Smoothed Finite Element Method

Based on Incremental Implicit Equilibrium Equation

for Large Deformation Mesh Rezoning Analysis

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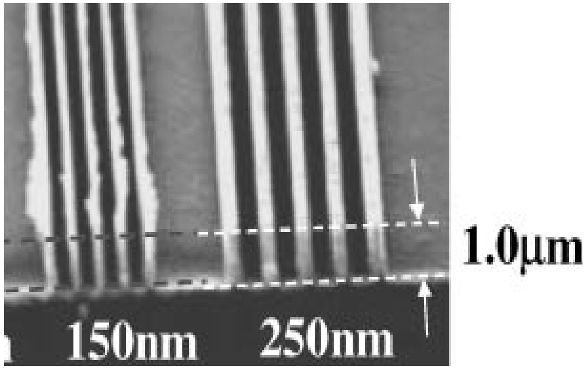
Motivation and Background

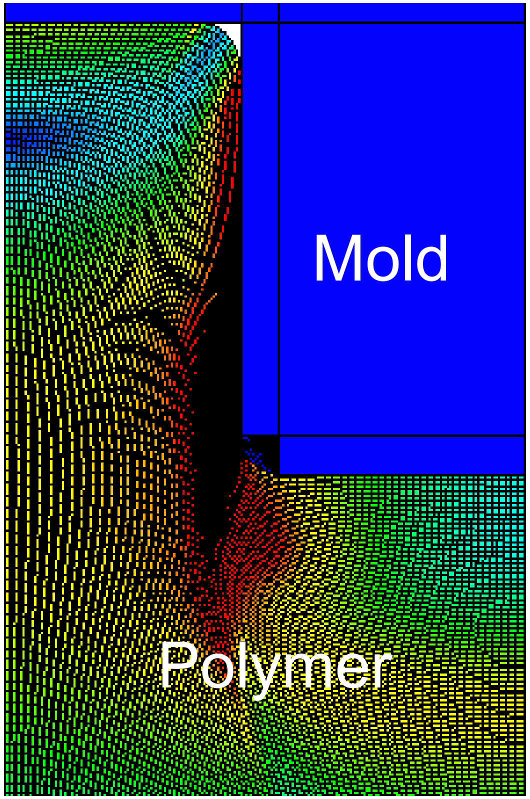
Motivation

We want to solve **severely large deformation** problems **accurately and stably**!
(Final target: thermal nanoimprinting)



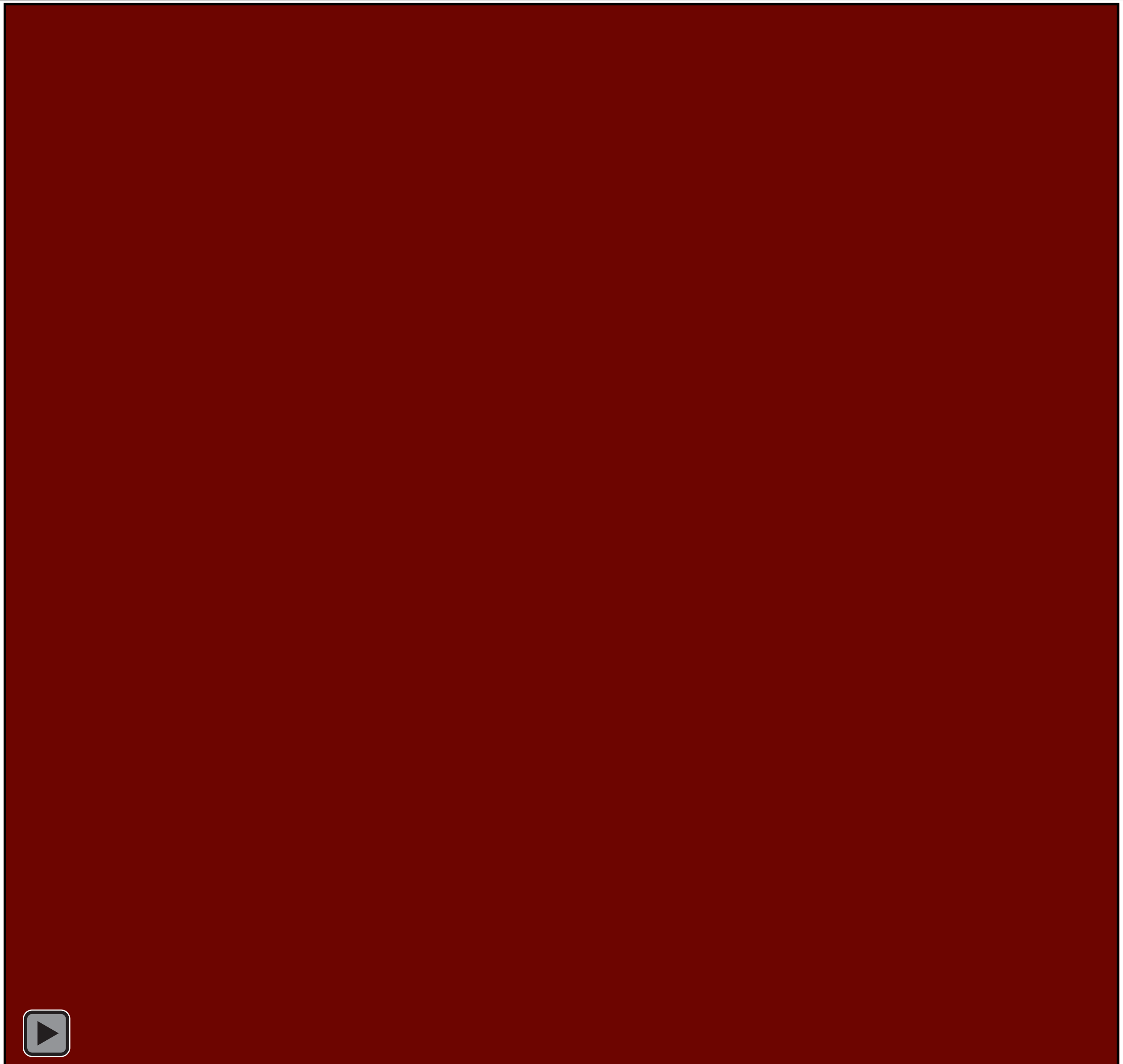
Background

Finite elements are **distorted** in a short time, thereby resulting in convergence failure.



Mesh rezoning method (*h*-adaptive mesh-to-mesh solution mapping) is indispensable.

Our First Result in Advance

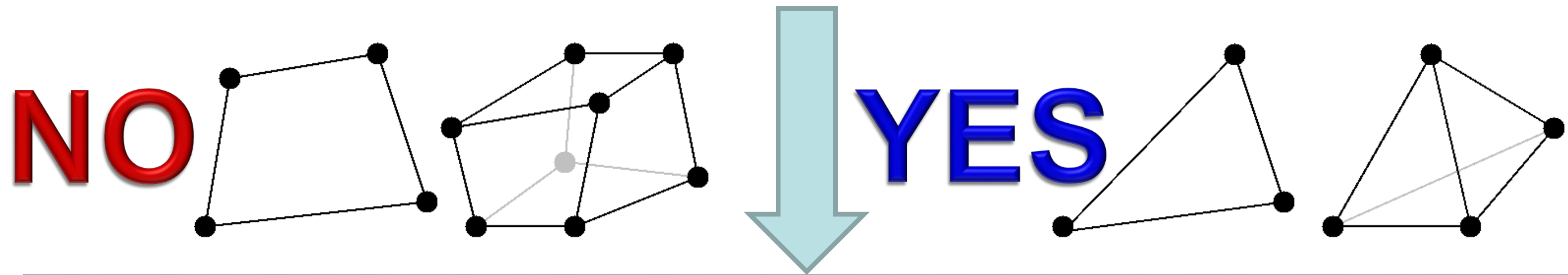


static-implicit
large deformation
analysis
with
mesh rezoning

Issues

The biggest issue
in large deformation mesh rezoning

It is impossible to remesh arbitrary deformed 2D or 3D domains with quadrilateral or hexahedral elements.



We have to use triangular or tetrahedral elements...

However, the *standard* (constant strain) triangular or tetrahedral elements induce shear and volumetric locking easily, which leads to inaccurate results.

Conventional Methods

- Higher order elements:
  - ✗ Not volumetric-locking-free; Not effective in large deformation due to intermediate nodes.
- EAS elements:
  - ✗ Unstable.

Can selective S-FEMs really be a solution?

My answer is "Let's try".

  - ✗ Difficult to construct patches
- u/p hybrid elements:
  - ✗ No sufficient formulation for triangular/tetrahedral is presented so far.

Objective

Develop a locking-free
modified selective S-FEM
for large deformation problems
with mesh rezoning

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- Part 2: Procedure of our mesh rezoning method
- Part 3: Examples of demonstrative analysis
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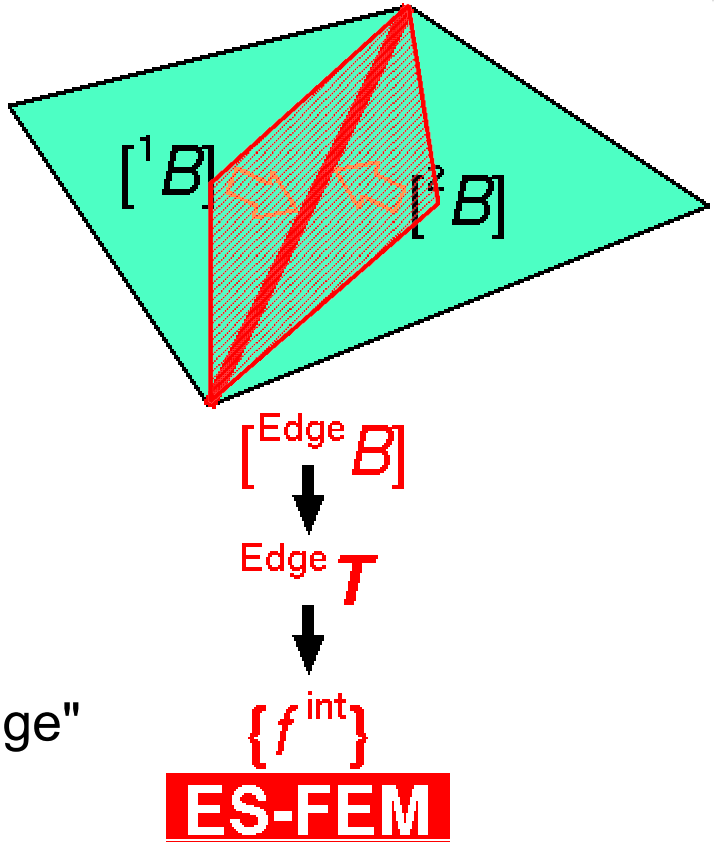
Part 1:

Introduction of Our *Modified* selective S-FEM Formulation

Review of Edge-based S-FEM (ES-FEM)

- Calculate $[B]$ at element as usual.
- Distribute $[B]$ to the connecting edges and make $[{}^{\mathrm{Edge}}B]$.
- ***F*, *T*** etc and $\{f^{\mathrm{int}}\}$ are calculated on smoothed edge domains.

Generally accurate but induces volumetric locking.



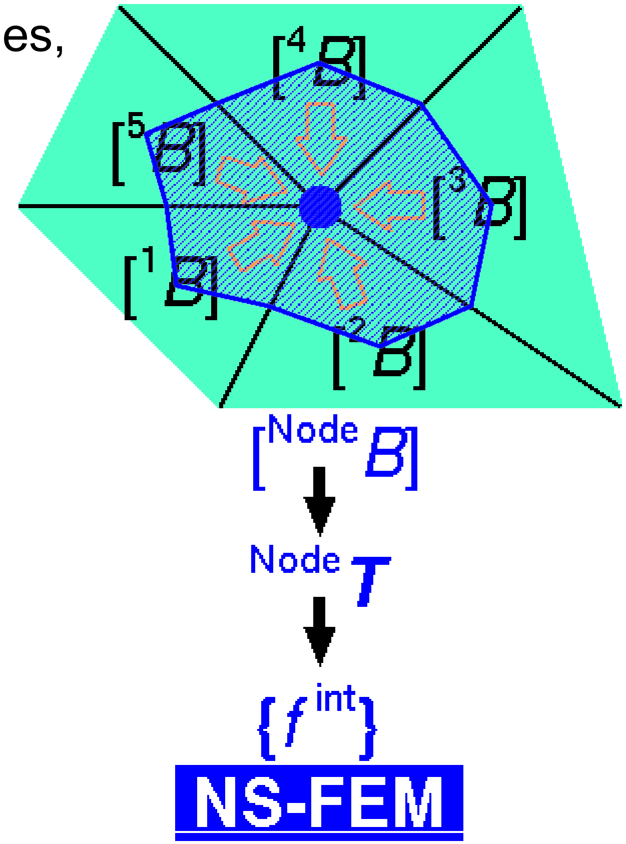
Substituting "face" for "edge" gives FS-FEM for 3D

Review of Node-based S-FEM (NS-FEM)

- Calculate $[B]$ at element as usual.
- Distribute $[B]$ to the connecting nodes and make $[{}^{\text{Node}}B]$
- ***F*, *T*** etc and $\{f^{\text{int}}\}$ are calculated on smoothed node domains.

Generally not accurate but volumetric locking free.

(due to zero-energy modes, which are arisen in reduced integration finite elements as hour-glass modes)



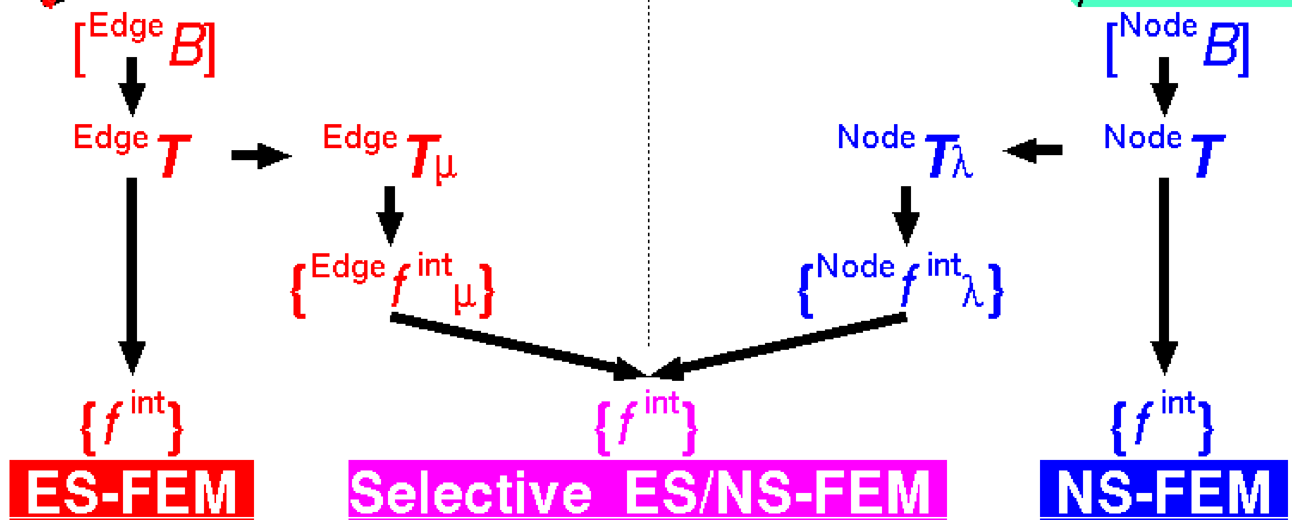
close to FVM with vertex-based control volume

Review of Selective ES/NS-FEM

- Separate stress into "μ part" and "λ part", where μ and λ are the Lame's parameters.
- ***F*, *T*** etc and $\{f^{\text{int}}\}$ are calculated on both smoothed domains.

Only applicable to elastic constitutive models.

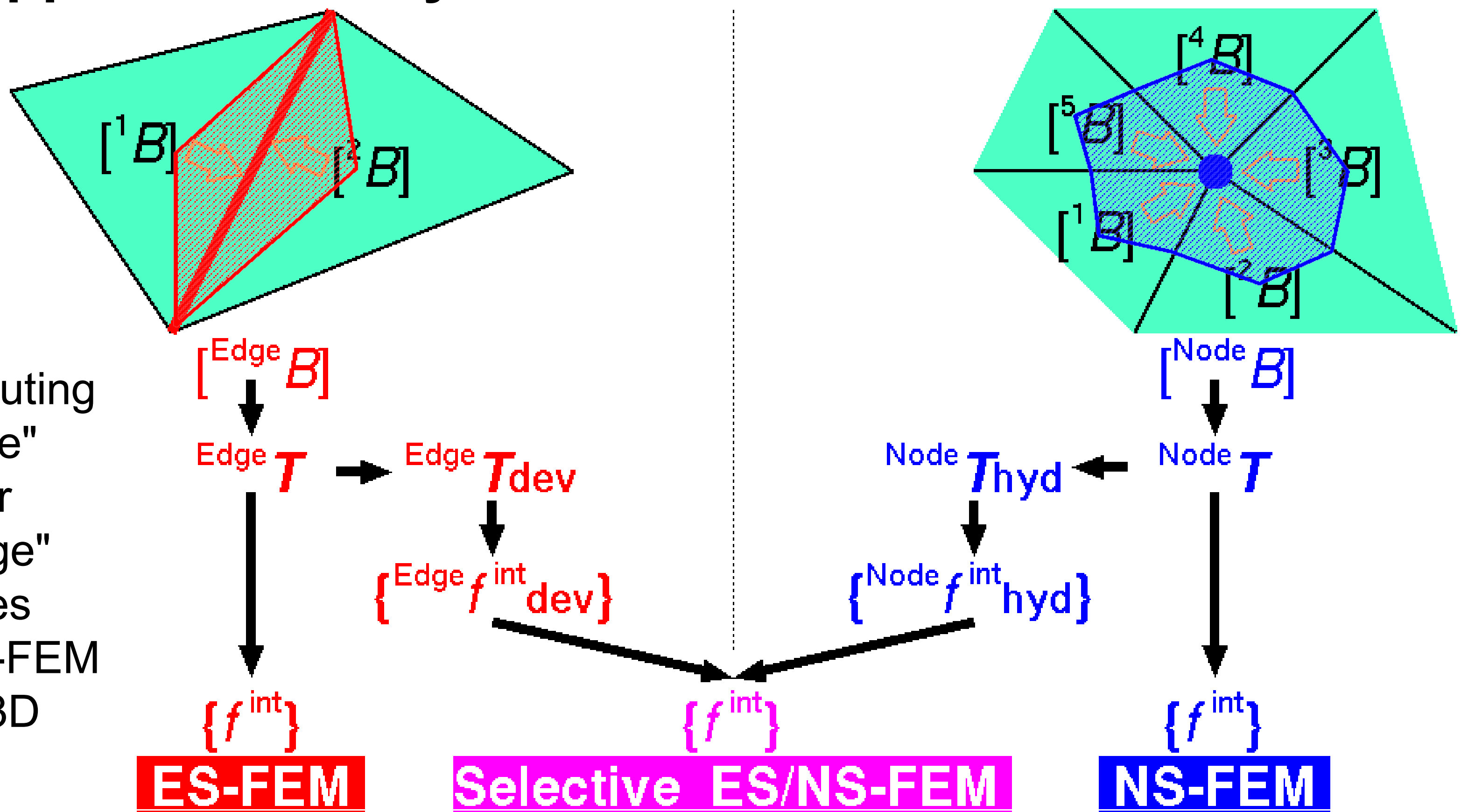
Substituting "face" for "edge" gives FS/NS-FEM for 3D



Our Modified Selective ES/NS-FEM

- Separate stress into "deviatoric part" and "hydrostatic part" instead of "μ part" and "λ part".
- ***F*, *T*** etc and $\{f^{\text{int}}\}$ are calculated on both smoothed domains.

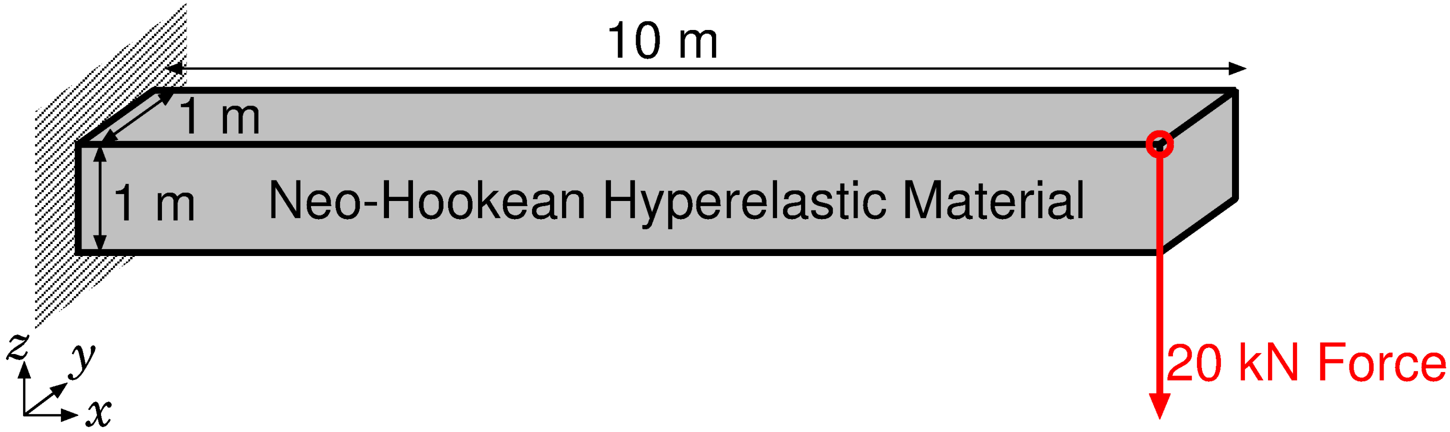
Applicable to any kind of material constitutive models.



Substituting "face" for "edge" gives FS/NS-FEM for 3D

Verification of Our Selective S-FEM

(1) Cantilever Bending Test



- Neo-Hookean hyperelastic material

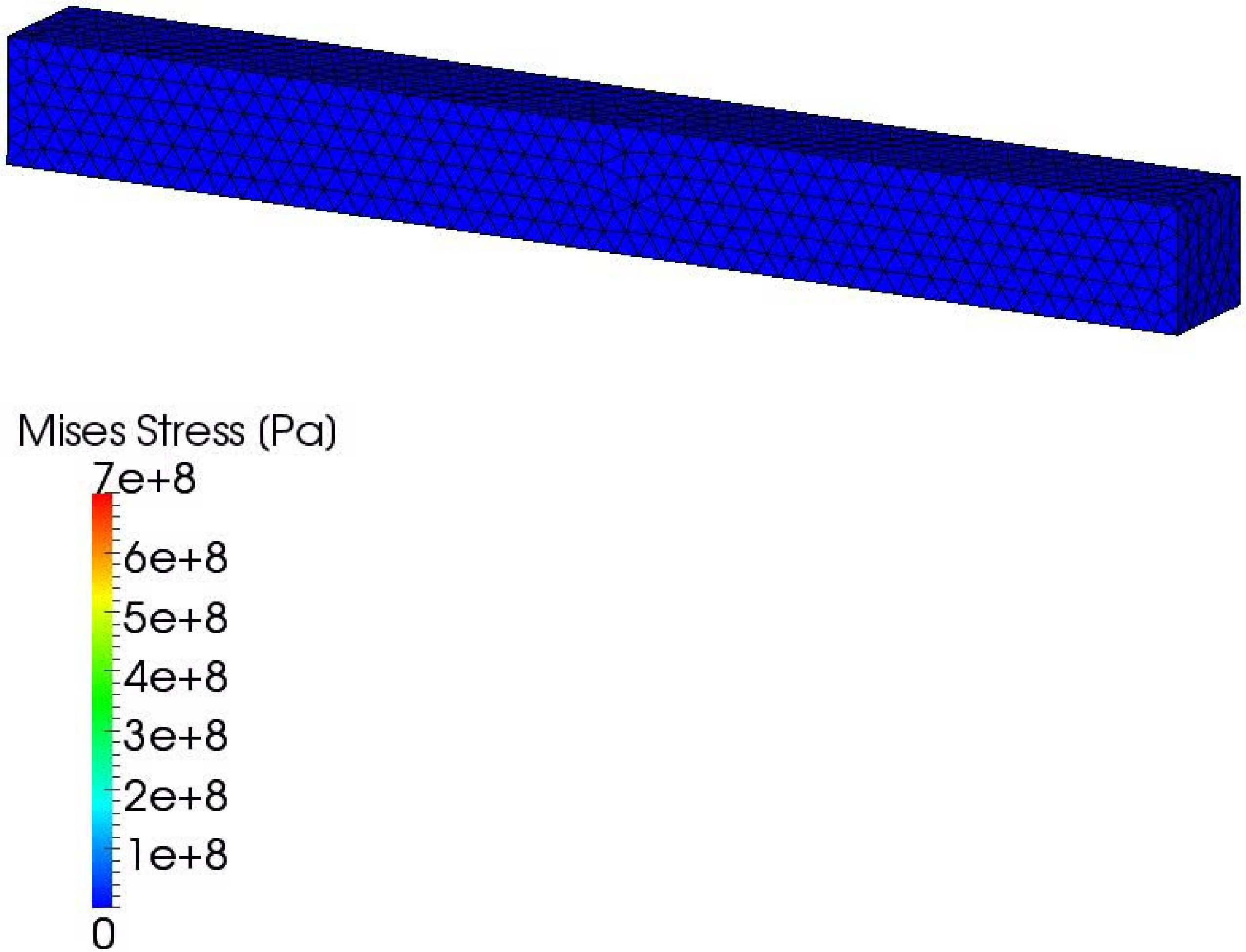
$$[T] = 2C_{10}\frac{\mathrm{Dev}(\bar{B})}{J} + \frac{2}{D_1}(J-1)[I]$$

with a constant C_{10}(=1 GPa) and various D_1s.

- Compared to ABAQUS/Standard with *C3D20H* (2nd-order hybrid hexahedral) elements.
- No mesh rezoning is taken place for this test.

Verification of Our Selective S-FEM

Results with $D_1 = 2 \times 10^{-15}$ $[\mathrm{Pa}^{-1}]$ (ν_{ini}=0.499999)

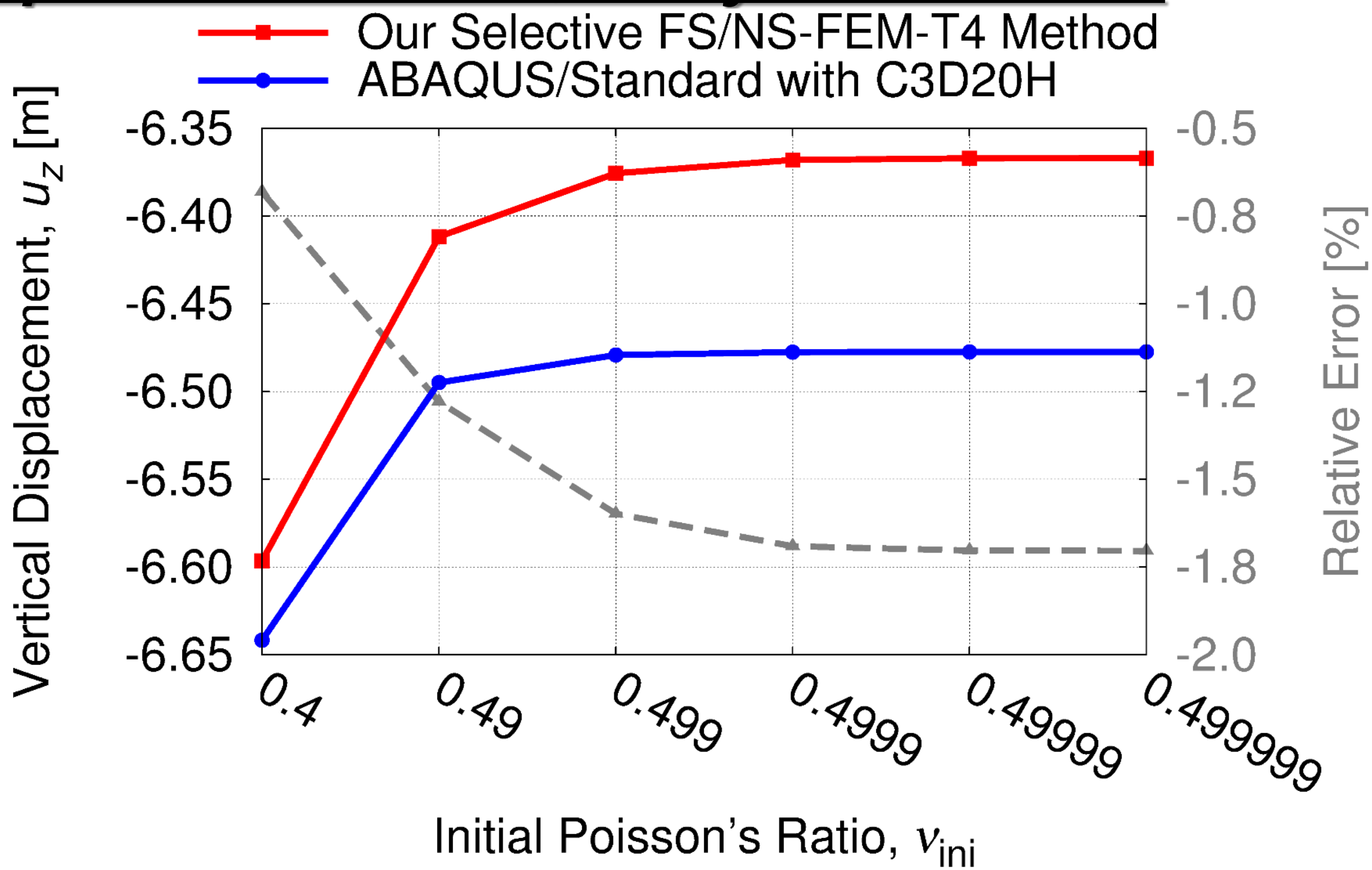


The amount of vertical deflection is about 6.5 m.

If we use constant strain tetrahedral, the amount of vertical deflection is about only 0.1 m.

Verification of Our Selective S-FEM

Comparison to ABAQUS Hybrid Element



Our selective S-FEM is free from shear locking!!

Verification of Our Selective S-FEM

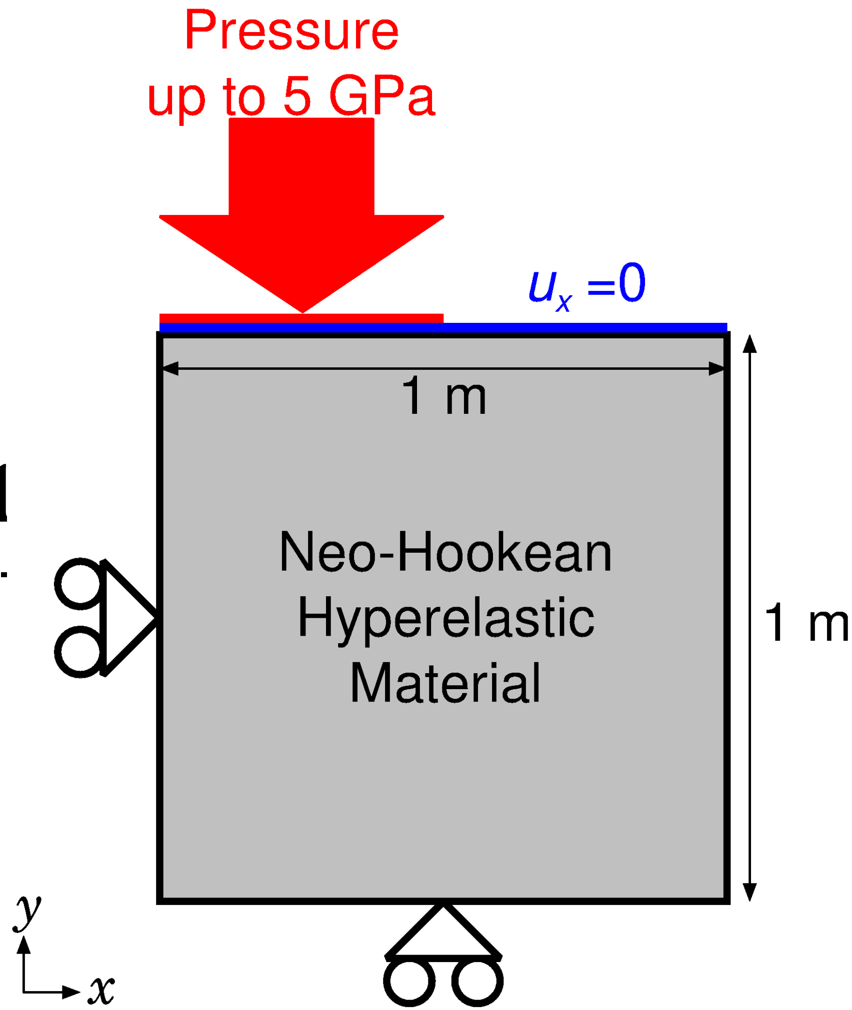
(2) Block Pressing Test

- Plane-strain condition
- Neo-Hookean hyperelastic material with

$$C_{10} = 40 \times 1$$
$$D_1 = 5 \times 10$$

$(\nu_{\mathrm{ini}} = 0.4999)$

- Pressure is applied to the top-left part up to 5 GPa

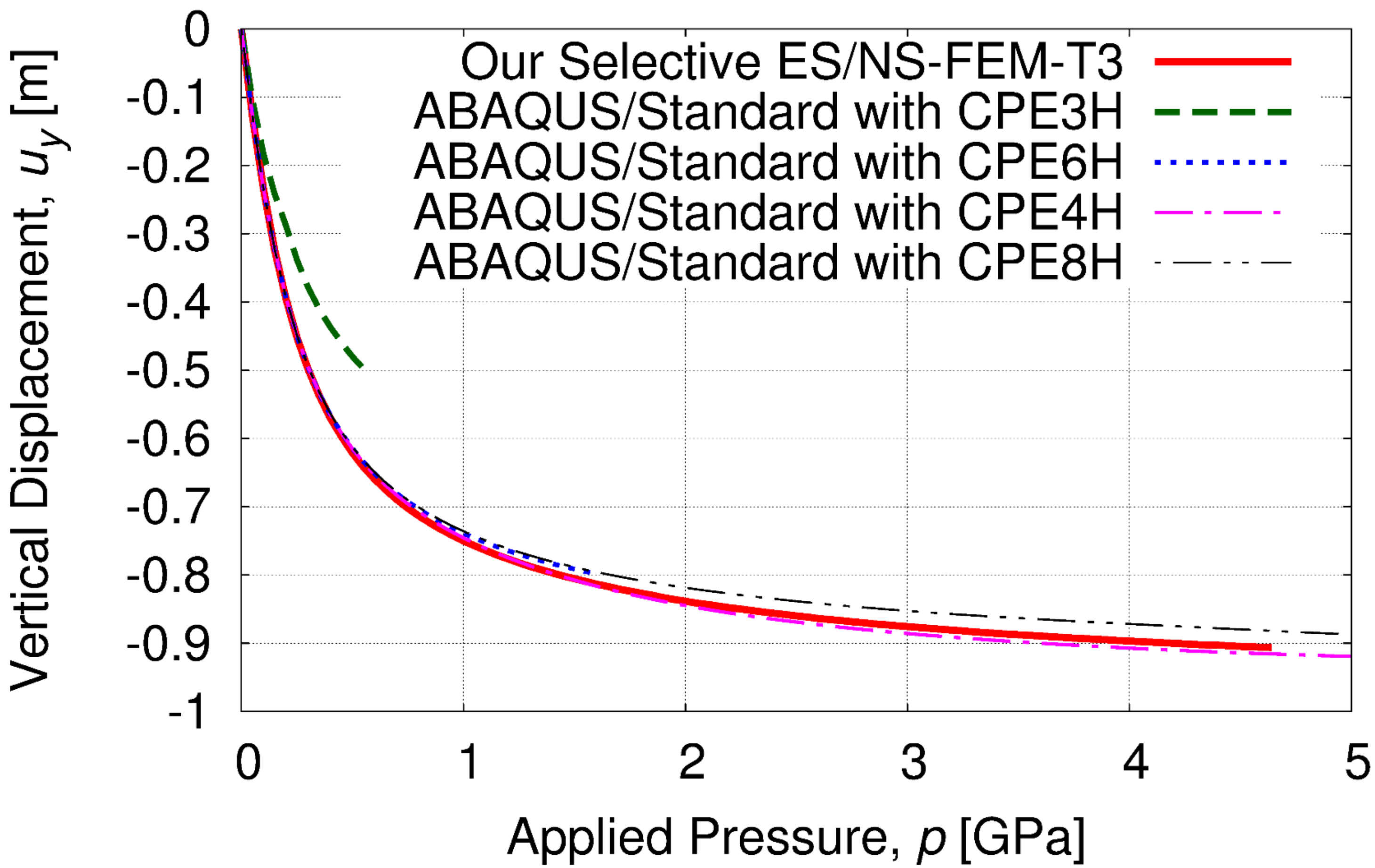


Verification of Our Selective S-FEM

Results

Verification of Our Selective S-FEM

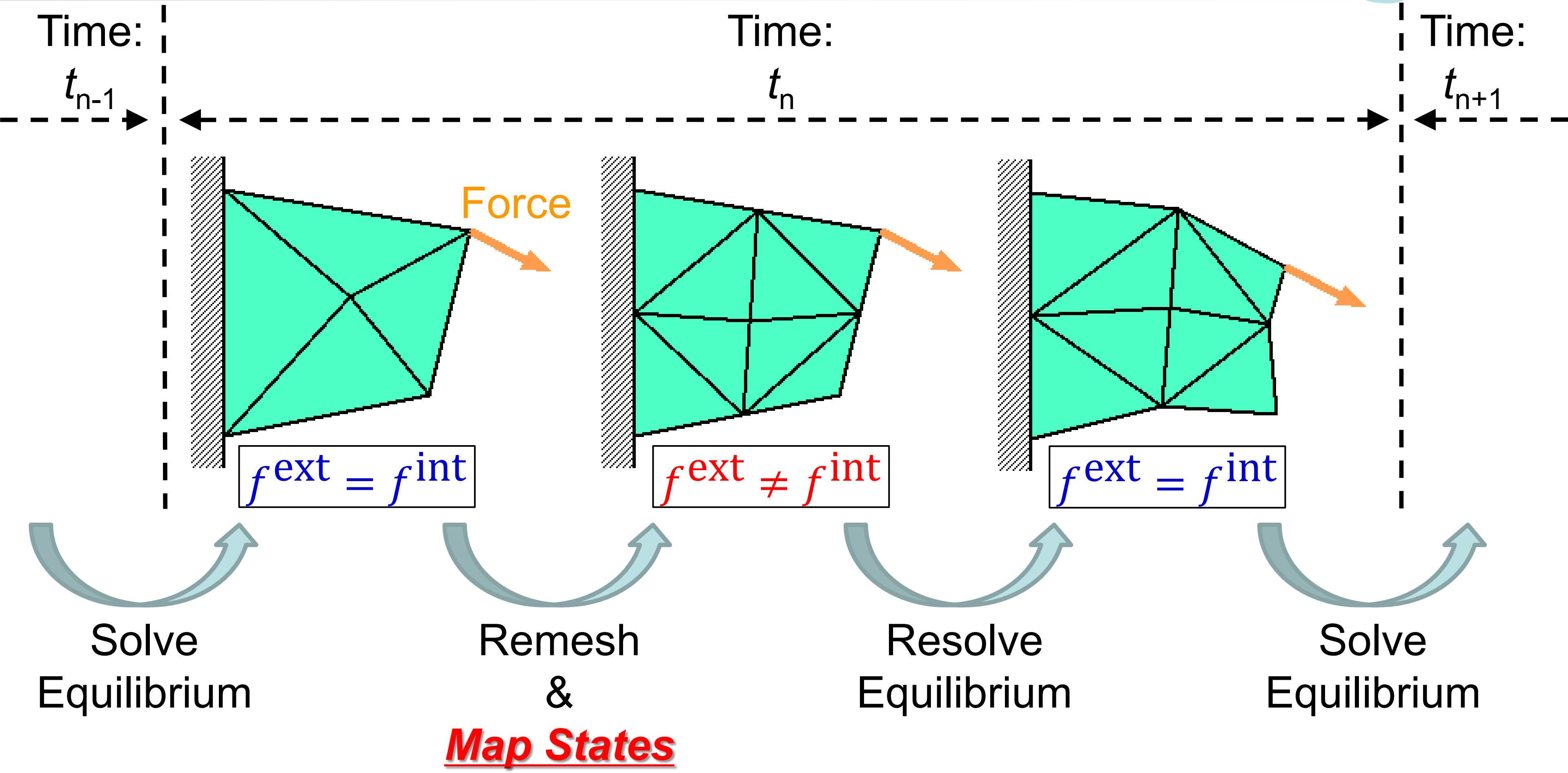
Comparison to ABAQUS Hybrid Elements



Our selective S-FEM is free from volumetric locking!!

Part 2:
Procedure of our mesh rezoning method

Procedure of Mesh Rezoning



The way of mapping varies with the material constitutive model. (e.g. Elasto-plastic models necessitate some kind of correction.)

Mapping of Stress/Strain States

For Elastic or Hyperelastic Materials

i.e., $[T] = [T([F])]$

- Map initial position $\{x^{\mathrm{initial}}\}$ at nodes, and then remake deformation gradient $[F]$ at edges & nodes.

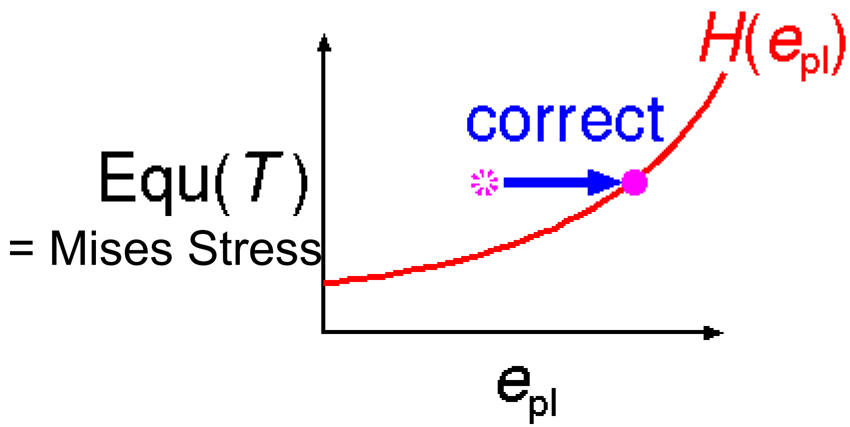
Each node preserve its initial position so that the domain can spring back to the initial shape after unloading.

Mapping of Stress/Strain States

For Elasto-Plastic Material in Total Strain Form

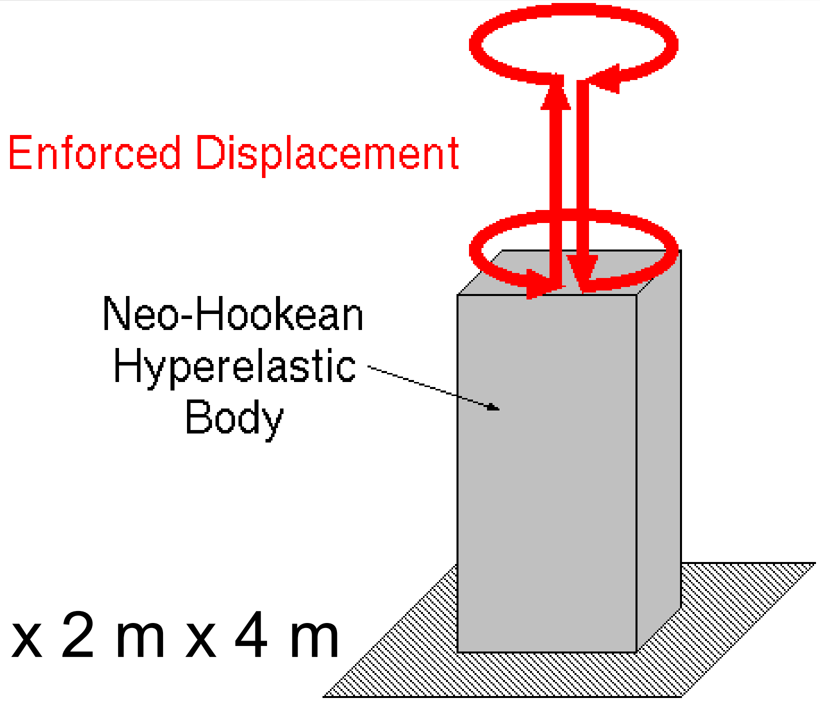
$$\text{e.g., } [T] = [T([F], [E_{\text{pl}}], e_{\text{pl}}; H(e_{\text{pl}}))]$$

- Map initial position $\{x^{\text{initial}}\}$ at nodes, and then remake deformation gradient $[F]$ at edges & nodes.
- Map history dependent variables, plastic strain $[E_{\text{pl}}]$ and equivalent plastic strain e_{pl}.
- Correct e_{pl} to satisfy $\text{Equ}([T]) = H(e_{\text{pl}})$



Part 3:
Examples of Demonstrative Analysis

Twist and Stretch of Hyperelastic Body



- Static, 1 m x 2 m x 4 m
- Neo-Hookean hyperelastic body of $C_{10} = 1\ \mathrm{GPa}$ and $D_1 = 400\ \mathrm{GPa}^{-1}$ ($\nu_0 = 0.48$)
- Twist up to 360 deg. $\Longrightarrow$ Stretch up to 100% nominal strain $\Longrightarrow$ Twist back $\Longrightarrow$ Shrink back
- Our selective FS/NS-FEM with tetrahedral elements
- Global mesh rezoning every 90 deg. and 50% stretch/shrink

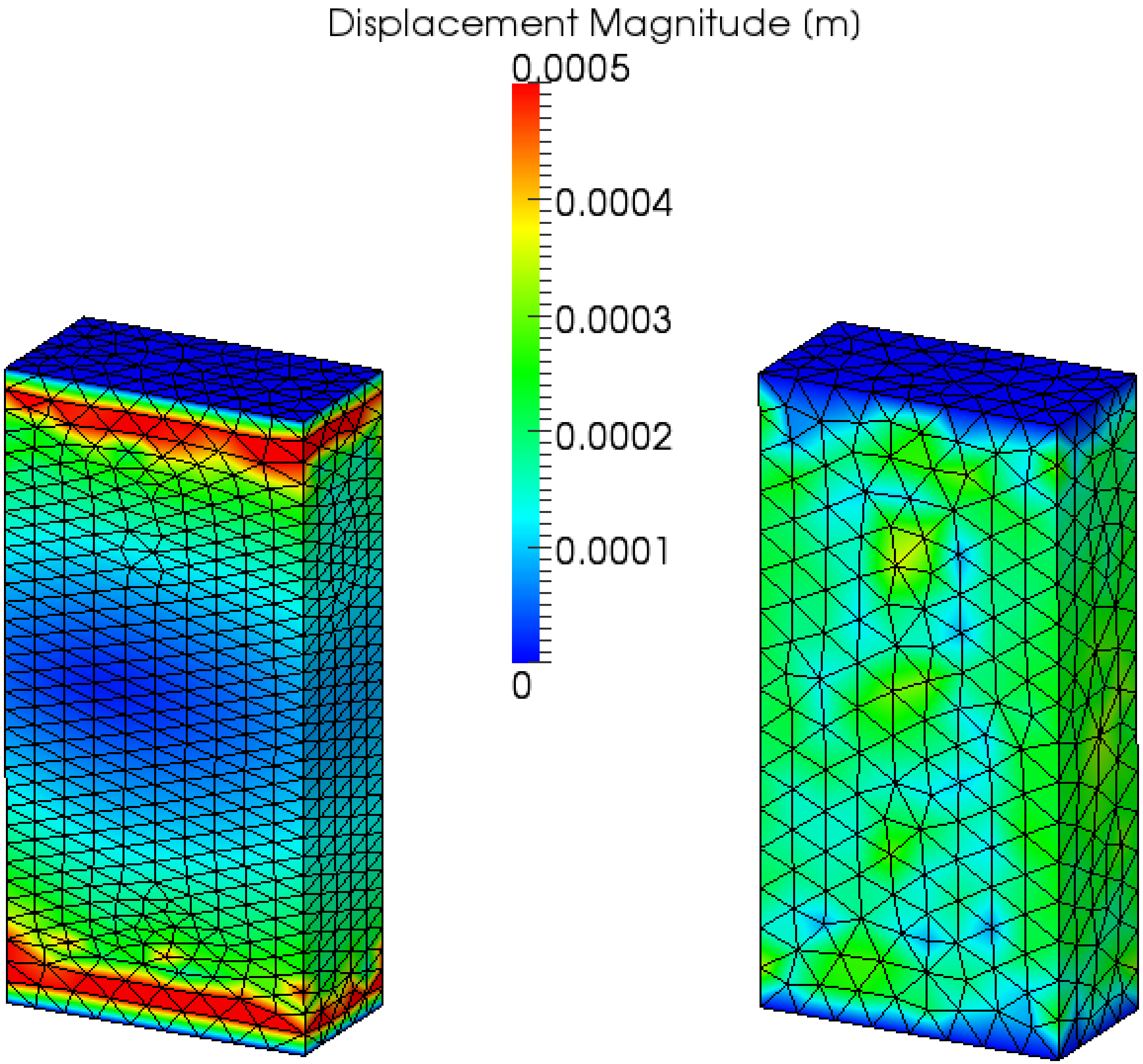
Twist and Stretch of Hyperelastic Body

Our selective FS/NS-FEM **with** mesh rezoning

Our selective FS/NS-FEM **without** mesh rezoning

Twist and Stretch of Hyperelastic Body

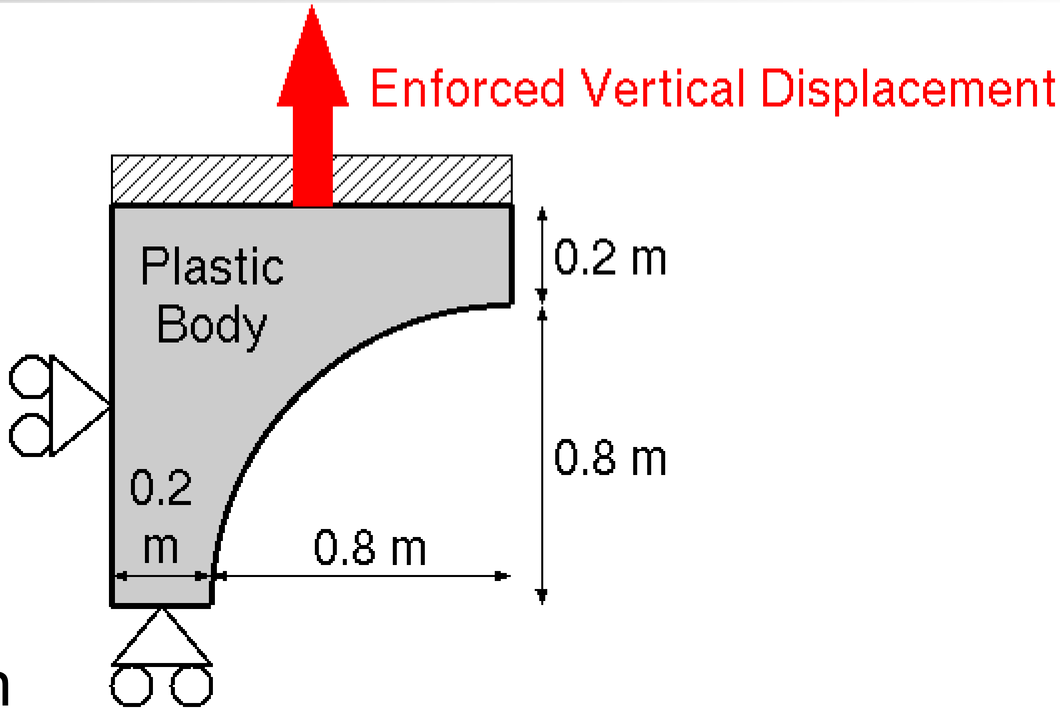
Residual Displacement



It spring backed almost perfectly.

Necking of 2D Elasto-Plastic Body

Outline

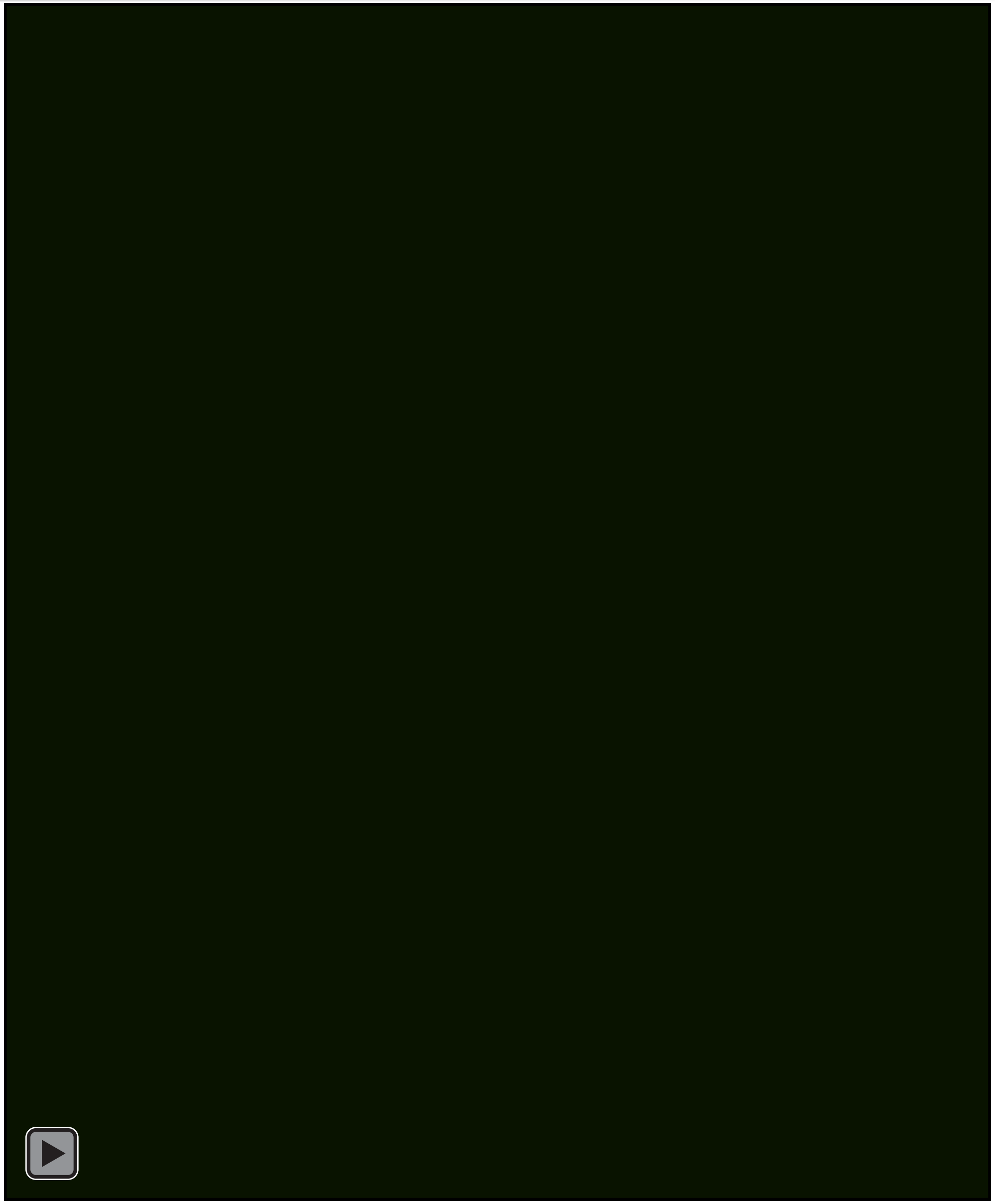


- Static, Plane-strain
- Hencky's elasto-plastic material, $\boldsymbol{T} = \boldsymbol{C} : \boldsymbol{h}_{\mathrm{el}}/J$,
  with von Mises yield criterion and isotropic hardening.
  Young's Modulus: 1 GPa, Poisson's Ratio: 0.3,
  Yield Stress: 1 MPa, Hardening Coeff.: 0.5 MPa.
- Mesh rezoning every 0.05 m displacement

Necking of 2D Elasto-Plastic Body

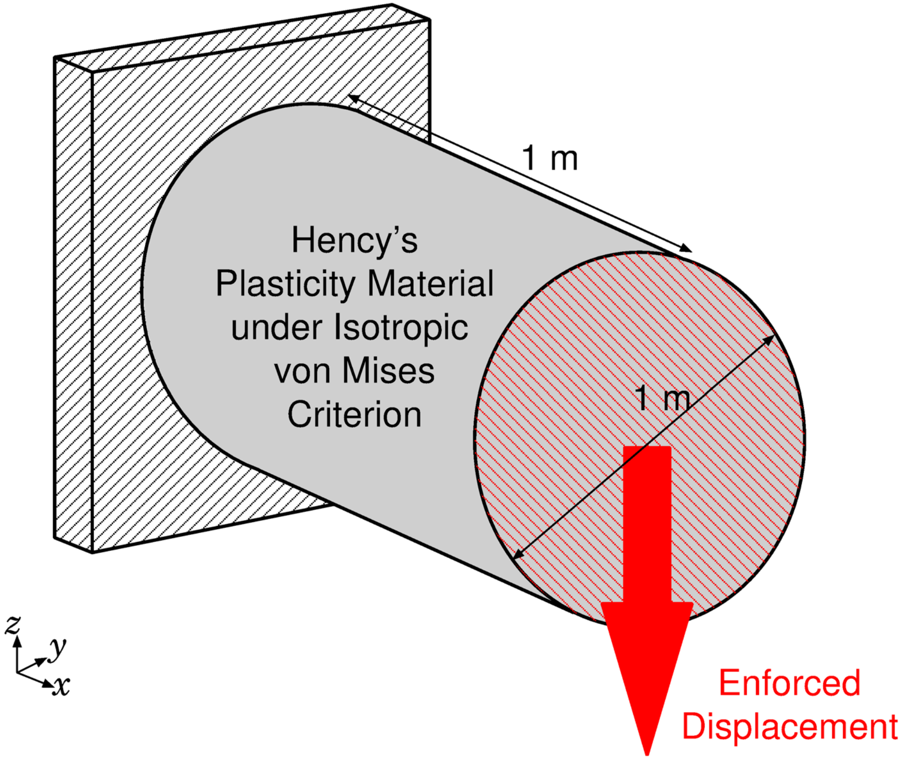
Result

Zoom-in view around the center



Shearing and Necking of 3D Plastic Rod

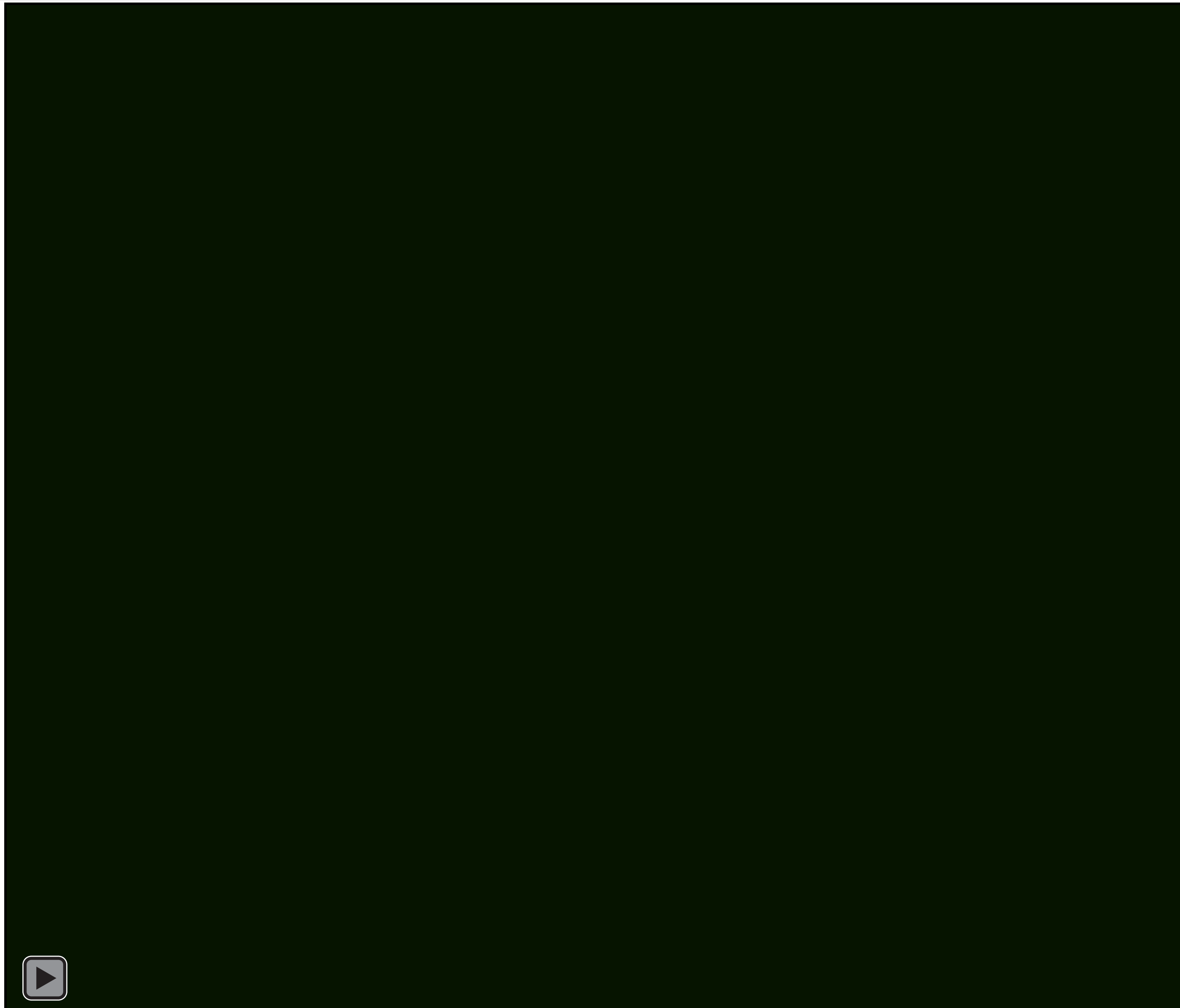
Outline



- Static, 3D
- Hencky's Plasticity Material
  with von Mises yield criterion and isotropic hardening.
  (same as 2D case)

Shearing and Necking of 3D Plastic Rod

3D Result



The deformation seems
to be valid.

After 2.8 m disp.,
mesh rezoning error
occurred.

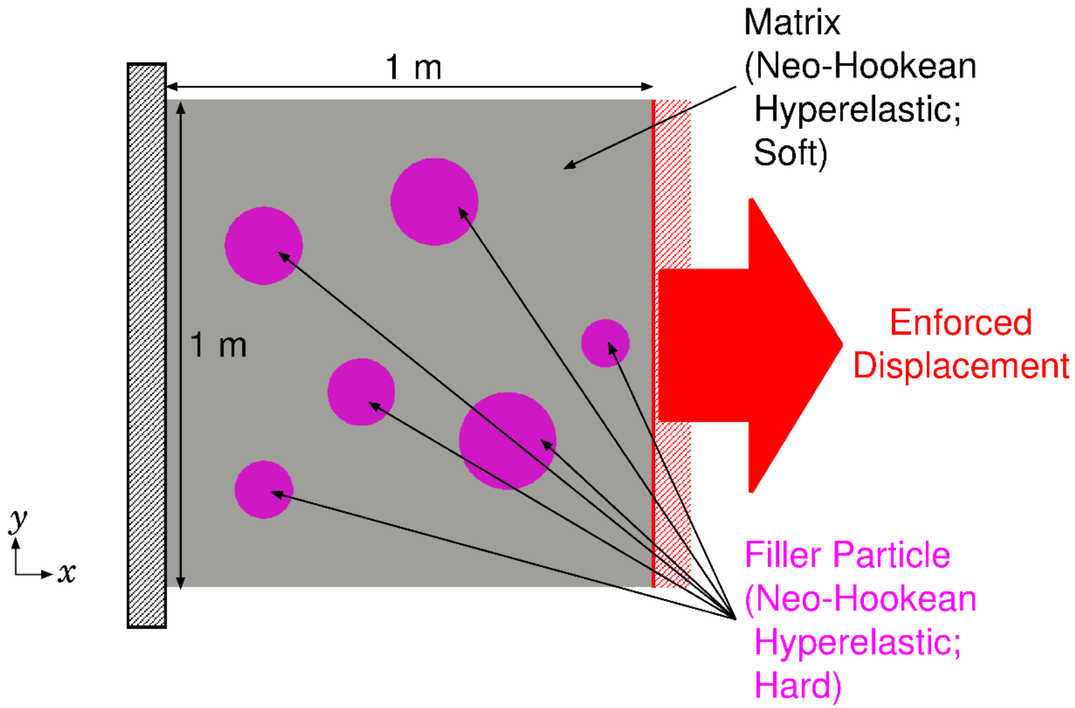
Shearing and Necking of 2D Plastic Bar

2D

Result

Tension of 2D Filler Particle Composite

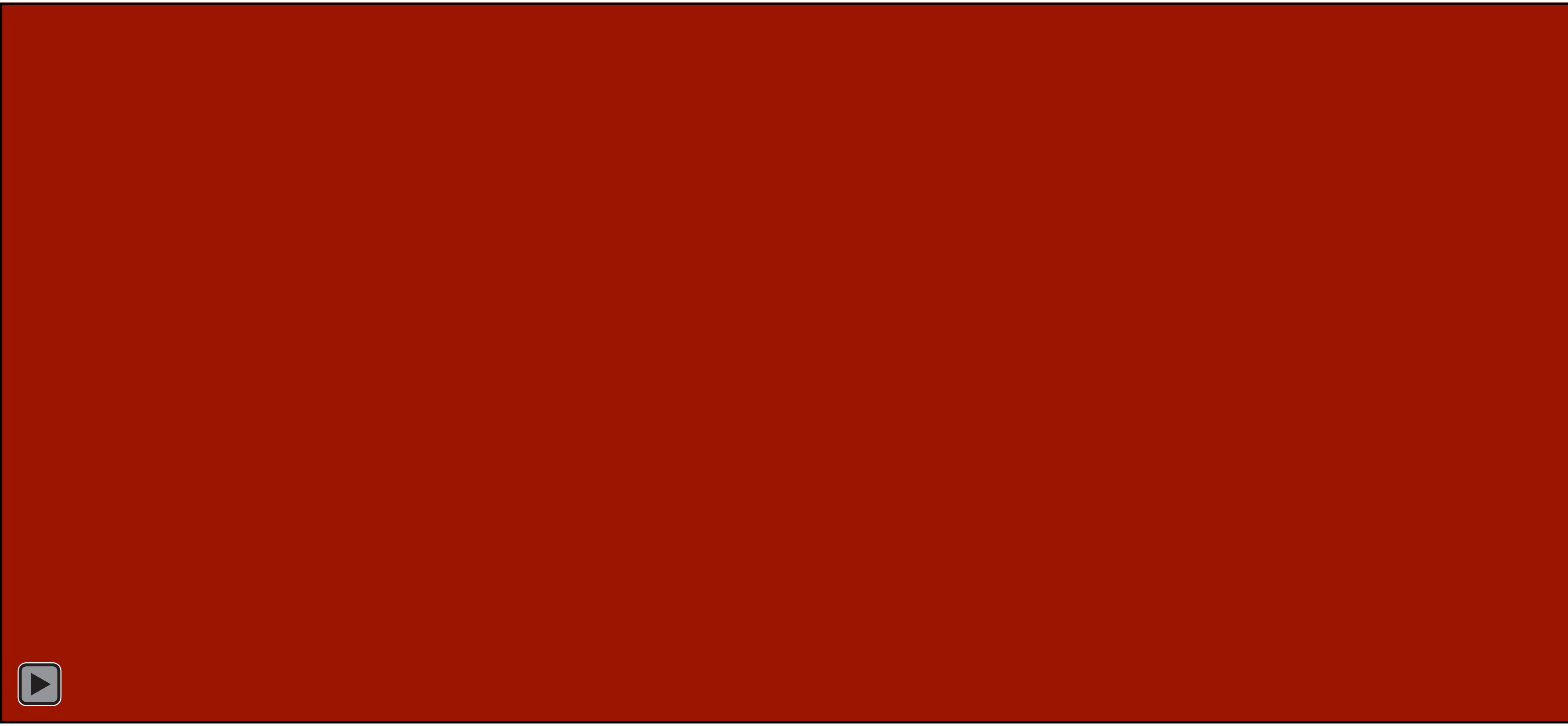
Outline



- Plane-strain static
- Neo-Hookean Hyperelastic
  - Filler: hard rubber （$E^{\text{initial}} = 100$ GPa, $\nu^{\text{initial}} = 0.49$）
  - Matrix: soft rubber （$E^{\text{initial}} = 1$ GPa, $\nu^{\text{initial}} = 0.49$）

Tension of 2D Filler Particle Composite

2D Result



The deformation seems to be valid.
After 1.8 m disp.,
analysis is stopped due to mesh rezoning error.

Tension of 3D Filler Particle Composite

3D Result without Mesh Rezoning

- Analysis resulted in convergence failure due to the singularity of stiffness after 1.58 m disp..

Summary

Take-Home Messages

1. Our modified selective S-FEM with triangular or tetrahedral elements is locking free and very easy to implement.

2. Our S-FEM goes well together with mesh rezoning.

3. Our S-FEM is worth using even without mesh rezoning.

Summary and Future Work

Summary

- A new static-implicit mesh rezoning method for severely large deformation analysis is proposed.
- It adopts our modified selective S-FEM, which separates stress into deviatoric part and hydrostatic part.
- Its accuracy are verified with hyperelastic material and elasto-plastic material.

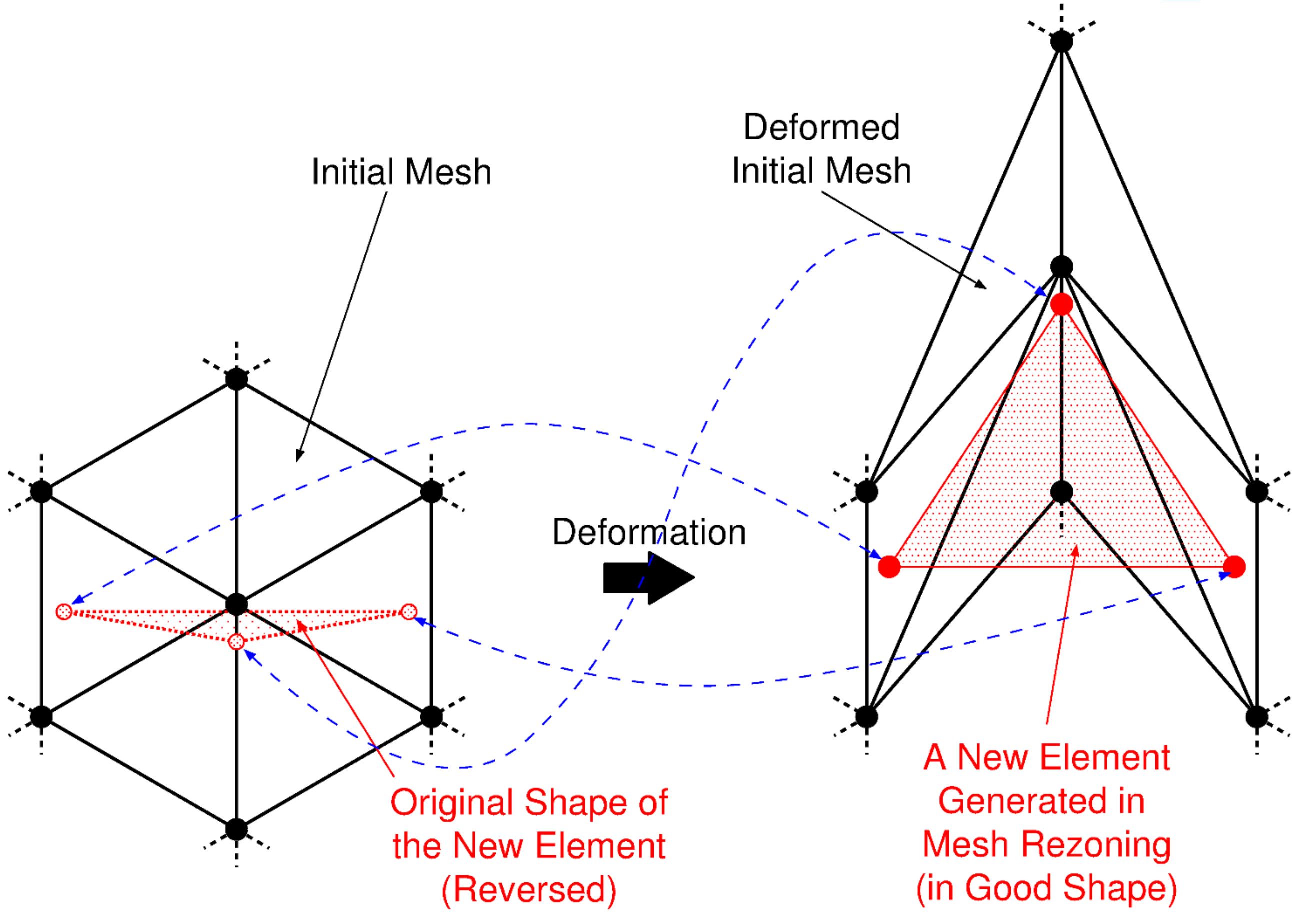
Future Work

- More V&V
- Local mesh rezoning
- Apply to contact forming, crack propagation, etc.

Thank you for your kind attention.
I appreciate your question in slow and easy English!!

Appendix

Reason for Mesh Rezoning Error



Remeshing on too distorted mesh brings "originally reversed elements".

Characteristics of Our S-FEM

Advantage

- Locking free
- No increase in DOF
  (The unknown is only the displacement vector, $\{u\}$.)
- Easy to implement

Disadvantage

- Increase in matrix band width
- Cannot treat perfectly incompressible material
- No smoothing effect with super coarse mesh