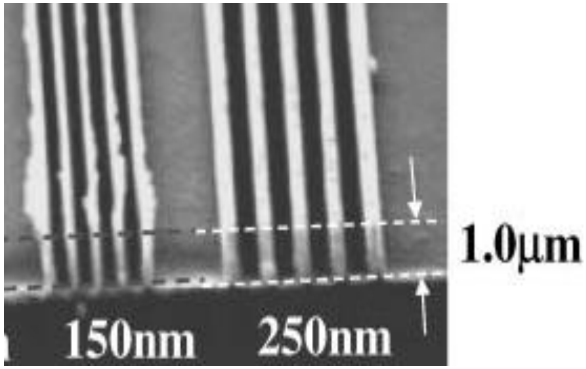
Locking-Free
Smoothed Finite Element Method
with Tetrahedral/Triangular Mesh Rezoning
in Severely Large Deformation Problems

Yuki ONISHI, **Kenji AMAYA**
Tokyo Institute of Technology (Japan)

Motivation and Background

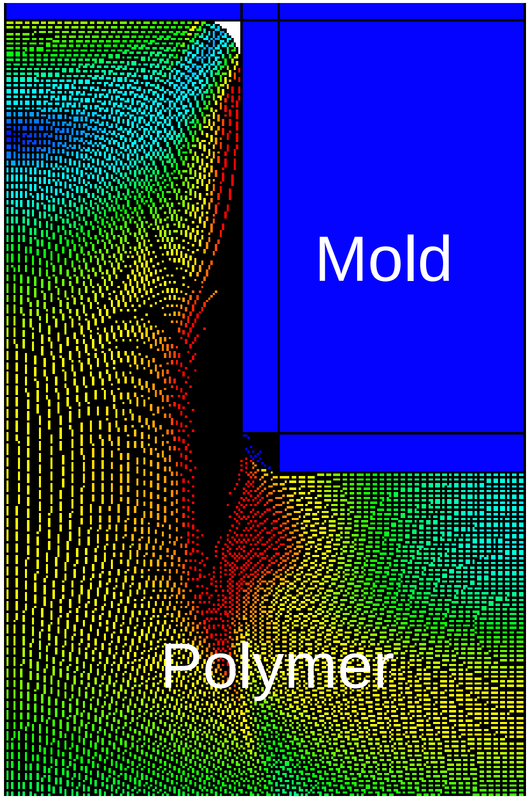
Motivation

We want to solve **severely large deformation** problems ***accurately and stably!***
(Target: automobile tire, thermal nanoimprint, etc.)



Background

Finite elements are **distorted** in a short time, thereby resulting in convergence failure.



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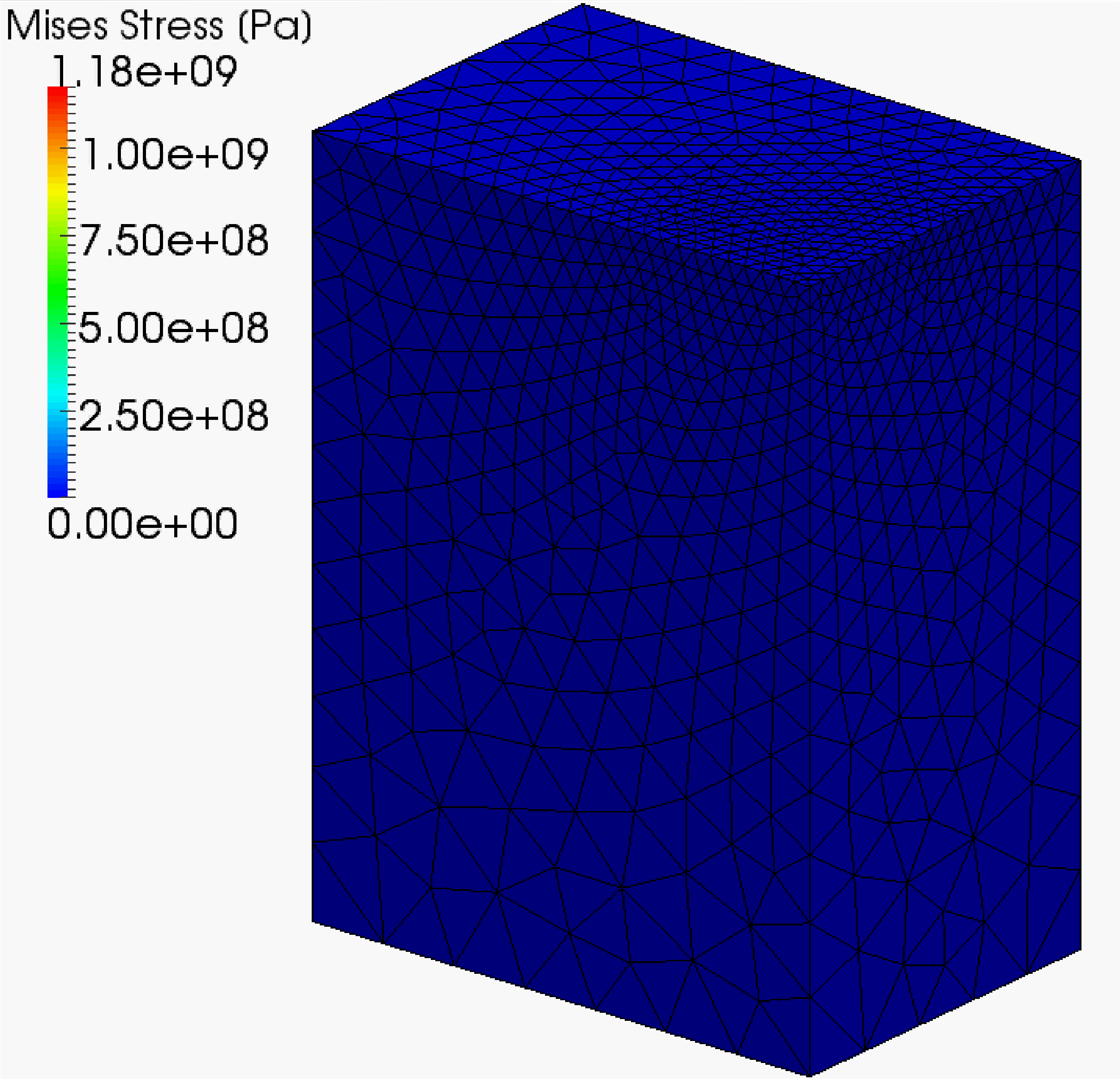
Mesh rezoning method (*h*-adaptive mesh-to-mesh solution mapping) is indispensable.

Our First Result in Advance

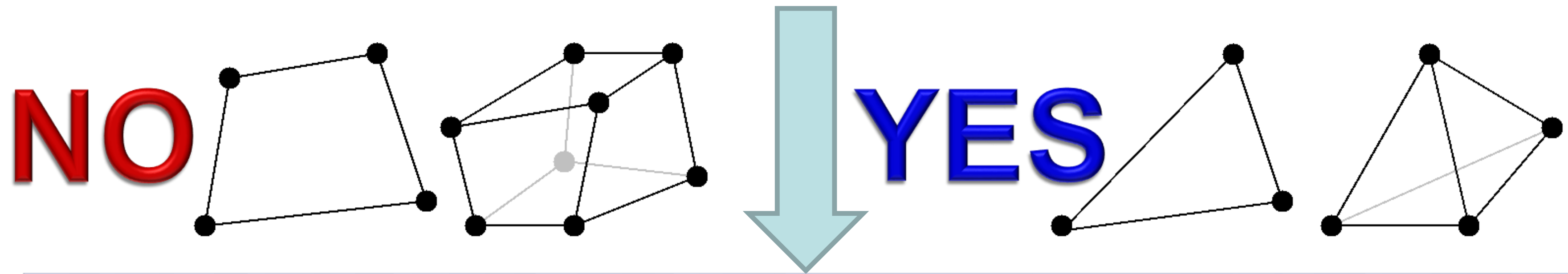


static-implicit
large deformation
analysis
with
mesh rezoning

Issues

<u>The biggest issue in large deformation mesh rezoning</u>

It is impossible to remesh arbitrary deformed 2D or 3D domains with quadrilateral or hexahedral elements.



We have to use triangular or tetrahedral elements...

However, the *standard* (constant strain) triangular or tetrahedral elements induce shear and volumetric locking easily, which leads to inaccurate results.

Conventional Methods

- Higher order elements:
  - ✗ Not volumetric-locking-free; Not effective in large deformation due to intermediate nodes.
- EAS elements:
  - ✗ Unstable.
- B-bar, F-bar and selective integration elements:
  - ✗ Not applicable to triangular/tetrahedral.
- F-bar patch elements:
  - ✗ Difficult to construct patches
- u/p hybrid elements
  - ✗ No sufficient formulation for triangular/tetrahedral is presented so far. (There are almost acceptable hybrid elements such as C3D4H of ABAQUS.)
- Selective smoothed finite elements:
  - ? Unknown potential. ***Let's try!***

Objective

Develop a locking-free
modified selective S-FEM
for large deformation problems
with mesh rezoning

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  without mesh rezoning
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  with mesh rezoning
- Summary

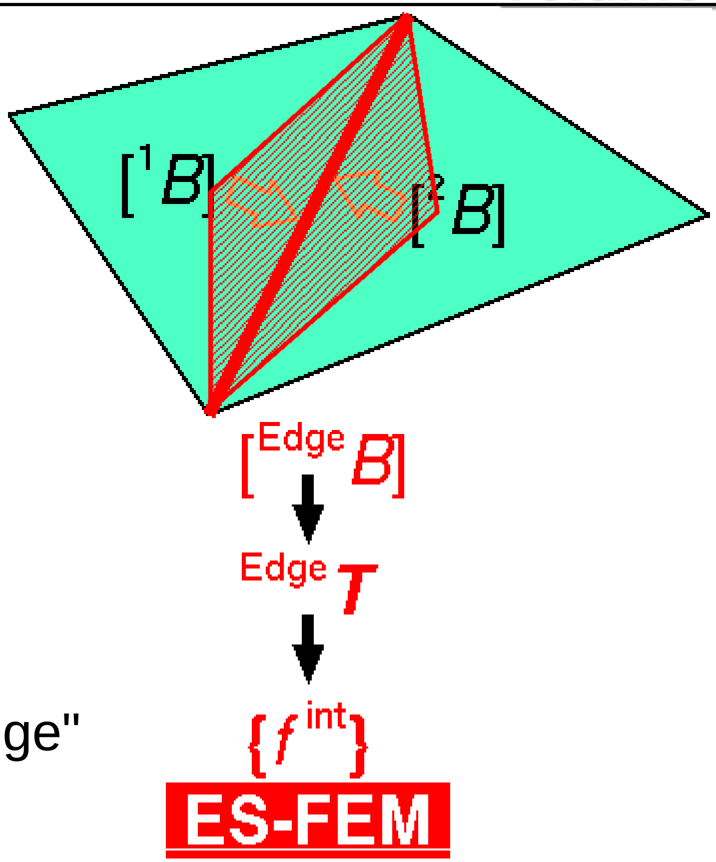
Part 1:

Introduction of Our *Modified* selective S-FEM without Mesh Rezoning

Review of Edge-based S-FEM (ES-FEM)

- Calculate $[B]$ at element as usual.
- Distribute $[B]$ to the connecting edges and make $[\,^{\mathrm{Edge}}B]$.
- ***F*, *T*** etc and $\{f^{\mathrm{int}}\}$ are calculated on smoothed edge domains.

Generally accurate but induces volumetric locking.



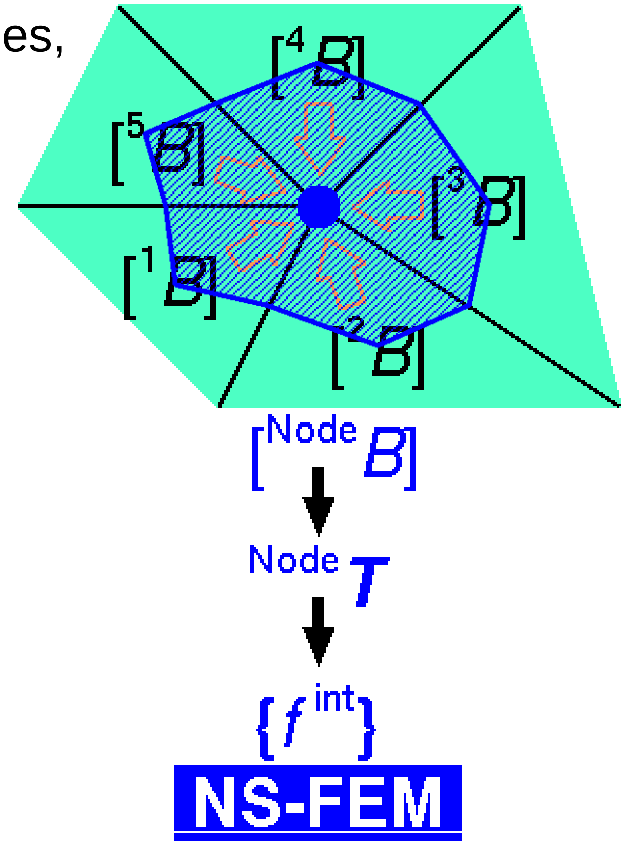
Substituting "face" for "edge" gives FS-FEM for 3D

Review of Node-based S-FEM (NS-FEM)

- Calculate $[B]$ at element as usual.
- Distribute $[B]$ to the connecting nodes and make $[{}^{\mathrm{Node}}B]$
- ***F*, *T*** etc and $\{f^{\mathrm{int}}\}$ are calculated on smoothed node domains.

Generally not accurate but volumetric locking free.

(due to zero-energy modes, which are arisen in reduced integration finite elements as hour-glass modes)

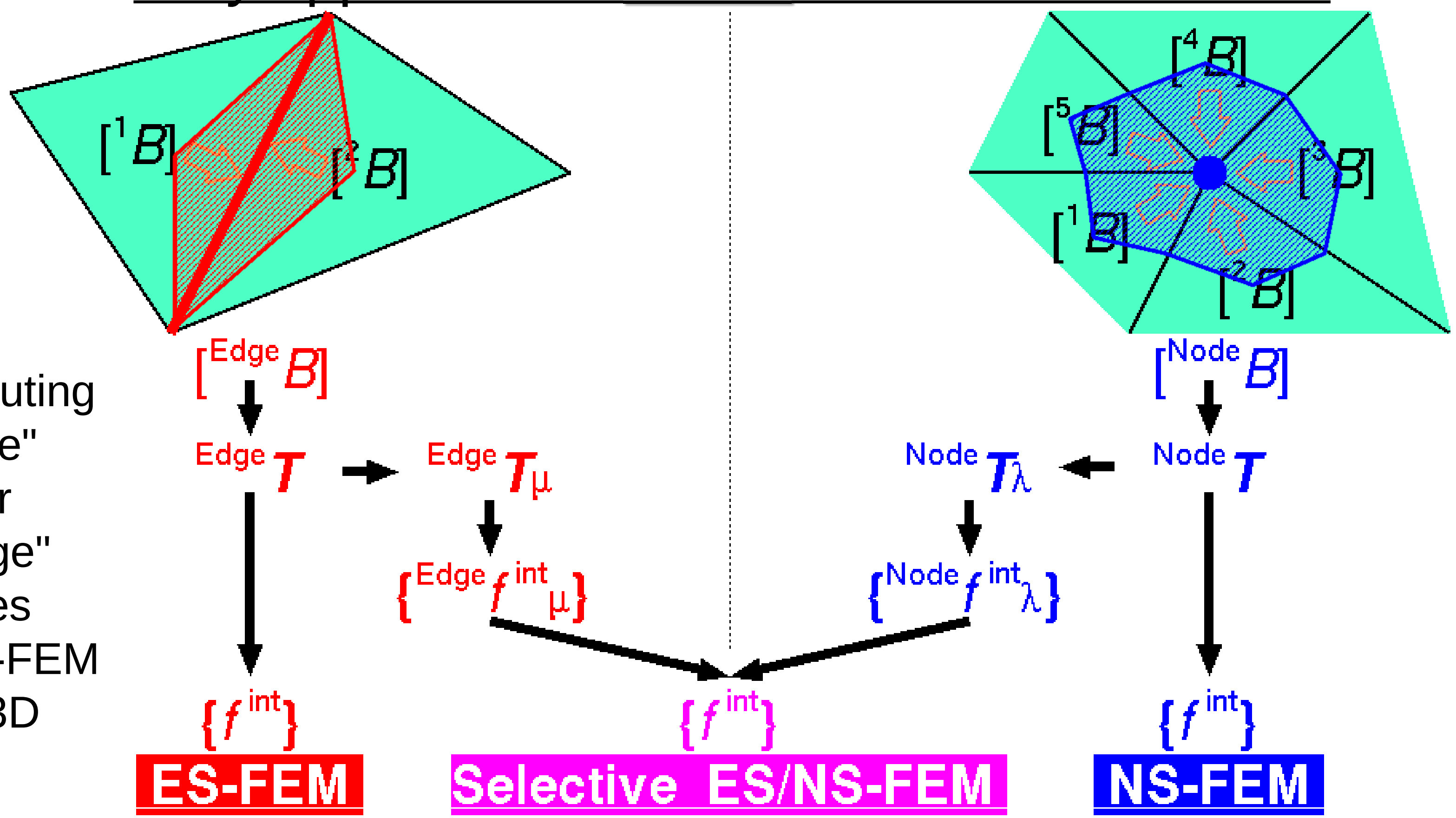


close to FVM with vertex-based control volume

Review of Selective ES/NS-FEM

- Separate stress into "μ part" and "λ part", where μ and λ are the Lame's parameters.
- ***F*, *T*** etc and {f^{int}} are calculated on both smoothed domains.

Only applicable to elastic constitutive models.



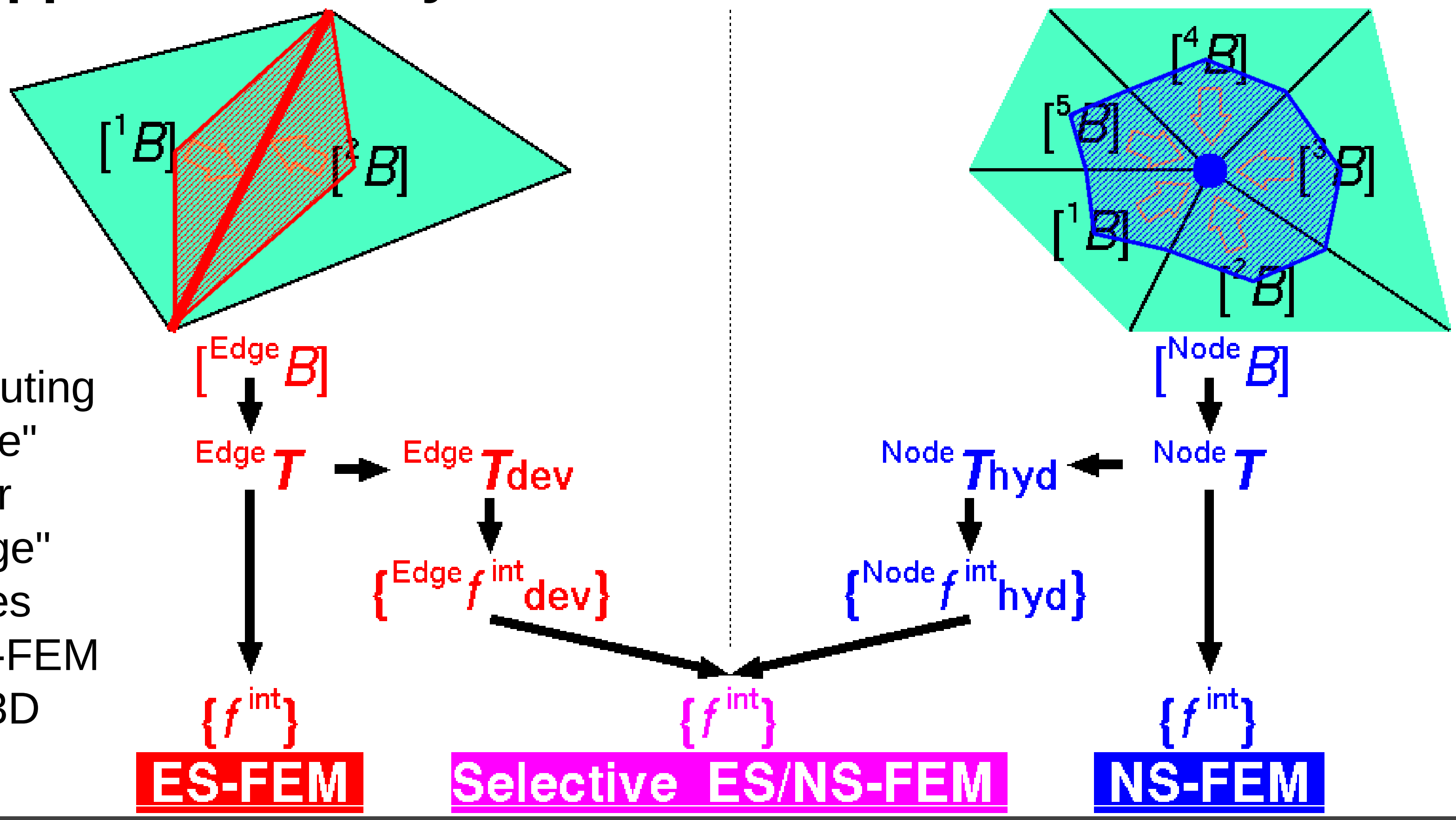
Substituting "face" for "edge" gives FS/NS-FEM for 3D

Our Modified Selective ES/NS-FEM

- Separate stress into "deviatoric part" and "hydrostatic part" instead of "μ part" and "λ part".
- ***F*, *T*** etc and $\{f^{\text{int}}\}$ are calculated on both smoothed domains.

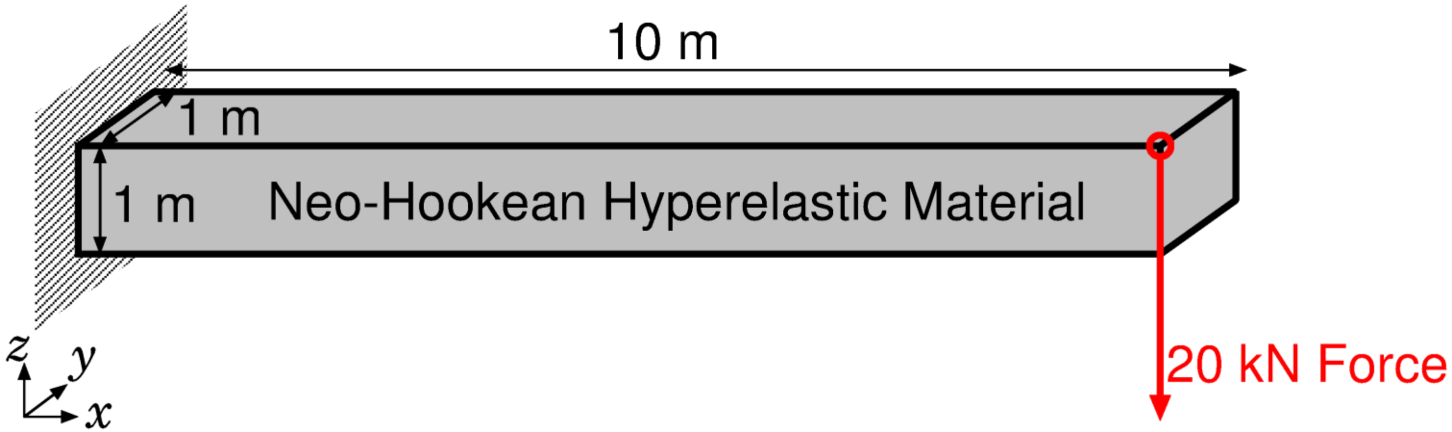
Applicable to any kind of material constitutive models.

Substituting "face" for "edge" gives FS/NS-FEM for 3D



Bending of Cantilever

Outline



- Neo-Hookean hyperelastic material

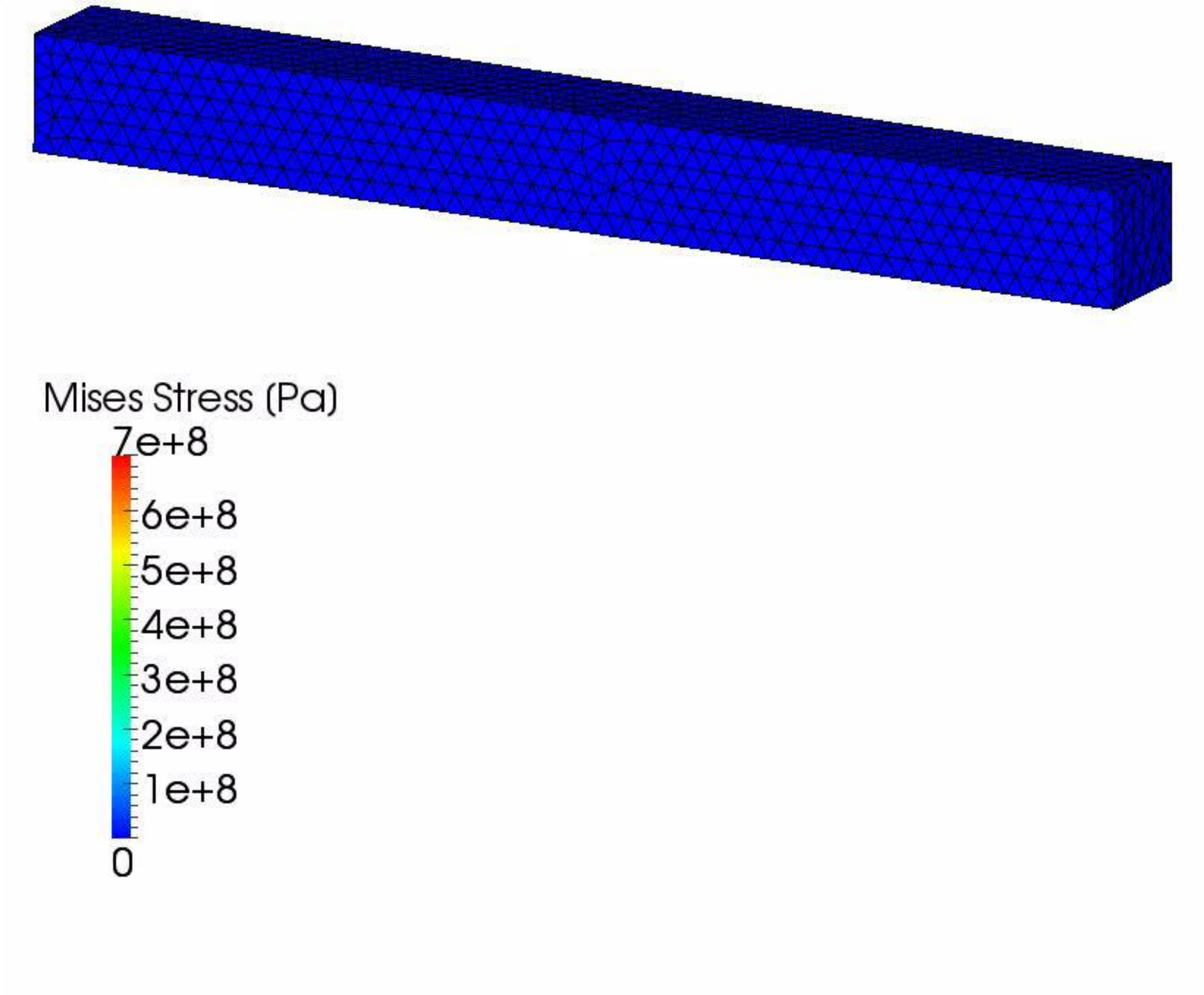
$$[T] = 2C_{10}\frac{\mathrm{Dev}(\bar{B})}{J} + \frac{2}{D_1}(J-1)[I]$$

with a constant C_{10}(=1 GPa) and various D_1s.

- Compared to ABAQUS/Standard with *C3D20H* (2nd-order hybrid hexahedral) elements.
- No mesh rezoning is taken place for this test.

Verification of Our Selective S-FEM

Results with $D_1 = 2 \times 10^{-15}$ [Pa^{-1}] (ν_{ini}=0.499999)

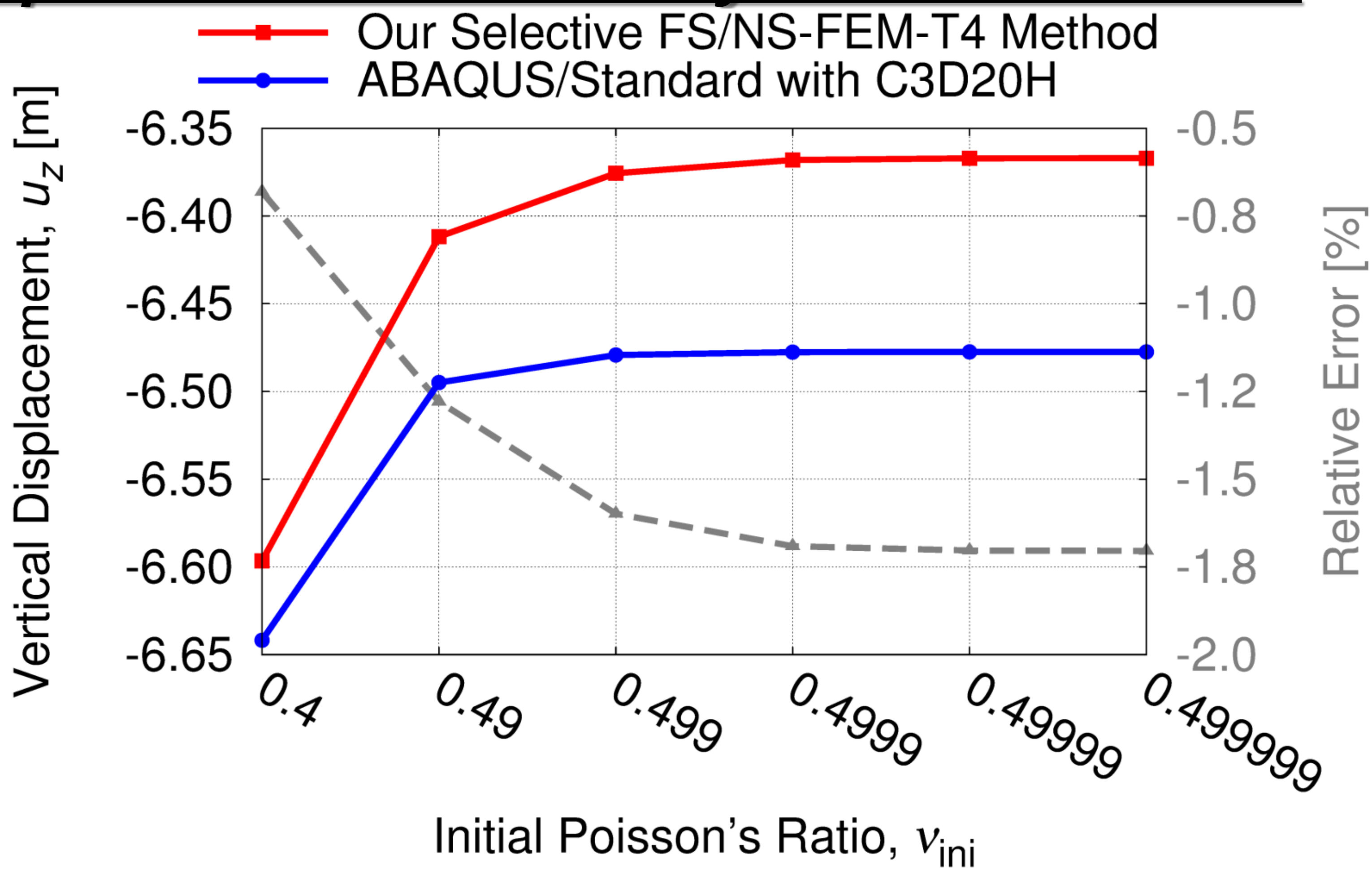


The amount of vertical deflection is about 6.5 m.

If we use constant strain tetrahedral, the amount of vertical deflection is about only 0.1 m.

Verification of Our Selective S-FEM

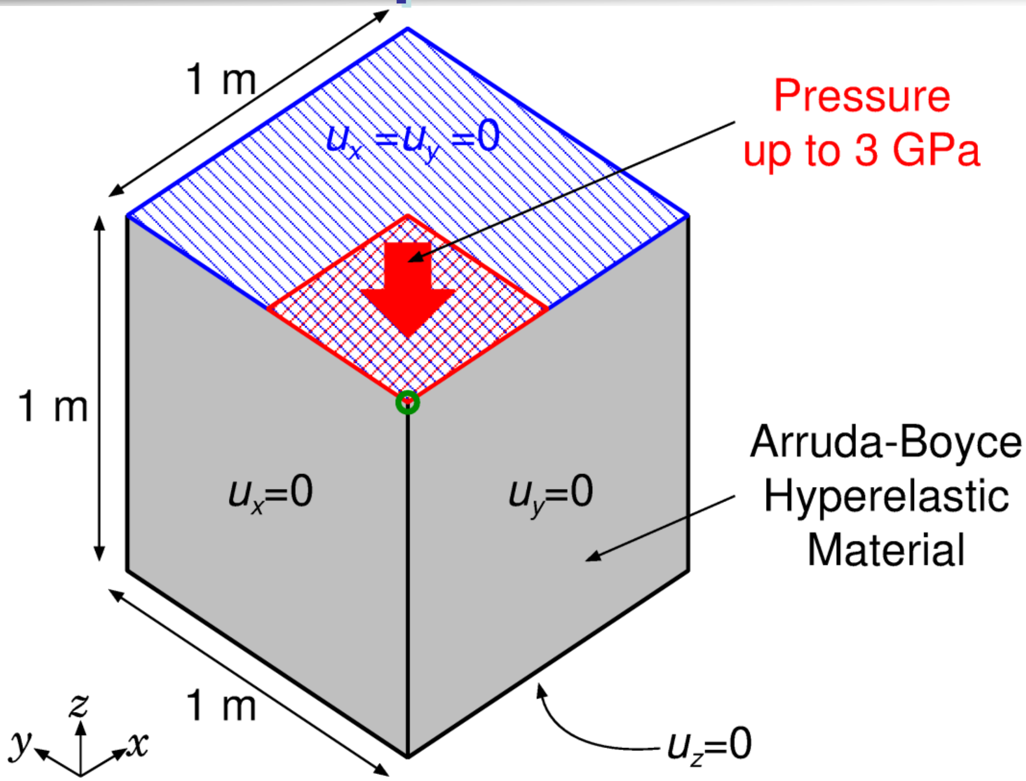
Comparison to 2nd-order Hybrid Hex Element



Our selective S-FEM is free from shear locking!!

Partial Compression of Block

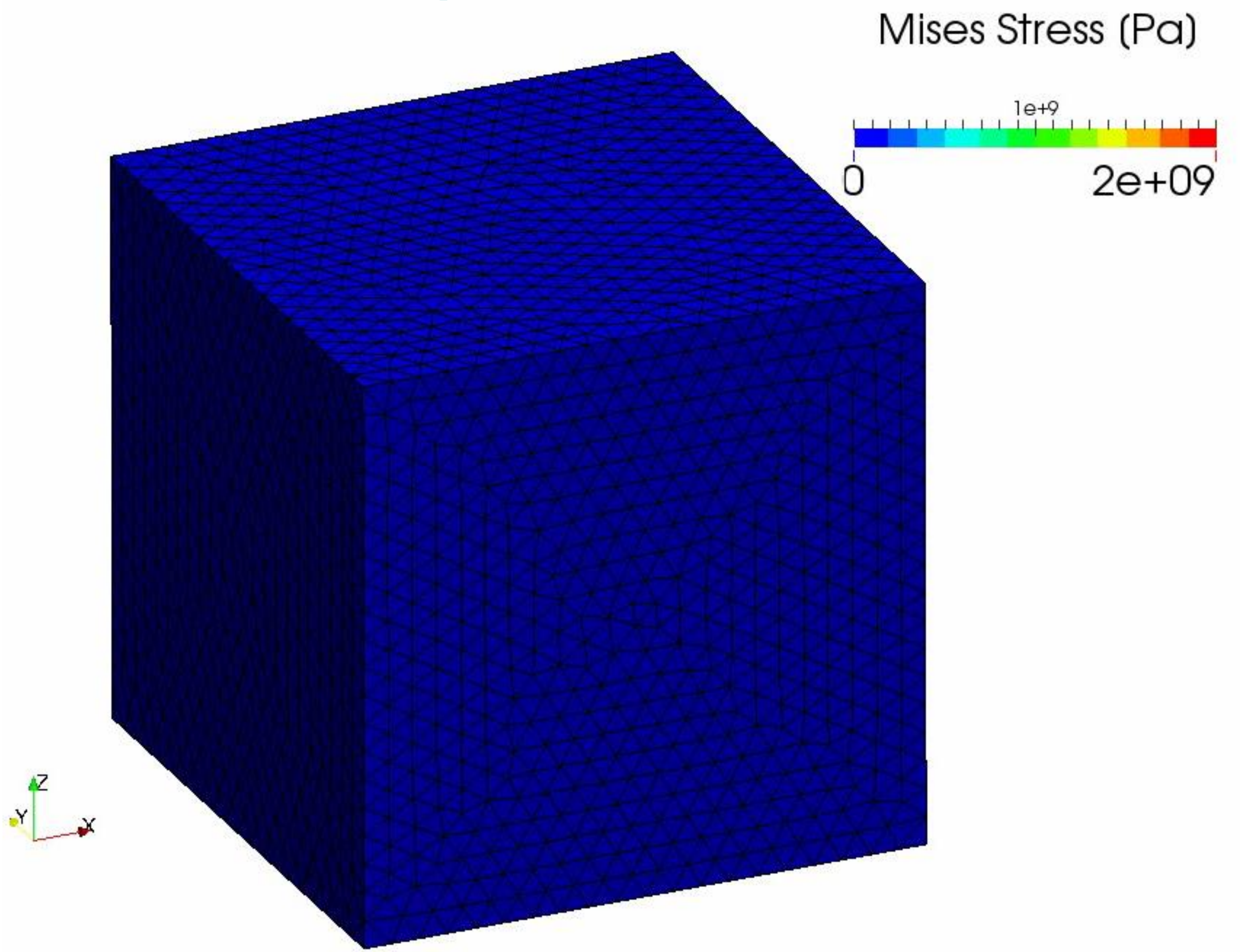
Outline



- Arruda-Boyce Hyper elastic Material with $\nu_{\mathrm{ini}} = 0.4999$
- Applying pressure on ¼ of the top face

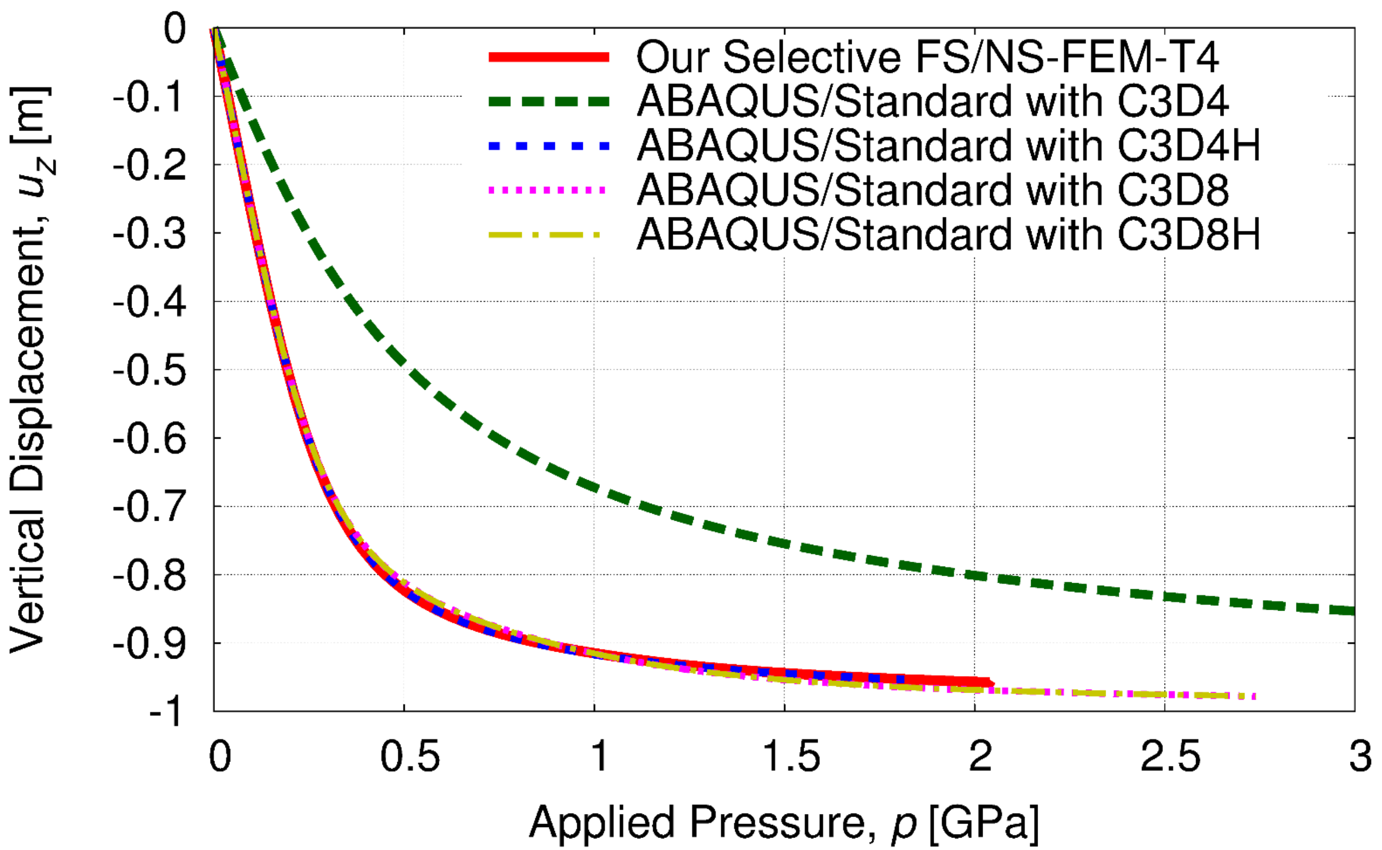
Partial Compression of Block

Result of our method



Partial Compression of Block

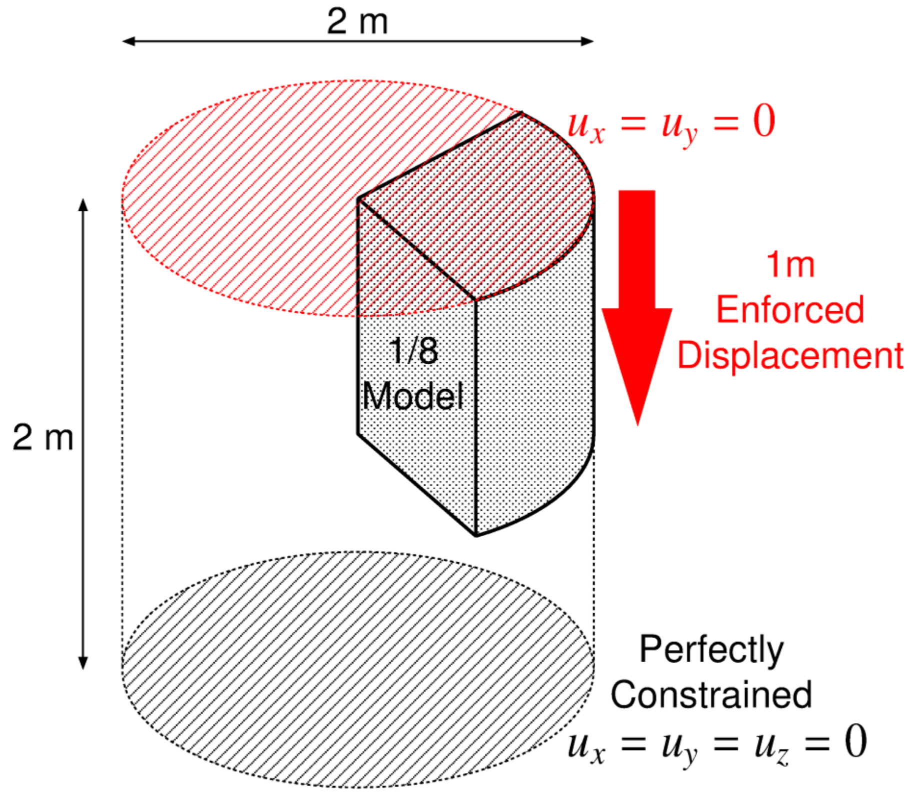
Vertical Displacements vs. Applied Pressure



- Constant strain element (C3D4) locks quickly.
- Other elements including our method do not lock.
- Result of our method is almost identical to that of C3D4H.

Compression of 1/8 Cylinder

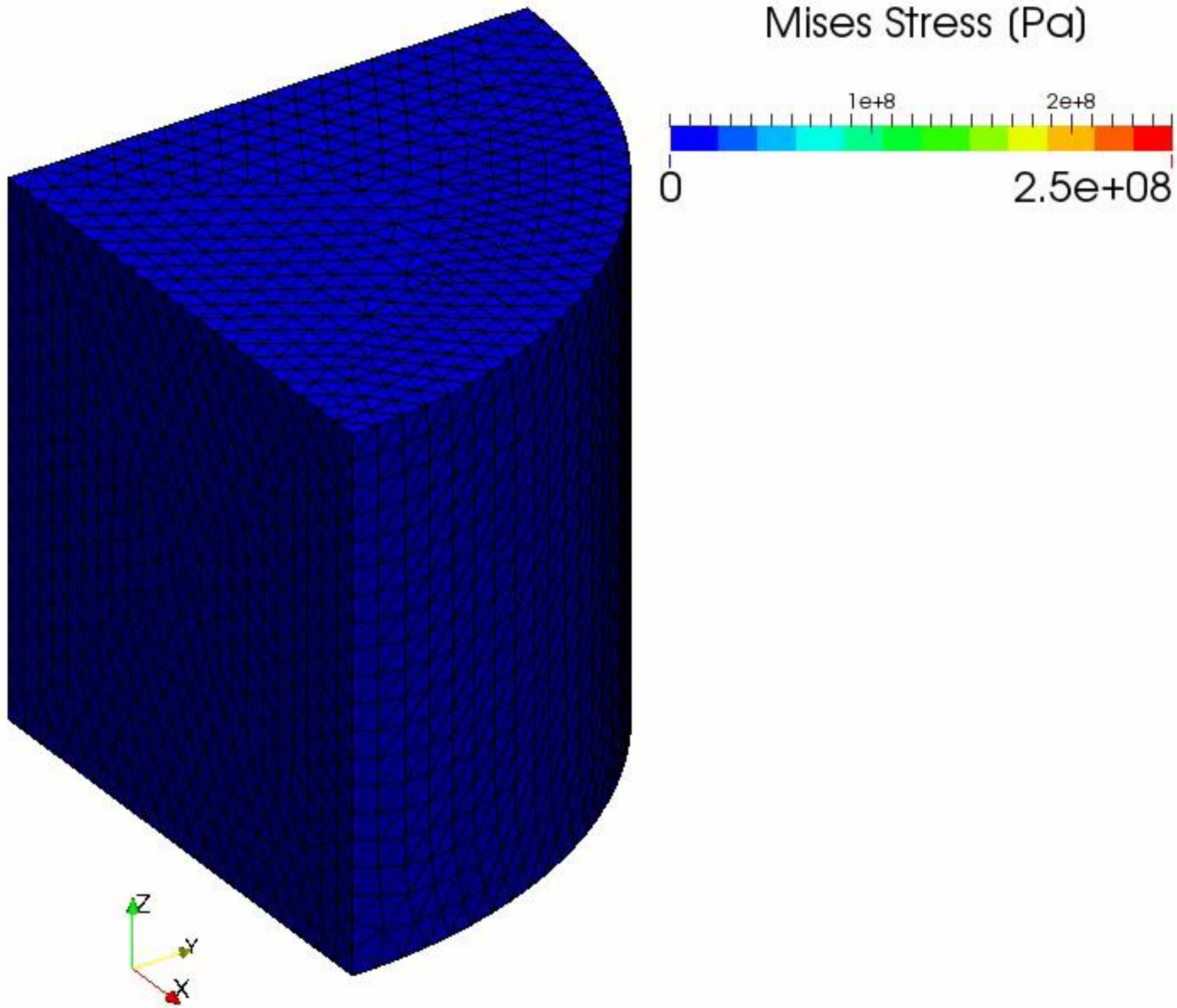
Outline



- 50% axial compression.
- Neo Hookean hyper elastic material of $C_{10} = 40 \times 10^6$ Pa, $D = 5 \times 10^{-12}\ \mathrm{Pa}^{-1}$ (i.e., $\nu_{\mathrm{ini}} = 0.4999$).
- Compared to C3D4H element of ABAQUS/Standard with exactly same mesh.

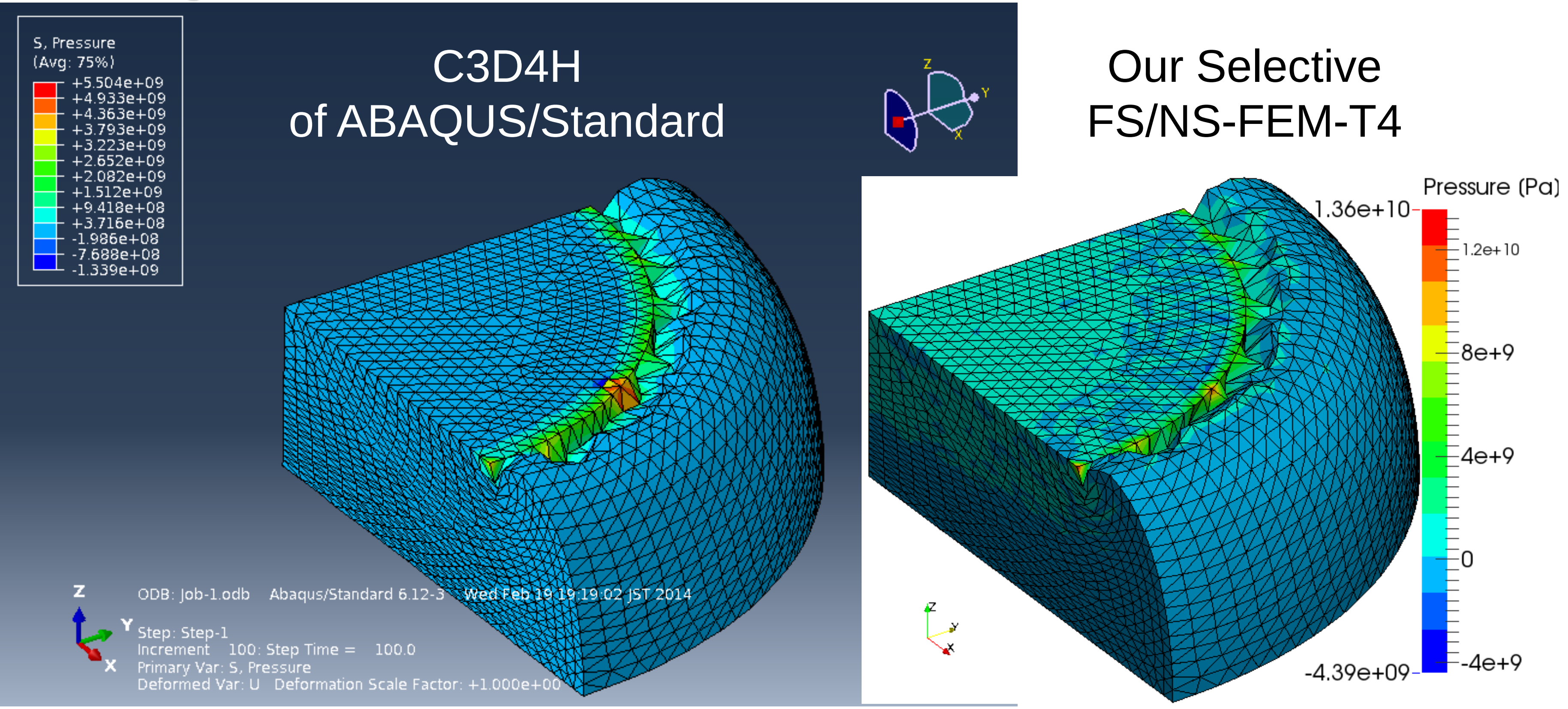
Compression of 1/8 Cylinder

Result of our method



Compression of 1/8 Cylinder

Comparison to ABAQUS

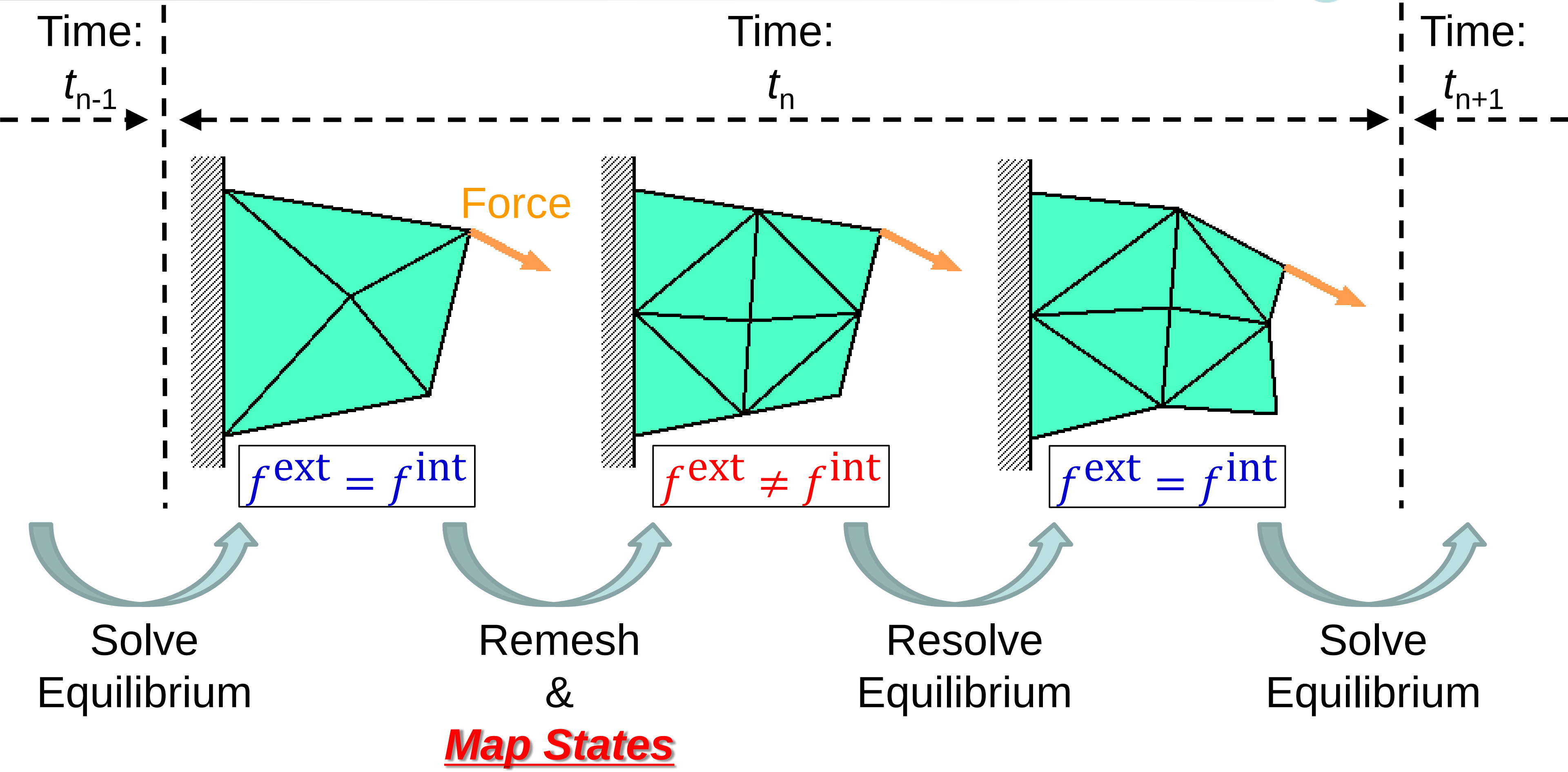


■ Deformation is almost the same each other.

■ Pressure oscillation is about double in our result.

Part 2:
Introduction of Our *Modified* selective S-FEM with Mesh Rezoning

Procedure of Mesh Rezoning



The way of mapping varies with the material constitutive model.
(e.g. Elasto-plastic models necessitate some kind of correction.)

Mapping of Stress/Strain States

For Elastic or Hyperelastic Materials

i.e., $[T] = [T([F])]$

- Map initial position $\{x^{\text{initial}}\}$ at nodes, and then remake deformation gradient $[F]$ at edges & nodes.

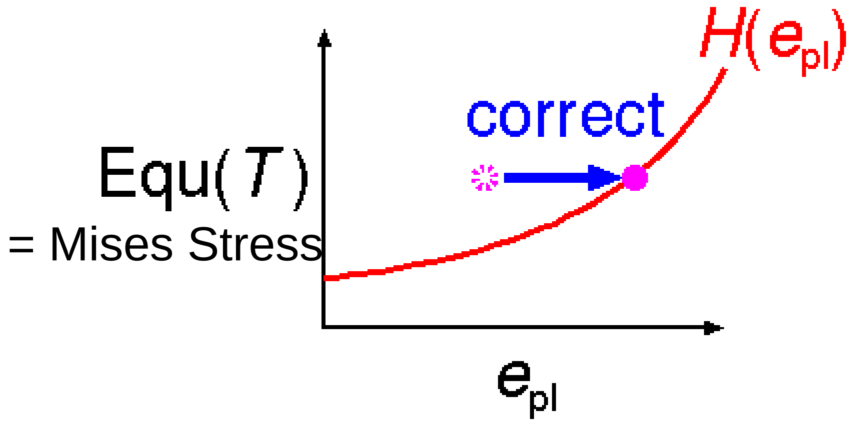
Each node preserve its initial position so that the domain can spring back to the initial shape after unloading.

Mapping of Stress/Strain States

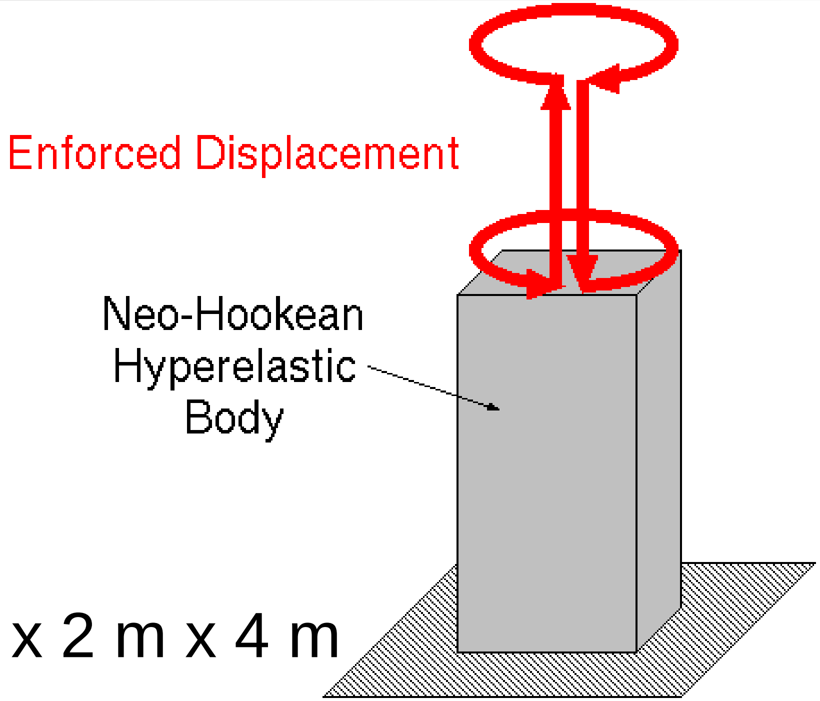
For Elasto-Plastic Material in Total Strain Form

$$\text{e.g., } [T] = \left[T\left([F], [E_{\mathrm{pl}}], e_{\mathrm{pl}}; H(e_{\mathrm{pl}})\right)\right]$$

- Map initial position $\{x^{\mathrm{initial}}\}$ at nodes, and then remake deformation gradient $[F]$ at edges & nodes.
- Map history dependent variables, plastic strain $[E_{\mathrm{pl}}]$ and equivalent plastic strain e_{pl}.
- Correct e_{pl} to satisfy $\mathrm{Equ}([T]) = H(e_{\mathrm{pl}})$



Twist and Stretch of Hyperelastic Body

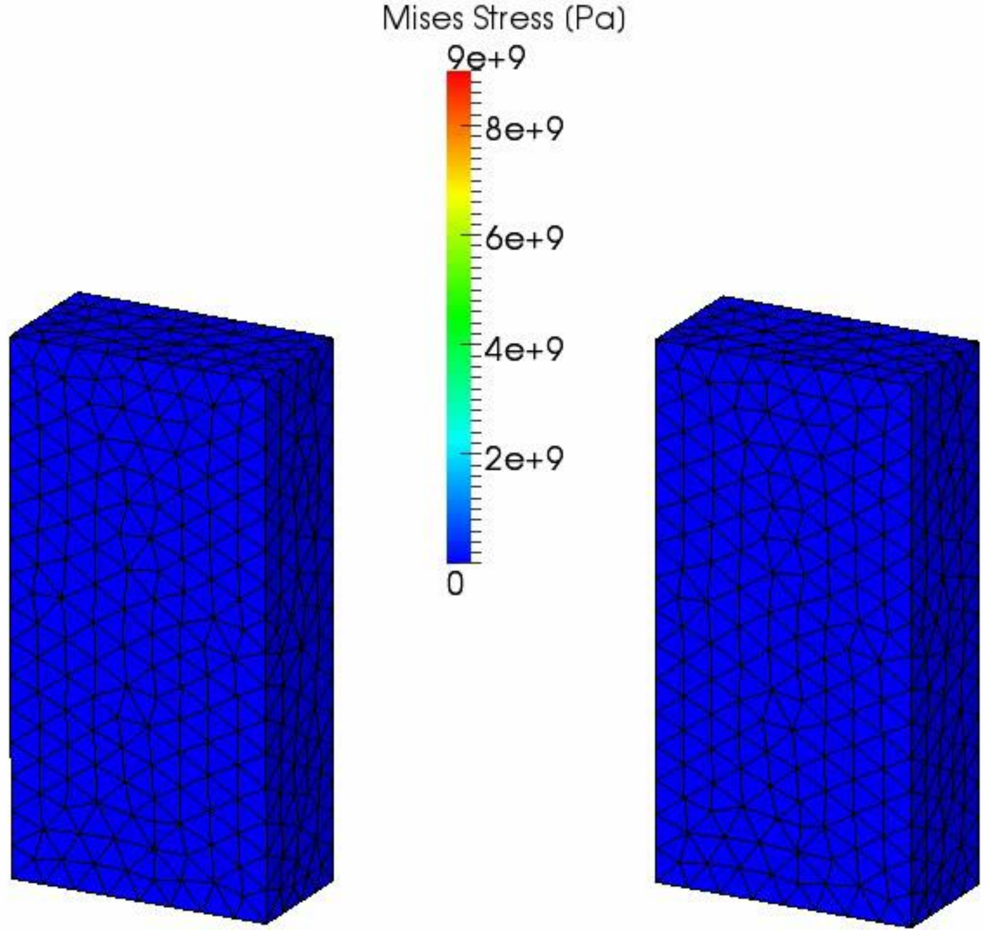


- Static, 1 m x 2 m x 4 m
- Neo-Hookean hyperelastic body of $C_{10} = 1\ \mathrm{GPa}$ and $D_1 = 400\ \mathrm{GPa}^{-1}$ ($\nu_0 = 0.48$)
- Twist up to 360 deg. ⟹ Stretch up to 100% nominal strain ⟹ Twist back ⟹ Shrink back
- Our selective FS/NS-FEM with tetrahedral elements
- Global mesh rezoning every 90 deg. and 50% stretch/shrink

Twist and Stretch of Hyperelastic Body

Our selective FS/NS-FEM **with** mesh rezoning

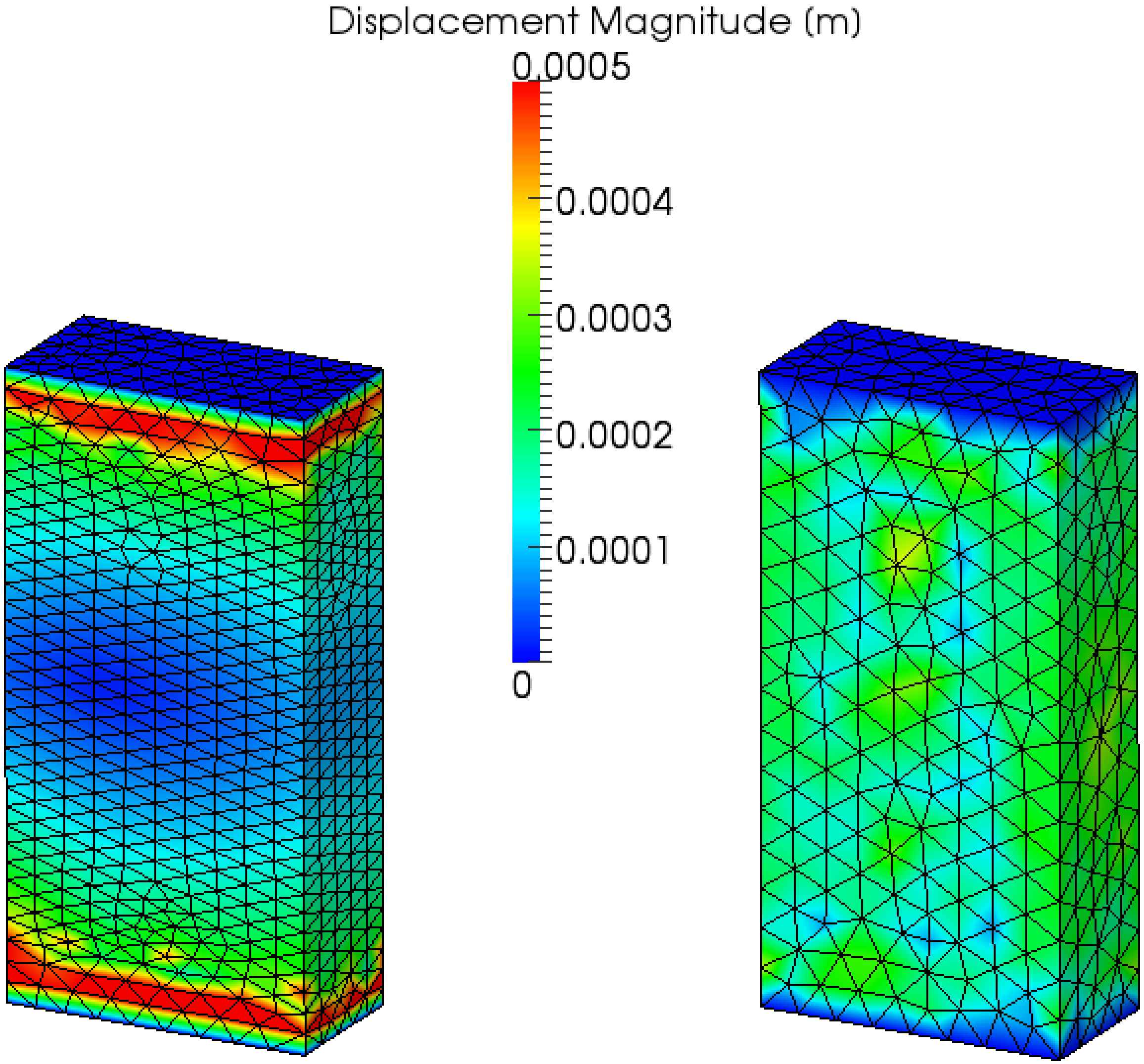
Our selective FS/NS-FEM **without** mesh rezoning



Twist and Stretch of Hyperelastic Body

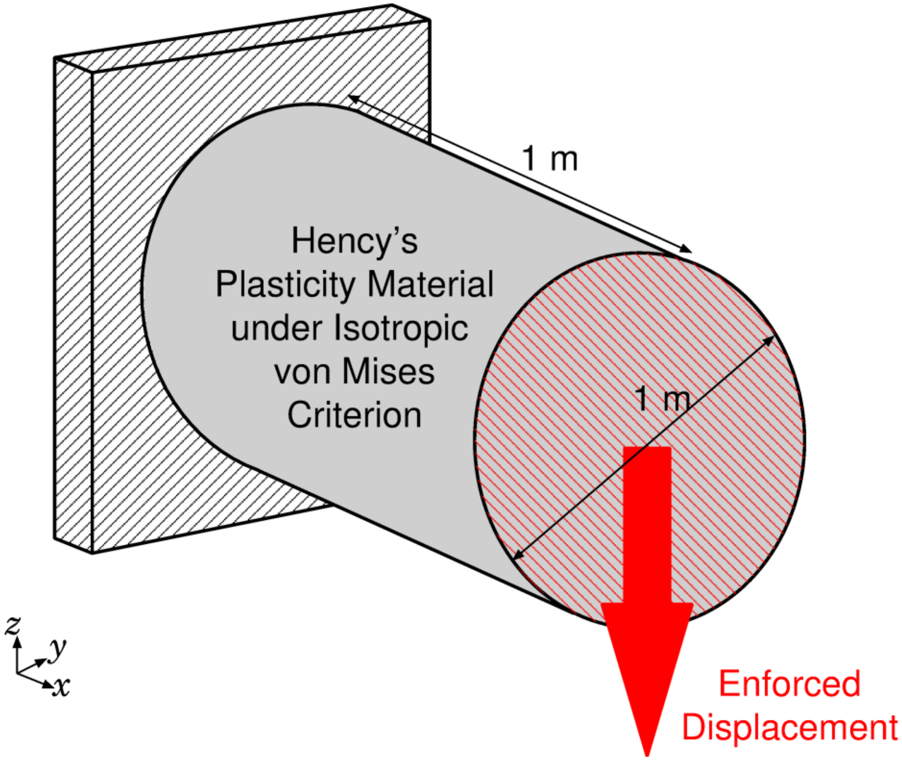
Residual Displacement

It spring backed almost perfectly.



Shearing and Necking of 3D Plastic Rod

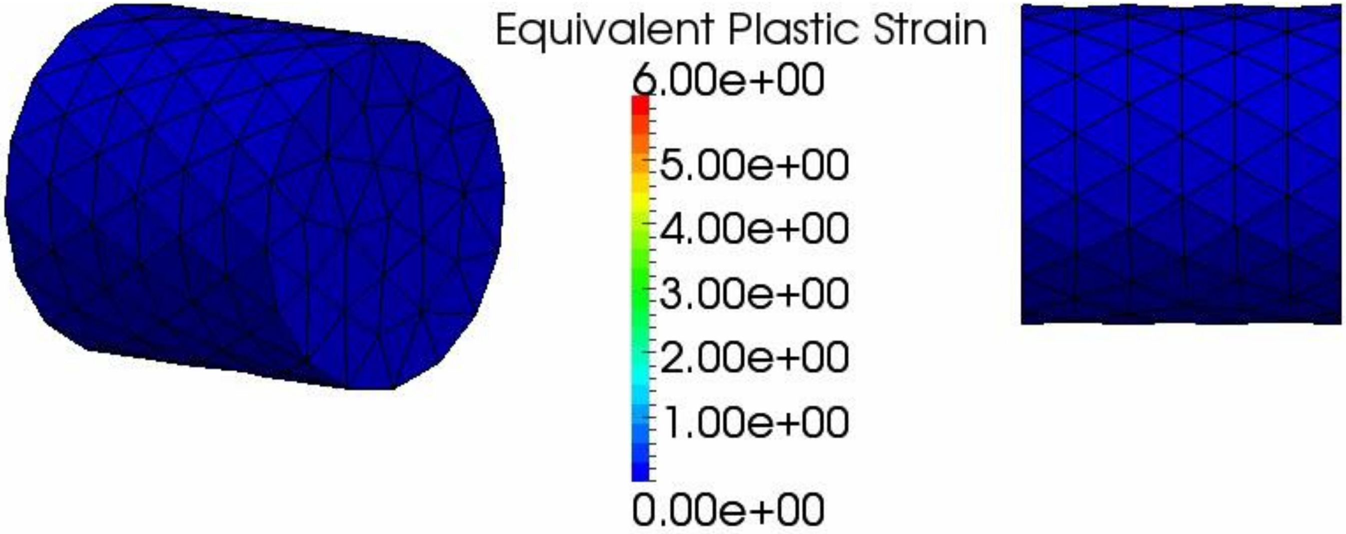
Outline



- Static, 3D
- Hencky's Plasticity Material
  with von Mises yield criterion and isotropic hardening.
  (same as 2D case)

Shearing and Necking of 3D Plastic Rod

3D Result



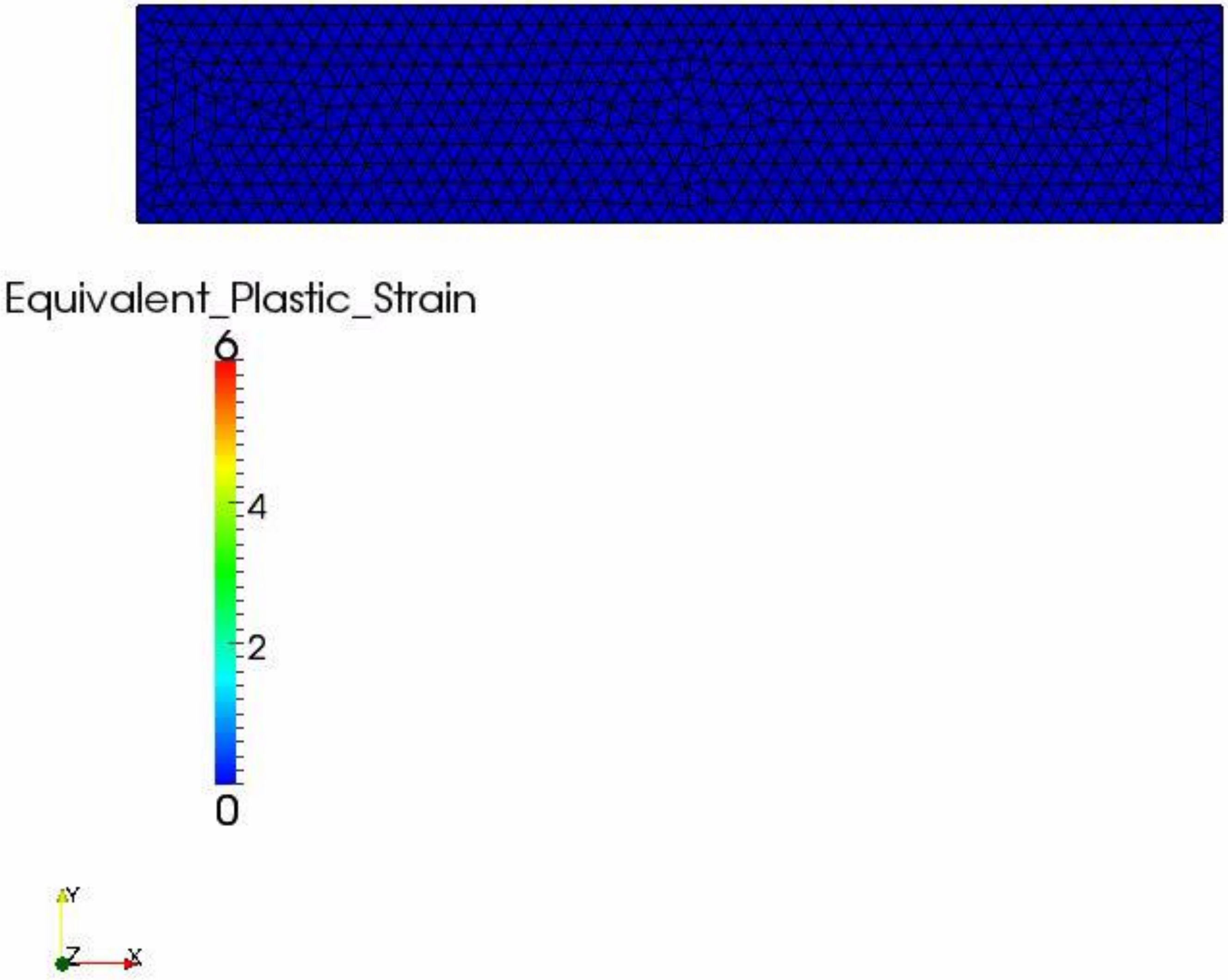
The deformation seems
to be valid.

After 2.8 m disp., mesh rezoning error occurred.

Shearing and Necking of 2D Plastic Bar

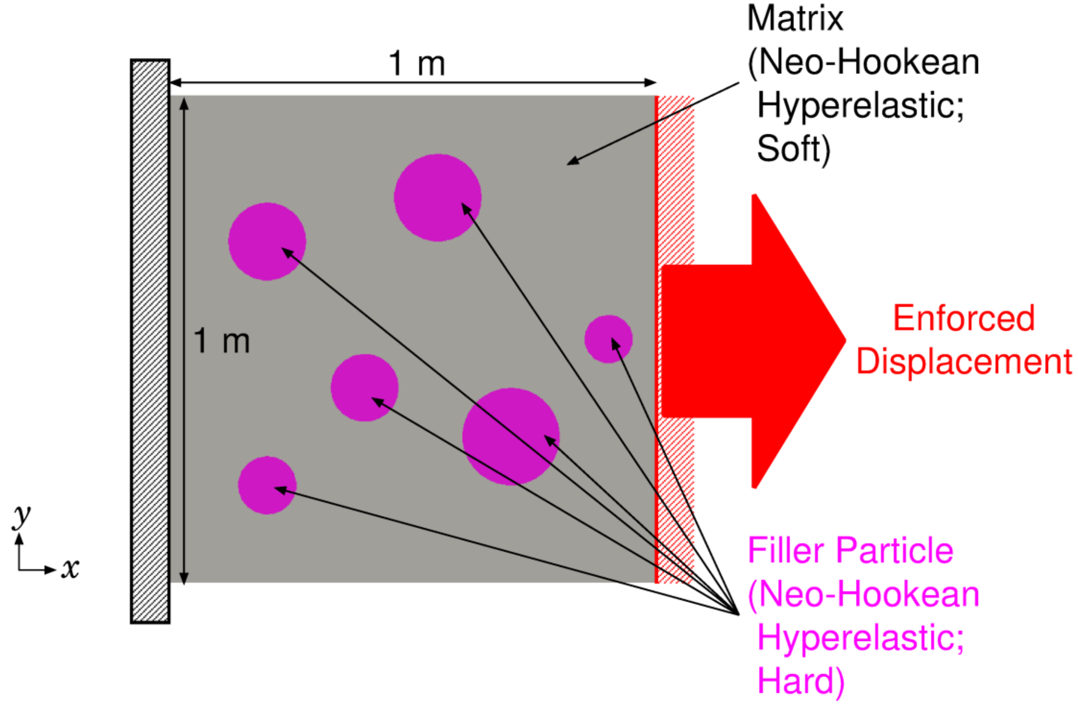
2D

Result



Tension of 2D Filler Particle Composite

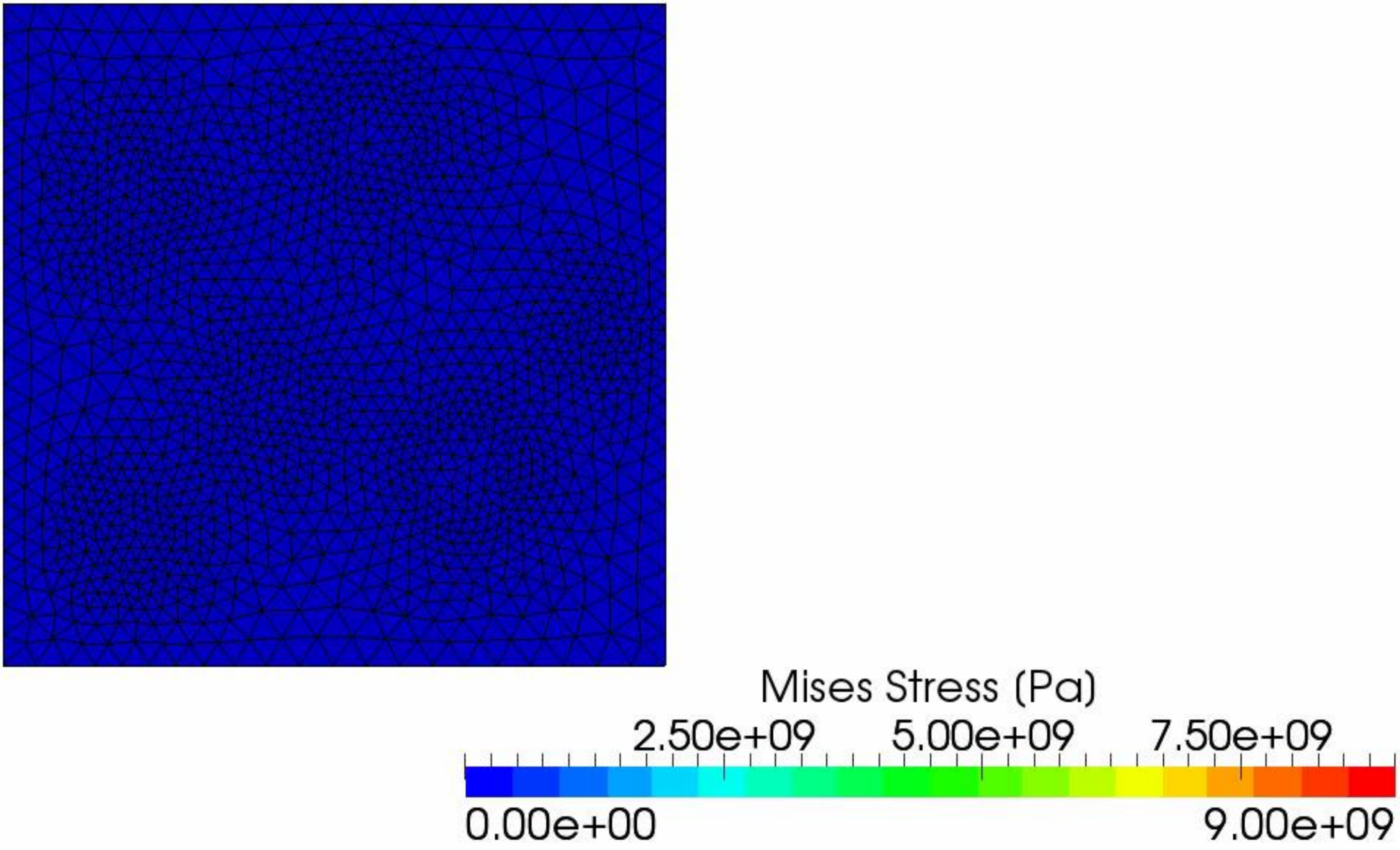
Outline



- Plane-strain static
- Neo-Hookean Hyperelastic
  - Filler: hard rubber $(E^{\mathrm{initial}} = 100\ \mathrm{GPa},\ \nu^{\mathrm{initial}} = 0.49)$
  - Matrix: soft rubber $(E^{\mathrm{initial}} = 1\ \mathrm{GPa},\ \nu^{\mathrm{initial}} = 0.49)$

Tension of 2D Filler Particle Composite

2D Result



The deformation seems to be valid.
After 1.8 m disp.,
analysis is stopped due to mesh rezoning error.

Summary

Take-Home Messages

1. Our modified selective S-FEM with triangular or tetrahedral elements is locking free and very easy to implement.

2. The accuracy of our method is almost the same as C3D4H of ABAQUS, which is one of the current best hybrid elements.

3. Our S-FEM goes well together with mesh rezoning.

Summary and Future Work

Summary

- A new static-implicit mesh rezoning method for severely large deformation analysis is proposed.
- It adopts our modified selective S-FEM, which separates stress into deviatoric part and hydrostatic part.
- Its accuracy are verified with hyperelastic material and elasto-plastic material.

Future Work

- Explicit dynamic simulation for safety engineering (e.g., car crash simulation)
- Local mesh rezoning
- Apply to contact forming, crack propagation, etc.

Thank you for your kind attention.
I appreciate your question in slow and easy English!!